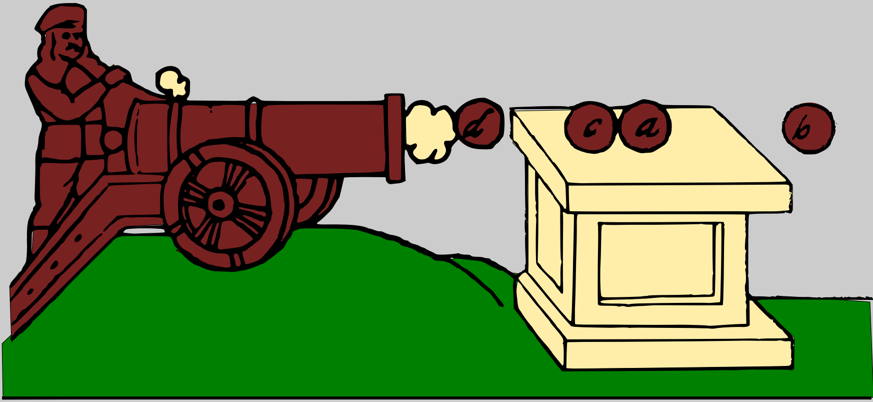
Book 1

Physics for Everyone

L.D. Landau, A.I. Kitaigorodsky



PHYSICAL BODIES

Mir Publishers Moscow

Physics for Everyone Book 1

The aim of the authors, the late Lev Landau, Nobel and Lenin winner, and Prof, Alexander Kitaigorodsky, was to provide a clear understanding of the fundamental ideas and latest discoveries in physics, set forth in readily understandable language for the layman. The new and original manner in which the material is presented enables this book to be enjoyed by a wide range of readers, from those making the first acquaintance with physics to college graduates and others interested in this branch of science. The book may serve as an excellent instructional aid for physics teachers,

In this first of four, the motion of bodies is dealt with from the points af view of two observers: one in an inertial and the other in a non-inertial frame of reference. The law universal gravitation is discussed in detail, including its application for calculating space velocities and for interpreting lunar tides and various geophysical phenomena.

ИЗДАТЕЛЬСТВО • МИР •

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the Russian
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Mir Publishers Moscow

Физика для всех
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PREFACE TO THE FOURTH RUSSIAN EDITION

After many years I decided to return to an unfinished book that I wrote together with Dau, as his friends called the remarkable scientist and great-hearted man Lev Davidovich Landau. The book was *Physics for Everyone.*

Many readers in letters had reproached me for not continuing the book. But I found it difficult because the book was a truly joint venture.

So here now is a new edition of *Physics for Everyone*, which I have divided into four small books, each one taking the reader deeper into the structure of matter. Hence the titles *Physical Bodies*, *Molecules*, *Electrons*, and *Photons and Nuclei*. The books encompass all the main laws of physics. Perhaps there is a need to continue *Physics for Everyone* and to devote subsequent issues to the basics of various fields of science and technology.

The first two books have undergone only slight changes, but in places the material has been considerably augmented. The other two were written by me.

The careful reader, I realize, will feel the difference. But I have tried to preserve the presentation principles that Dau and I followed. These are the deductive principle and the logical principle rather than the historical. We also felt it would be well to use the language of everyday life and inject some humour. At the same time we did not oversimplify. If the reader wants to fully understand the subject, he must be prepared to read some places many times and pause for thought.

The new edition differs from the old in the following way. When Dau and I wrote the previous book, we viewed it as a kind of primer in physics; we even thought it might compete with school textbooks. Reader's comment and the experience of teachers, however, showed that the users of the book were teachers, engineers, and school students who wanted to make physics their profession. Nobody considered it a textbook. It was thought of as a popular science book intended to broaden knowledge gained at school and to focus attention on questions that for some reason are not included in the physics syllabus.

Therefore, in preparing the new edition I thought of my reader as a person more or less acquainted with physics and thus felt freer in selecting the topics and believed it possible to choose an informal style.

The subject matter of *Physical Bodies* has undergone the least change. It is largely the first half of the previous edition of *Physics for Everyone*.

Since the first book of the new edition contains phenomena that do not require a knowledge of the structure of matter, it was natural to call it *Physical Bodies*. Of course, another possibility was to use, as is usually done, the title *Mechanics* (i.e. the science of motion). But the theory of heat, which is covered in the second book, *Molecules*, also studies motion except that what is moving is the invisible molecules and atoms. So I think the title *Physical Bodies* is a better choice.

