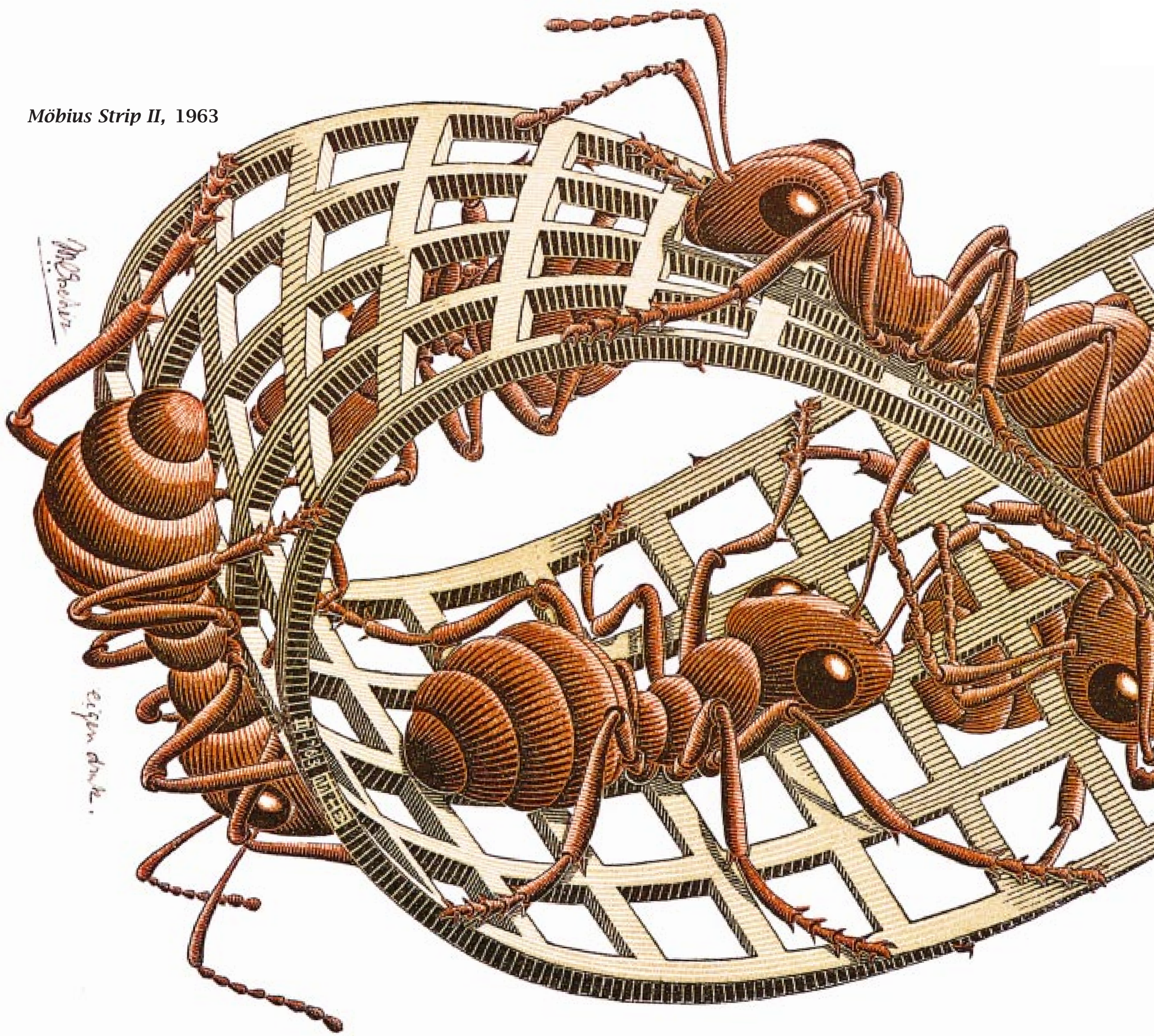
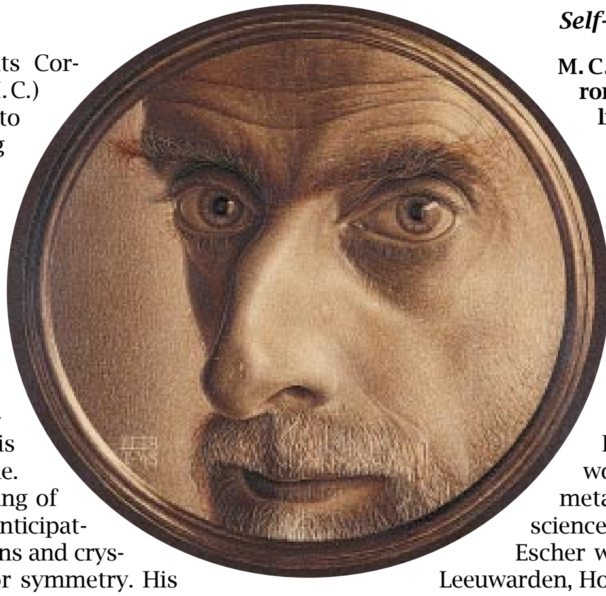
Escher's Metaphors

The prints and drawings of M.C. Escher give expression to abstract concepts of mathematics and science

by Doris Schattschneider



Möbius Strip II, **1963**



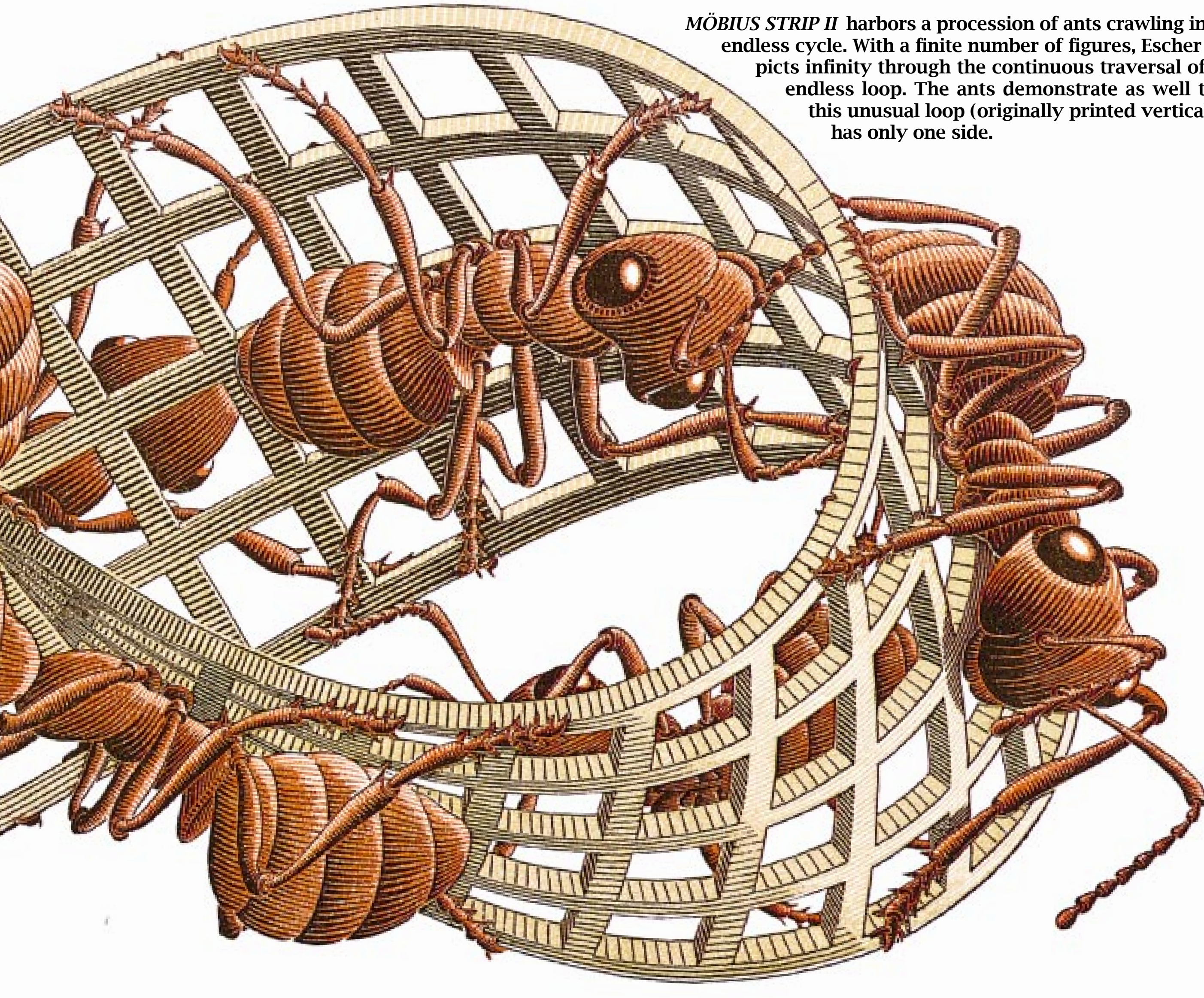
Self-Portrait, 1943

M. C. ESCHER views himself in a mirror in this "scratch" drawing with lithographic ink.

Throughout his life Maurits Cornelis Escher (he used only M. C.) remarked on his inability to understand mathematics, declaring himself "absolutely innocent of training or knowledge in the exact sciences." Yet even as a child, Escher was intrigued by order and symmetry. The fascination later led him to study patterns of tiles at the Alhambra in Granada, to look at geometric drawings in mathematical papers (with the advice of his geologist brother) and ultimately to pursue his own unique ideas for tiling a plane.

Escher's attention to the coloring of his drawings of interlocked tiles anticipated the later work of mathematicians and crystallographers in the field of color symmetry. His works are now commonly used to illustrate these concepts. His exhibit in conjunction with the 1954 International Congress of Mathematicians in Amsterdam and the publication of his first book (*The Graphic Work of M. C. Escher*) in 1959 struck a chord with mathematicians and scientists that still resonates strongly. He wrote that a main impetus for his work was "a keen interest in the geometric laws contained by nature around us." In expressing his ideas in graphic works, he provided arresting visual metaphors for fundamental ideas in science.