Physical Bodies deals largely with the laws of motion and gravitational attraction. These laws will always remain the foundation of physics and for this reason of science as a whole.

September 1977

A. I. Kitaigorodsky

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I. Basic Concepts

The Centimetre and the Second

Everyone has to measure lengths, reckon time and weigh various bodies. Therefore, everyone knows just what a centimetre, a second and a gram are. But these measures are especially important for a physicist—they are necessary for making judgements about most physical phenomena. People try to measure distances, intervals of time and mass, which are called the basic concepts of physics, as accurately as possible.

Modern physical apparatuses permit us to determine a difference in length between two-metre long rods, even if it is less than one-billionth of a metre. It is possible to distinguish intervals of time differing by one-millionth of a second. Good scales can determine the mass of a poppy seed with a very high degree of accuracy.

Measurement techniques started developing only a few hundred years ago, and agreement on what segment of length and what mass of a body to take as units has been reached relatively recently.

But why were the centimetre and the second chosen to be such as we know them? As a matter of fact, it is clear that there is no special significance to whether the centimetre or the second be longer.

A unit of measurement should be convenient—we require nothing further of it. It is very good for a unit of measurement to be at hand, and simplest of all to take the hand itself for such a unit. This is precisely what was done in ancient times; the very names of the units testify to this: for example, an "ell" or "cubit" is the distance between the elbow and the fingertips of a stretched-out hand, an "inch" is the width of a thumb at its base. The foot was also used for measurement—hence the name of the length "foot"

Although these units of measurement are very convenient in that they are always part of oneself, their disadvantages are obvious: there are just too many differences between individuals for a hand or a foot to serve as a unit of measurement which does not give rise to controversy.

With the development of trade, the need for agreeing on units of measurement arose. Standards of length and mass were at first established within a separate market, then for a city, later for an entire country and, finally, for the whole world. A standard is a model measure: a ruler, a weight. Governments carefully preserve these standards, and other rulers and weights must be made to correspond exactly to them.

The basic measures of weight and length in tsarist Russia—they were called the pound and the arshin—were first made in 1747. Demands on the accuracy of measurements increased in the 19th century, and these standards turned out to be imperfect. The complicated and responsible task of creating exact standards was carried out from 1893 to 1898 under the guidance of Dmitri Ivanovich Mendeleev. The great chemist considered the establishment of exact standards to be very important. The Central Bureau of Weights and Measures, where the standards are kept and their copies made, was founded at the end of the 19th century on his initiative.

Some distances are expressed in large units, others in smaller ones. As a matter of fact, we wouldn't think of expressing the distance from Moscow to Leningrad in centimetres, or the mass of a railroad train in grams.

People therefore agreed on definite relationships between large and small units. As everyone knows, in the system of units which we use, large units differ from smaller ones by a factor of 10, 100, 1000 or, in general, a power of ten. Such a condition is very convenient and simplifies all computations. However, this convenient system has not been adopted in all countries. Metres, centimetres and kilometres as well as grams and kilograms are still used infrequently in England and the USA in spite of the obviousness of the metric system's conveniences.*

In the 17th century the idea arose of choosing a standard which exists in nature and does not change in the course of years and even centuries. In 1664 Christiaan Huygens proposed that the length of a pendulum making one oscillation a second be taken as the unit of length. About a hundred years later, in 1771, it was suggested that the length of the path of a freely falling body during the first second be regarded as the standard. However, both variants proved to be inconvenient and were not accepted. A revolution was necessary for the emergence of the modern units of measurement—the Great French Revolution gave birth to the kilogram and the metre.

In 1790 the French Assembly created a special commission containing the best physicists and mathematicians for the establishment of a unified system of measurements. From all the suggested variants of a unit of length, the commission chose one-ten-millionth of the Earth's meridian quadrant, calling this unit the *metre*.

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*The following measures of length were officially adopted in England: the nautical mile (equals 1852 m); the ordinary mile (1609 m); the foot (30.48 cm), a foot is equal to 12 inches; an inch is 2.54 cm; a yard, 0.9144 m, is the "tailors' measure" used to mark off the amount of material needed for a suit.

In Anglo-Saxon countries, mass is measured in pounds (454 g). Small fractions of a pound are an ounce (1/16 pound) and a grain (1/7000 pound); these measures are used by druggists in weighing out medicine.

Its standard was made in 1799 and given to the Archives of the Republic for safe keeping.

Soon, however, it became clear that the theoretically correct idea about the advisability of choosing models for our measures by borrowing them from nature cannot be fully carried out in practice. More exact measurements performed in the 19th century showed that the standard made for the metre is approximately 0.08 of a millimetre shorter than one-forty-millionth of the Earth's meridian. It became obvious that new corrections would be introduced as measurement techniques developed. If the definition of the metre as a fraction of the Earth's meridian were to be retained, it would be necessary to make a new standard and recalculate all lengths anew after each new measurement of the meridian. It was therefore decided after discussions at the International Congresses of 1870, 1872 and 1875 to regard the standard of the metre, made in 1799 and now kept at the Bureau of Weights and Measures at Sèvres, near Paris, rather than one-forty-millionth of a meridian, as the unit of length.

Together with the metre, there arose its fractions: one-thousandth, called the *millimetre*, one-millionth, called the *micron*, and the one which is used most frequently, one-hundredth—the *centimetre*.

Let us now say a few words about the *second*. It is much older than the centimetre. There were no disagreements in establishing a unit for measuring time. This is understandable: the alternation of day and night and the eternal revolution of the Sun suggest a natural means of choosing a unit of time. The expression "determine time by means of the Sun" is well known to everyone. When the Sun is high up in the sky, it is noon, and, by measuring the length of the shadow cast by a pole, it is not difficult to determine the moment when it is at its summit. The same instant of the next day can be marked off in the same way. The interval of time which elapses constitutes a day. And the further division of a day into hours, minutes and seconds is all that remains to be done.

The large units of measurement—the year and the day—were given to us by nature itself. But the hour, the minute and the second were devised by man.

The modern division of the day goes far back to antiquity. The sexagesimal, rather than the decimal, number system was prevalent in Babylon. Since 60 is divisible by 12 without any remainder, the Babylonians divided the day into 12 equal parts.

The division of the day into 24 hours was introduced in Ancient Egypt. Minutes and seconds appeared later. The fact that 60 minutes make an hour and 60 seconds make a minute is also a legacy of Babylon's sexagesimal system.

In Ancient Times and the Middle Ages, time was measured with the aid of sun dials, water clocks (by the amount of time required for water to drip out of large vessels) and a series of subtle but rather imprecise devices.

With the aid of modern clocks it is easy to convince oneself that the duration of a day is not exactly the same at all times of the year. It was therefore stipulated that the average solar day for an entire year would be taken as the unit of measurement. One-twenty-fourth of this yearly average interval of time is what we call an hour.

But in establishing units of time—the hour, the minute and the second—by dividing the day into equal parts, we assume that the Earth rotates uniformly. However, lunar-solar ocean tides slow down, although to an insignificant degree, the rotation of the Earth. Thus, our unit of time—the day—is incessantly becoming longer.

This slowing down of the Earth's rotation is so insignificant that only recently, with the invention of atomic clocks measuring intervals of time with great accuracy—up to a millionth of a second—has it become possible to measure it directly. The change in the length of a day amounts to 1-2 milliseconds in 100 years.

But a standard should exclude, when possible, even such an insignificant error. On p. 20 we shall show how this is done.

Weight and Mass

Weight is the force with which a body is attracted by the Earth. This force can be measured with a spring balance. The more the body weighs, the more the spring on which it is suspended will be stretched. With the aid of a weight taken as the unit it is possible to calibrate the spring—make marks which will indicate how much the spring has been stretched by a weight of one, two, three, etc., kilograms. If, after this, a body is suspended on such a scale, we shall be able to find the force (gravity) of its attraction by the Earth, by observing the stretching of the spring (Figure 1.1*a*). For measuring weights, one uses not only stretching but also contracting springs (Figure 1.1*b*). Using springs of various thickness, one can make scales for measuring very large and also very small weights. Not only coarse commercial scales are constructed on the basis of this principle but also precise instruments used for physical measurements.

A calibrated spring can serve for measuring not only the force of the Earth's attraction, i.e. weight, but also other forces. Such an instrument is called a dynamometer, which means a measurer of forces. You may have seen how a dynamometer is used for measuring a person's muscular force. It is also convenient to measure the tractive force of a motor by means of a stretching spring (Figure 1.2).