Escher was born in 1898 in the town of Leeuwarden, Holland. The youngest son of a civil engineer, he grew up with four brothers in Arnhem. Although three of his brothers pursued science or engineering, Escher was a poor mathematics student. With the encour-



MÖBIUS STRIP II **harbors a procession of ants crawling in an endless cycle. With a finite number of figures, Escher depicts infinity through the continuous traversal of an endless loop. The ants demonstrate as well that this unusual loop (originally printed vertically) has only one side.**

agement of his high school art teacher, he became interested in graphic arts, first making linoleum cuts.

In 1919 he entered the School for Architecture and Decorative Arts in Haarlem, intending to study architecture. But when he showed his work to Samuel Jessurun de Mesquita, who taught graphic arts there, he was invited to concentrate in that field. De Mesquita had a profound influence on Escher, both as a teacher (particularly of woodcut techniques) and later as a friend and fellow artist.

After finishing his studies in Haarlem, Escher settled in Rome and made many extensive sketching tours, mostly in southern Italy. His eyes discerned striking visual effects in the ordinary—architectural details of monumental buildings from unusual vantage points, light and shadow cast by warrens of staircases in tiny villages, clusters of houses clinging to mountain slopes that plunged to distant valleys and, at the opposite scale, tiny details of nature as if viewed through a magnifying glass. In his studio, he would transform the sketches into woodcuts and lithographs.