The weight of a body is one of its very important properties. However, the weight depends not only on the body itself. As a matter of fact, the Earth attracts it. And what if we were on the Moon? It is obvious that

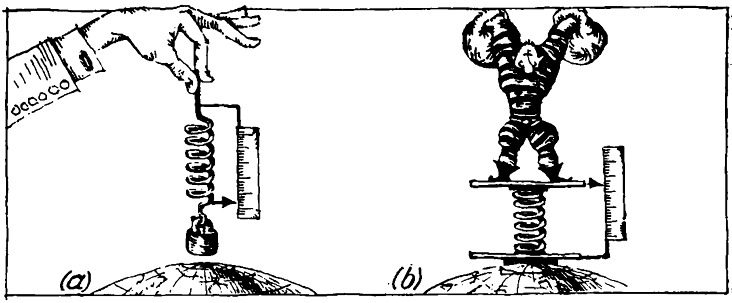

Figure 1.1

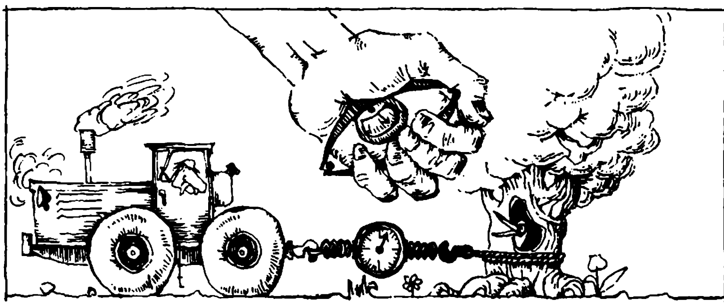

Figure 1.2

its weight would be different—about six times less, as shown by computations. In fact, even on the Earth, weight is different at various latitudes. At a pole, for example, a body weighs 0.5% more than at the equator.

However, for all its changeability, weight possesses a remarkable peculiarity—the ratio of the weights of two bodies remains unchanged under any conditions, as experiments have shown. If two different loads stretch a

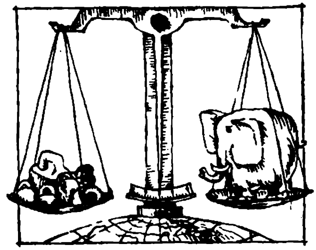

Figure 1.3

spring identically at a pole, this identity is completely preserved even at the equator.

In measuring weight by comparing it with the weight of a standard, we find a new property of bodies, which is called *mass*.

The physical meaning of this new concept—mass—is related in the most intimate way to the identity in comparing weights which we have just noted.

Unlike weight, mass is an invariant property of a body depending on nothing except the given body.

A comparison of weights, i.e. measurement of mass, is most conveniently carried out with the aid of ordinary balance scales (Figure 1.3). We say that the masses of two bodies are equal if the balance scale on whose pans these bodies are placed is in perfect equilibrium. If a load is in equilibrium on a balance scale at the equator, and then the load and the weights are transported to a pole, the load and the weights change their weight identically. Weighing at the pole will therefore yield the same result: the scale will remain balanced.

We can even verify this state of affairs on the Moon. Since the ratio of bodies' weights will not change there either, a load placed on a scale will be balanced by the same weights there. The mass of a body remains the same no matter where it is.

Units of mass and weight are related to the choice of a standard weight. Just as in the case of the metre and the second, people tried to find a natural standard of mass. The same commission used a definite alloy to make a weight which balanced one cubic decimetre of water at four degrees Centigrade*. This standard was called the *kilogram.*

Later, however, it became clear that it isn't so easy to "take" one cubic decimetre of water. Firstly, the decimetre, as a fraction of the metre, changed along with the refinement of the metre's standard. Secondly, what kind of water should we take? Chemically pure water? Twice distilled? Without any trace of air? And what should be done about admixtures of "heavy water"? To top off all our misfortunes, accuracy in measuring a volume is noticeably less than that in weighing.

It again became necessary to reject a natural unit and accept a specially made weight as the unit of mass. This weight is also kept in Paris together with the standard for the metre.

One-thousandth and one-millionth of a kilogram—the *gram* and the *milligram*—are widely used for measuring mass. The Tenth and Eleventh General Conferences of Weights and Measures developed the International System of Units (SI), which was then ratified by most countries as national standards. The name "kilogram" (kg) is retained by mass in this system. Every force, including of course weight, is measured in newtons (N) in this system. We shall find out a bit later why this unit was given such a name and how it is defined.

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*This temperature was not chosen by chance. Its significance lies in the fact that the volume of water changes with heating in a very peculiar manner, unlike most bodies. A body ordinarily expands when heated, but water contracts as its temperature rises from 0 to 4 °C, and only starts expanding after it gets above 4 °C. Thus, 4 °C is the temperature at which water stops to contract and begins to expand.

The new system will undoubtedly not be immediately and universally applied, and so it is still helpful to recall that the kilogram of mass (kg) and the kilogram of force (kgf) are units of different physical quantities, and it is impossible to perform arithmetical operations on them. Writing 5 kg + 2 kgf = 7 is just as meaningless as adding metres to seconds.

The International System of Units and Standards of Measurement

We began our discourse from the simplest things. For what can be simpler than measuring distances, time intervals and mass? Indeed, this was so in the early days of physics, but today the methods used in measuring length, time and mass are so sophisticated that they require a knowledge of all branches of physics. What we are going to discuss now in more or less detail is studied in the fourth book, *Photons and Nuclei*. With this in mind, I suggest that if this is your first book in physics, postpone reading this section until later.

The International System of Units, abbreviated SI from the French "Le Système International d'Unites", was adopted in 1960. Slowly but surely it is gaining recognition. But even now when these lines are being written (on the threshold of 1977) the good "old" units of measurement are still in use. If you ask a car owner what engine his car has, his first reaction will be "a 100 horsepower" (just as, say, ten years ago) but not "a 74 kilowatt". I believe that the SI system will become predominant only after two generations have passed and the books whose authors refuse to recognize it have gone out of print.

The SI system is based on seven *base* units: the metre, the kilogram, the second, the mole, the ampere, the kelvin and the candela.

Let us start with the first four. My purpose is to emphasize a significant tendency of a general nature rather than to expound the details of measuring the corresponding quantities. The tendency is to discard material (i.e. man-made) standards and instead use natural standards, that is, standards whose values do not depend on the measuring devices and do not change with time, at least from the viewpoint of today's physics.

We will begin with the metre. In the spectrum of a particular isotope of krypton, Kr^{86}, there is a sharp spectral line. By using methods which we will discuss later it was established that each spectral line is characterized by the initial and final energy levels. The line we are interested in is the transition from the $5d_5$ level to the $2p_{10}$ level. Specifically, one metre is 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom. There is no use in adding another significant digit to the above nine-digit number, since the accuracy in measuring this wavelength is not more than 4 parts in 10^9. We see that this definition is in no way connected with a material standard. There is also no reason to believe that the wavelength of a specific transition changes over the ages. So we have achieved our goal.

Well and good, my reader may say. But how does one calibrate an ordinary yardstick with the aid of such a non-material standard? Physicists know how to do this using interference methods, which we will examine in the fourth book.

There is every reason to assume that this definition will undergo a change in the near future. The point is that using a laser beam (say, of a helium-neon laser stabilized with iodine vapour) we can achieve an accuracy of 1 part in 10^{11} or even 1 part in 10^{12}. It may prove expedient to use another spectral line for the natural standard.

The definition of the second is quite similar. The transition used is between two close energy levels of the caesium-133 atom. The inverse of the frequency of such a transition gives the time needed for the completion of one vibration, the period. One second is taken as 9 192 631 770 such periods. Since these vibrations lie in the microwave range, we can apply radio methods to divide the frequency and thus calibrate any clock. The error is 1 second in 300 000 years.

It was the dream of metrologists to use one energy transition for defining the unit of length (expressed in a certain number of wavelengths) and the unit of time (expressed in a certain number of vibration periods).

In 1973 scientists showed how this could be done. The measurements were made using a helium-neon laser stabilized with methane. The wavelength was 3.39 millimicrons, and the frequency was 88×10^{-12} cycles per second. The precision was so high that the product of these two numbers gave the speed of light in vacuum as 299 792 458 metres per second with an accuracy of 4 parts in 10^9.

In contrast to these brilliant achievements and still greater prospects, the precision in measuring mass leaves much to be desired. The "material" kilogram is still in use, unfortunately. True, balances are constantly being perfected, but still a precision of 1 part in 10^9 is achieved only in rare cases and only in comparing two masses. The accuracy in measuring the mass of a body in grams and in measuring the gravitational constant in the law of universal gravitation still does not exceed 1 part in 10^5.

The Fourteenth General Conference of Weights and Measures (1971) introduced into the SI system a new base unit of amount of substance, the *mole*. The introduction of the mole as an independent unit of amount of substance is due to the new definition of the Avogadro number.