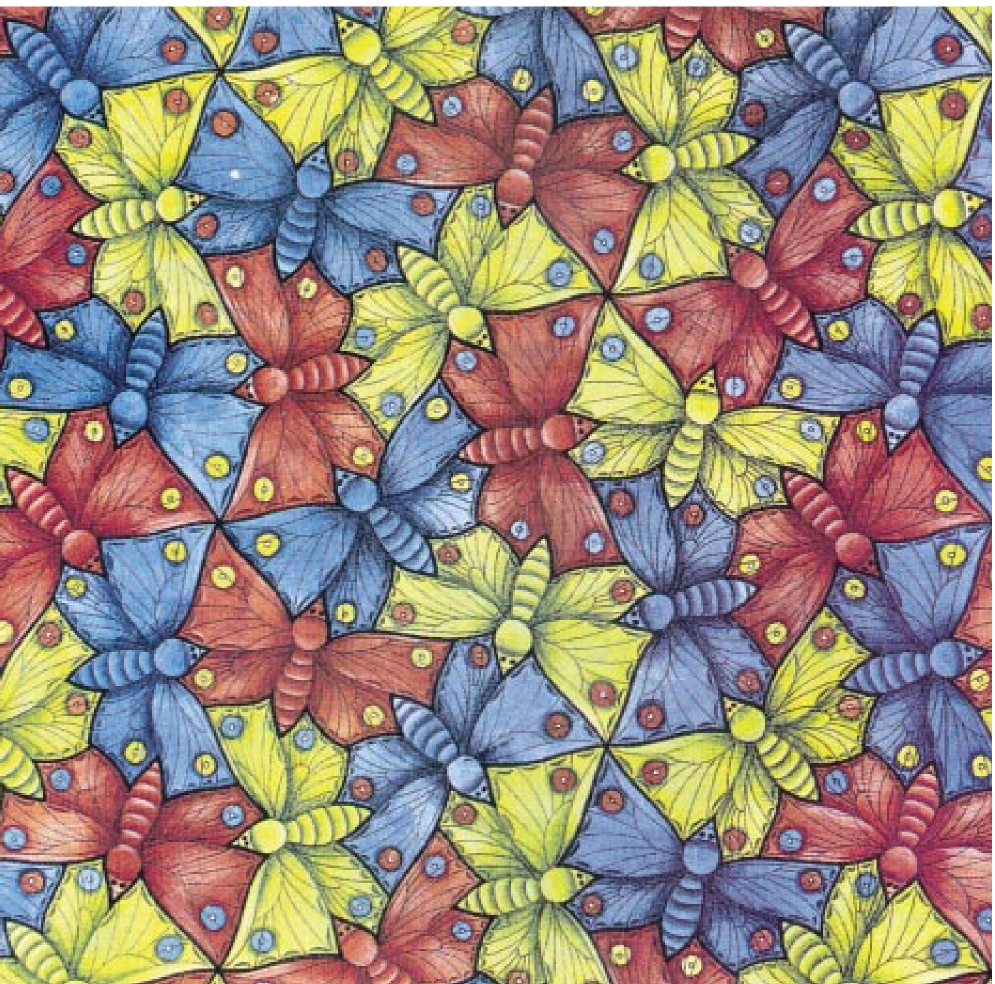
In 1935 the political situation became unendurable, and with his wife and young sons, Escher left Italy forever. After two years in Switzerland and then three years in Uccle, near Brussels, they settled permanently in Baarn, Holland. These years also brought an abrupt turn in Escher's work. Almost all of it from this time on would draw its inspiration not from what his eyes observed but rather from his mind's eye. He sought to give visual expression to concepts and to portray the ambiguities of human observation and understanding. In doing so, he often found himself in a world governed by mathematics.

Escher was fascinated, almost obsessed, with the concept of the "regular division of the plane." In his lifetime, he produced more than 150 color drawings that testified to his ingenuity in creating figures that crawled, swam and soared, yet filled the plane with their clones. These drawings illustrate symmetries of many different kinds. But for Escher, division of the plane was also a means of capturing infinity. Although a tiling such as the one using butterflies [*see illustration below*] can in principle be continued indefinitely, thus giving a suggestion of infinity, Escher was challenged to contain infinity within the confines of a single page.