It was agreed that the *Avogadro number* was not just the number of atoms in one gram-atom but the number of atoms in 12 grams of the isotope of carbon with mass number 12, that is C^{12}. If we denote the number of atoms in 12 grams of C^{12} as N_A, we define a mole as the amount of substance that contains N_A particles. The particle may be an atom, a molecule, an ion, a radical, an electron, etc., or a specified group of such entities.

What makes it necessary to introduce not only a new base unit but a new physical concept is the fact that we inadmissibly apply the concept of mass to elementary particles, whereas mass is a quantity measured with a beam balance.

Today the amount of substance (the Avogadro number and, hence, atomic mass) is determined with a lower accuracy than mass proper. But, understandably, the accuracy of measuring the amount of substance cannot exceed the accuracy of measuring mass.

My reader may think that the introduction of a new unit is no more than a formality. However, the existence of two concepts of mass is justified by the difference in precision of measurement. If it ever proves possible to express the kilogram as a multiple of the mass of an atom, the case will be reviewed and the kilogram will become a quantity of the same type as the metre or second.

Density

What do we mean when we say: as heavy as lead and as light as a feather? It is clear that a grain of lead will be light, while a mountain of feathers has considerable mass. Those who use such comparisons have in mind not the mass of a body but the density of the material of which it consists.

The mass of a unit volume of a body is called its *density*. It is evident that a grain of lead and a massive block of lead have the same density.

In denoting density, we usually indicate how many grams (g) a cubic centimetre (cm^3) of the body weighs—after this number we place the symbol g/cm^3. In order to determine the density, the number of grams must be divided by the number of cubic centimetres; the solidus in the symbol reminds us of this.

Certain metals are among the heaviest materials—osmium whose density is equal to 22.5 g/cm^3, iridium (22.4), platinum (21.5), tungsten and gold (19.3). The density of iron is 7.88, that of copper 8.93.

The lightest metals are magnesium (1.74), beryllium (1.83) and aluminium (2.70). Still lighter bodies should be sought among organic materials: various sorts of wood and plastic may have a density as low as 0.4.

It should be stipulated that we are dealing with continuous bodies. If there are pores in a solid, it will of course be lighter. Porous bodies—cork, foam glass—are frequently used in technology. The density of foam glass may be less than 0.5, although the solid matter from which it is made has a density greater than 1 g/cm^3. As all other bodies whose density is less than 1 g/cm^3, foam glass floats superbly on water.

The lightest liquid is liquid hydrogen; it can only be obtained at extremely low temperatures. One cubic centimetre of liquid hydrogen has a mass of 0.07 g. Organic liquids—alcohol, benzine, kerosene—do not differ significantly from water in density. Mercury is very heavy—it has a density of 13.6 g/cm^3.

And how can the density of gases be characterized? For gases, as is well known, occupy whatever volumes we let them. If we empty gas-bags with the same mass of gas into vessels of different volumes, the gas will always fill them up uniformly. How then can we speak of density?

We define the density of gases under so-called normal conditions—a temperature of 0 °C and a pressure of 1 atm. The density of air under normal conditions is equal to 0.001 29 g/cm³, of chlorine 0.003 22 g/cm³. Gaseous hydrogen, just as the liquid one, holds the record: the density of this lightest gas is equal to 0.000 09 g/cm³.

The next lightest gas is helium; it is twice as heavy as hydrogen. Carbon dioxide is heavier than air by a factor of 1.5. In Italy, near Naples, there is a famous "canine cave"; carbon dioxide continually exudes from its lower part, hangs low and slowly escapes from the cave. A person can enter this cave without difficulty, but such a stroll will end badly for a dog. Hence the cave's name.

The density of gases is extremely sensitive to external conditions—pressure and temperature. Without an indication of the external conditions, the values of the density of gases have no meaning. The densities of liquids and solids also depend on temperature and pressure, but the dependence is considerably weaker.

The Law of Conservation of Mass

If we dissolve some sugar in water, the mass of the solution will be precisely equal to the sum of the masses of the sugar and the water.

This and an infinite number of similar experiments show that the mass of a body is an invariable property. No matter how the body is crushed or dissolved, its mass remains fixed.