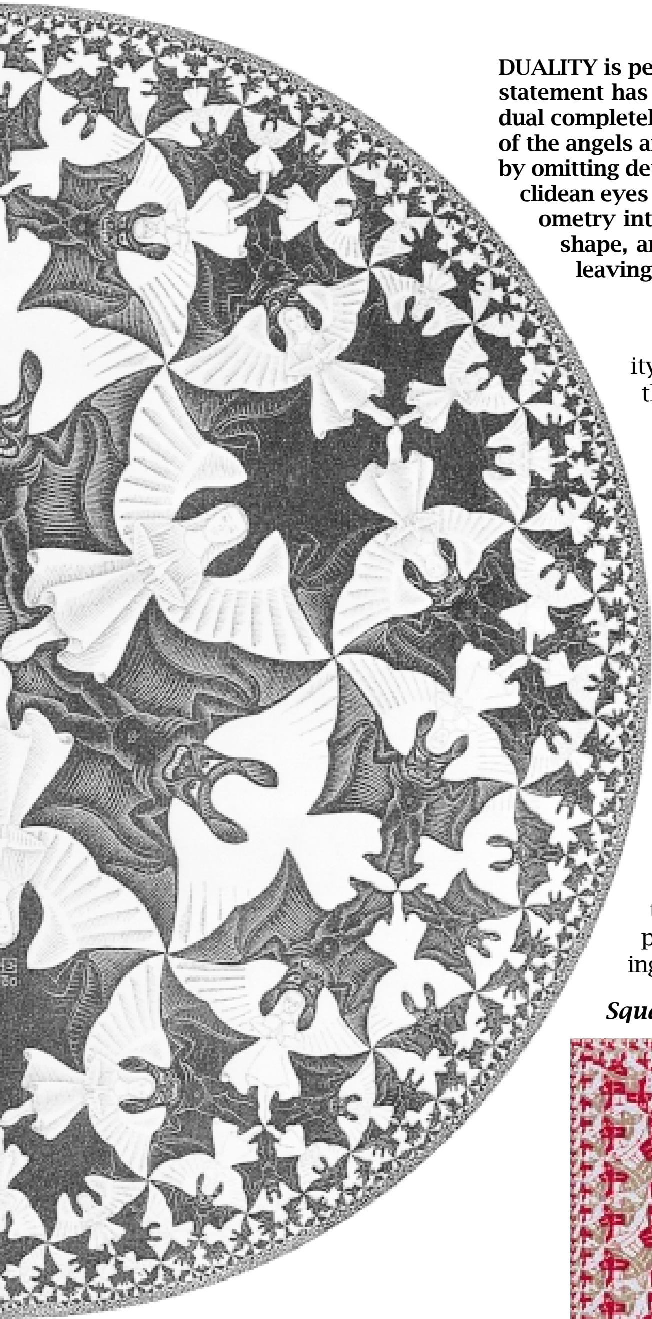
"Anyone who plunges into infin-

***Circle Limit IV,* 1960**

***Triangle System I B_3 Type 2,* 1948**



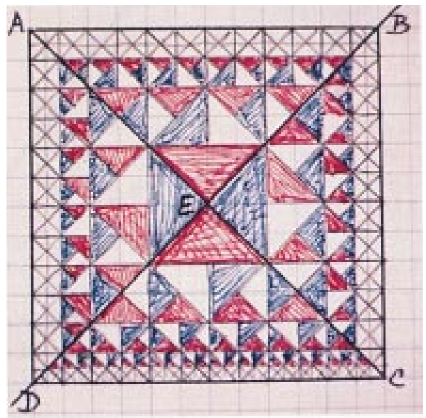
SYMMETRY is a structural concept that shapes many mathematical and physical models. In Escher's drawing the butterflies seem to fill the page randomly, yet each one is precisely placed and surrounded in exactly the same way. Always six (in alternating colors) swirl about a point where left front wingtips meet; always three (in different colors) spin about a point where right back wings touch; and always pairs (of different colors) line up the edges of their right front wings. Along with rotational symmetry, the drawing has translational symmetry based on a triangular grid. The pattern can continue forever in all directions and so provides an implicit metaphor of infinity. Escher's attention to coloring anticipated discoveries by mathematicians in the field of color symmetry.



DUALITY is perhaps the most prevalent theme in Escher's later prints. In mathematics, a statement has a negation, and a set has a complement; in every case, the object and its dual completely define each other. In *Circle Limit IV,* there are no outlines. The contours of the angels and devils define one another. Either is figure or ground (Escher reminds us by omitting detail in half the figures). In this hyperbolic tiling the figures appear to our Euclidean eyes to become more distorted as they diminish in size. Yet measured by the geometry intrinsic to the world of the print, every angel is exactly the same size and shape, and so is every devil. An infinite number of copies repeat forever, never leaving the confines of the circle.

ity, in both time and space, farther and farther without stopping, needs fixed points, mileposts as he flashes by, for otherwise his movement is indistinguishable from standing still," Escher wrote. "He must mark off his universe into units of a certain length, into compartments which repeat one another in endless succession."

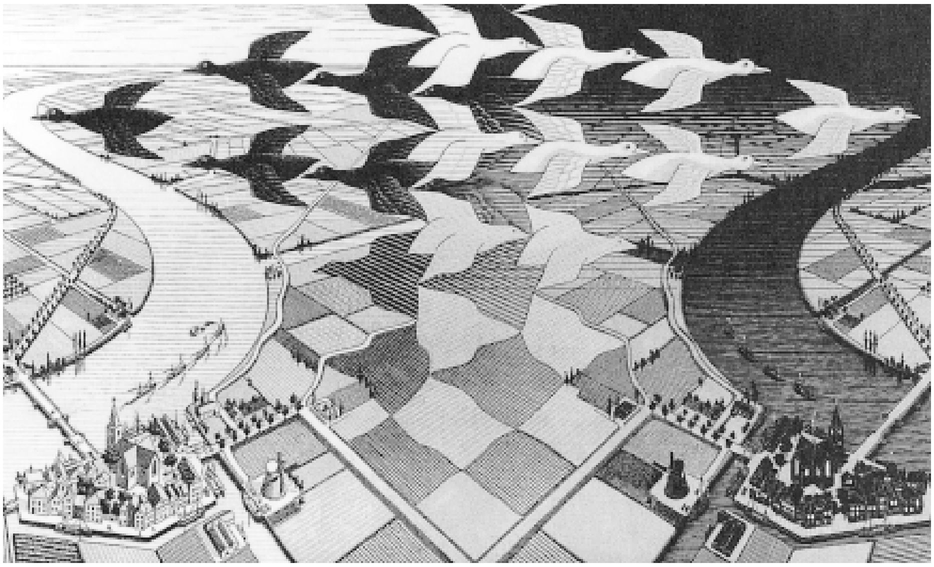
After completing several prints in which figures endlessly diminish in size as they approach a central vanishing point [*see* Whirlpools *on page 71*], Escher sought a device to portray progressive reduction in the opposite direction. He wanted figures that repeated forever, always approaching—yet never reaching—an encircling boundary. In 1957 the mathematician H.S.M. Coxeter sent Escher a reprint of a journal article in which he illustrated planar symmetry with some of Escher's drawings. There Escher found a figure that gave him "quite a shock"—a hyperbolic tessellation of tri-



***Square Limit,* 1964**

SELF-SIMILARITY is illustrated in the print *Square Limit,* constructed using a recursive scheme of Escher's own invention. A set of directions that is applied to an object to produce new objects, then applied to the new objects and so on, ad infinitum, is called a recursive algorithm. The end product is self-similar when all the final objects are the same as the original, except for changes of scale, orientation or position. A sketch (*top right*) sent by Escher to the mathematician H.S.M. Coxeter to explain the print shows that the underlying grid involves a recursive splitting of isosceles triangles. In executing the print, Escher carved the woodblock only for a triangle having its apex at the center of the square and its base as one side of the square—and printed the block four times.

Day and Night, 1938



DIMENSION is that concept which clearly separates point, line, plane and space. To illustrate the ambiguities in the perception of dimension, Escher exploited the printed page—which always must fool the viewer when it depicts a three-dimensional scene. In *Day and Night,* the flat checkerboard of farmland at the bottom of the print metamorphoses into two flocks of geese. The print also illustrates the concept of topological change, in which a figure is deformed without being cut or pierced. Reflection and duality are present as well: black geese fly over a sunlit village, whereas white ones wing over a night view of a mirror image of the same scene.

High and Low, 1947

RELATIVITY states that what an observer sees is influenced by context and vantage point. In the lithograph *High and Low,* Escher presents two different views of the same scene. In the lower half the viewer is on the patio; in the upper half the viewer is looking down. Now draw back from the print: Is that tiled diamond at the center of the print a floor or a ceiling? Escher uses it for both in order to marry the two views. It is impossible to see the entire print in a logical way. The scene also illustrates how pasting local views together to form a global whole can lead to contradictions.

angles that showed exactly the effect he sought. From a careful study of the diagram, Escher discerned the rules of tiling in which circular arcs meet the edge of an encompassing circle at right angles. During the next three years, he produced four different prints based on this type of grid, of which *Circle Limit IV* [*see top illustration on preceding two pages*] was the last.