The same also holds for arbitrary chemical transformations. Suppose that coal burns up. It is possible to estab-

Mikhail Lomonosov (1711-1765)—an outstanding Russian scientist, the initiator of science in Russia, a great educator. In the field of physics, Lomonosov struggled resolutely against the notions widespread in the 18th century of electrical and thermal "fluids", upholding the molecular-kinetic theory of matter. Lomonosov was lish by means of careful weighings that the mass of the coal and the oxygen from the air which was used up during the burning will be exactly equal to the mass of the end products of the combustion.

The law of conservation of mass was verified for the last time at the end of the 19th century, when the technique of fine weighing had already been highly developed. It turned out that mass does not even change by an insignificant fraction of its value during the course of any chemical transformation.

Mass was considered to be invariable as far back as Ancient Times. This law first underwent an actual experimental verification in 1756. This was done by Mikhail Lomonosov, who proved the conservation of mass during the sintering of metals by means of experiments in 1756, and demonstrated the scientific significance of the law.

Mass is the most important invariable characteristic of a body. The majority of the properties of a body is, so to say, in the hands of human beings. An iron bar that can be easily bent by hand can be made hard and brittle by tempering it. With the aid of ultrasonic waves, one can make a turbid solution transparent. Mechanical, electrical and thermal properties can be changed by means of external actions. If no matter is added to a body and not a single particle is separated from it, it is

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the first to experimentally prove the constancy of the mass of matter participating in chemical transformations. Lomonosov carried out extensive research in the field of atmospheric electricity and meteorology. He constructed a series of remarkable optical instruments and discovered the atmosphere on Venus. Lomonosov created the basis of scientific Russian; he succeeded in translating the basic physical and chemical terms from the Latin exceptionally well.

impossible* to change its mass, regardless of what external actions we resort to.

Action and Reaction

We frequently fail to notice that every action of a force is accompanied by a reaction. If a valise is placed on a bed with a spring mattress, the bed will sag. The fact that the weight of the valise acts on the bed is obvious to everyone. Sometimes, however, we forget that the bed also exerts a force on the valise. As a matter of fact, the valise lying on the bed does not fall; this means that there is a force acting on it equal to the weight of the valise and directed upwards.

Forces which are opposite in direction to gravity are often called reactions of the support. The word "reaction" means "counteraction" The action of a table on a book which is lying on it and the action of a bed on a valise which has been placed upon it are reactions of the support.

As we have just said, the weight of a body can be determined with the aid of a spring balance. The pressure of the body on the spring which has been placed under it, or the force stretching the spring on which the load has been suspended, is equal to the weight of the body. It is obvious, however, that the contraction or tension of the spring can just as well be used to obtain the value of the reaction of the support.

Thus, measuring the magnitude of some force by means of a spring, we measure the value of not one but of two forces opposite in direction. Spring balances measure the pressure exerted by the load on the pan, and also the reaction of the support—the action of the pan on the

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*The reader will later discover that there are certain limitations to this assertion.

load. Fastening a spring to a wall and pulling it by hand, we can measure the force with which our hand pulls the spring and, simultaneously, the force with which the spring pulls our hand.

Therefore, forces possess a remarkable property: they are always found in pairs and are, moreover, equal in magnitude and opposite in direction. It is these two forces which are usually called *action and reaction*.

"Single" forces do not exist in nature—only mutual reactions between bodies have a real existence; moreover, the forces of action and reaction are invariably equal—they are related to each other as an object is related to its mirror image.

One should not confuse balancing forces with forces of action and reaction. We say that forces are balanced if they are applied to a single body; thus, the weight of a book lying on a table (the action of the Earth on the book) is balanced by the reaction of the table (the action of the table on the book).

In contrast to the forces which arise in balancing two interactions, the forces of action and reaction characterize one interaction, for example, of a table with a book. The action is "table-book" and the reaction is "book-table" These forces, of course, are applied to different bodies.

Let us try to clear up the following traditional misunderstanding: "The horse is pulling the waggon, but the waggon is also pulling the horse; why then do they move?" First of all, we must recall that the horse will not move the waggon if the road is slippery. Hence, in order to explain the motion, we must take into account not one but two interactions—not only "waggon-horse" but also "horse-road". The motion will begin when the force of the interaction "horse-road" (the force with which the horse pushes off from the road) exceeds that of the interaction "waggon-horse" (the force with which the waggon pulls the horse). As for the forces "waggon pulls horse" and "horse pulls waggon", they characterize one and the same interaction, and will therefore be identical in magnitude when at rest and at any instant during the course of the motion.