Four years later Escher devised his own solution to the problem of infinity within a rectangle [*see bottom illustration on preceding page*]. His recursive algorithm—a set of directions repeatedly applied to an object—results in a self-similar pattern in which each element is related to another by a change of scale. Escher sent Coxeter a sketch of the underlying grid, apologizing: "I fear that the subject won't be very interesting, seen from your mathematical point of view, because it's really simple as a flat filling. None the less it was a headaching job to find an adequate method to realise the subject in the simplest possible way." In a lecture a few summers ago mathematician William P. Thurston, director of the Mathematical Sciences Research Institute at the University of California at Berkeley, illustrated the concept of self-similar tiling with just such a grid, unaware of Escher's earlier discovery.

Curiously, self-similar patterns provide examples of figures that

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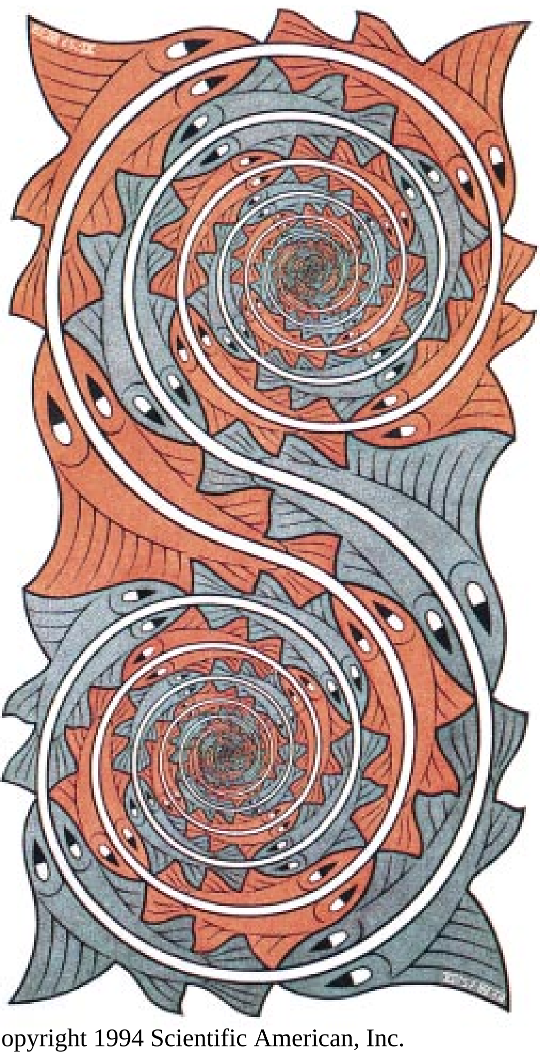
REFLECTION allows phenomena to be observed that are too small, too far away or too obscure to be seen directly. *Puddle* directs our eyes to a woodland trail imprinted by boots and tires—yet in the puddle are also revealed silhouetted trees arching overhead against a moonlit sky. Escher reminds us of the unseen worlds below, behind and above our limited gaze.

***Puddle,* 1952**

INFINITY is confined within the finite space of a print in *Whirlpools.* The artist draws a flat projection of the curve (a loxodrome) that is traced out on the globe by a path that cuts across all meridians at a constant angle. As any navigator knows, sailing such a "rhumb line" results in a never-ending, ever tightening spiral about the earth's pole. Escher used one woodblock for both colors. Printing once for the red, he turned it halfway around and printed for the gray.

***Whirlpools,* 1957**



have fractional, or fractal, dimension, an ambiguity that Escher would doubtless have enjoyed. In 1965 he confessed: "I cannot help mocking all our unwavering certainties. It is, for example, great fun deliberately to confuse two and three dimensions, the plane and space, and to poke fun at gravity." Escher was masterful at confusing dimensions, as in *Day and Night* [*see upper illustration on opposite page*], in which two-dimensional farm fields mysteriously metamorphose into three-dimensional geese. He also delighted in pointing out the ambiguities and contradictions inherent in a common practice of science: pasting together several local views of an object to form a global whole [*see lower illustration on opposite page*].

Near the end of his life (he died in 1972), Escher wrote, "Above all, I am happy about the contact and friendship of mathematicians that resulted from it all. They have often given me new ideas, and sometimes there is even an interaction between us. How playful they can be, these learned ladies and gentlemen!"

FURTHER READING

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