How Velocities Are Added

If I waited half an hour and then another hour, I would lose one and a half hours of time all told. If I were given a rouble and then two more, I would receive three roubles in all. If I bought 200 g of grapes and then another 400 g, I would have 600 g of grapes. We say that time, mass and other similar quantities are added arithmetically.

However, not every quantity can be added and subtracted so simply. If I say that it is 100 km from Moscow to Kolomna and 40 km from Kolomna to Kashira, it does not follow from this that Kashira is located at a distance of 140 km from Moscow. Distances are not added arithmetically.

How else can quantities be added? We shall easily find the required rule on the basis of our example. Let us draw three points on a piece of paper indicating the relative locations of the three places of interest to us (Figure 1.4). We can construct a triangle with these three points as vertices. If two of its sides are known, it is possible to find the third. For this, however, we must know the angle between the two given segments.

The trip from Moscow to Kolomna can be represented by an arrow whose direction shows where we are moving to. Such arrows are called vectors. So the trip from Kolomna to Kashira is represented by another vector.

Now, how do we show the trip from Moscow to Kashira? With a vector, of course. We will start this vector at the beginning of the first vector and end it at the end of the second. The sought path will be the line that completes the triangle.

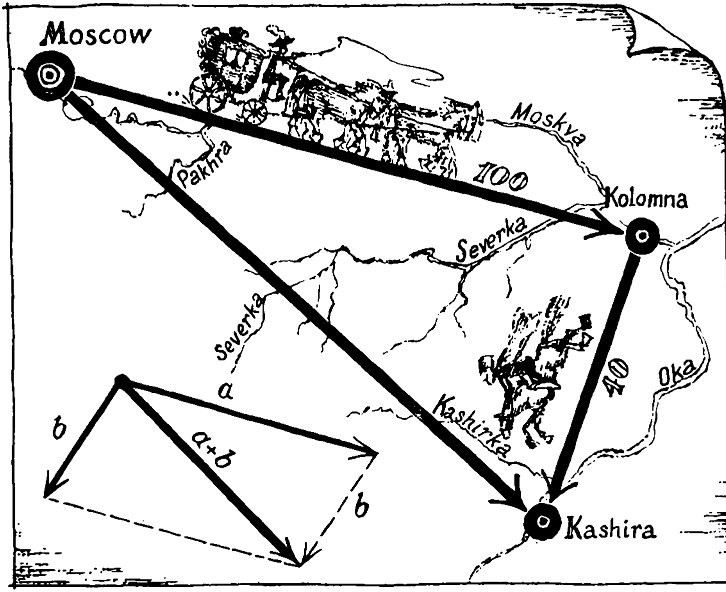

Figure 1.4

The kind of addition just described is called geometrical and the quantities which are added in this manner are called *vectors.*

In order to distinguish the initial point of a segment from its end point, we add an arrow to it. Such a segment—a vector—indicates a length and a direction.

This rule is also applied in adding several vectors. Passing from the first point to the second, from the second to the third, etc., we cover a path which can be represented by a broken line. But it is possible to go directly from the starting point to the terminal point. This segment closing up the polygon will be precisely the vector sum.

A vector triangle also shows, of course, how to subtract one vector from another. For this we draw them from one point. The vector drawn from the end point of the second vector to the end point of the first will be the vector difference.

Besides the triangle rule, one may make use of the equivalent parallelogram rule (Figure 1.4 in the lower left corner). This rule requires that we construct a parallelogram on the vectors we are adding, and draw the diagonal from the point of their intersection. It is clear from the figure that the diagonal of the parallelogram is precisely the segment which closes up the triangle. Hence, both rules are equally suitable.

Vectors are used for describing not only displacements. Vector quantities are frequently found in physics.

Consider, for example, a velocity of motion. *Velocity* is the displacement during a unit of time. Since the displacement is a vector, the velocity is also a vector, and it has the same direction. In the course of motion along a curve, the direction of displacement is changing all the time. How then can we answer the question about the direction? A small segment of a curve has the same direction as a tangent. Therefore, the displacement and velocity of a body are directed along the tangent to the path of motion at each given instant.

In many cases one must add and subtract velocities according to the rule for vectors. The need to add velocities arises when a body participates simultaneously in two motions. Such cases are not uncommon: a person walks inside a train and, in addition, moves together with the train; a drop of water trickling down the window pane of a train moves downwards under the action of its weight and travels along with the train; the Earth moves around the Sun and together with the Sun moves with respect to the other stars. In all these and other similar cases, velocities are added in accordance with the rule for adding vectors.

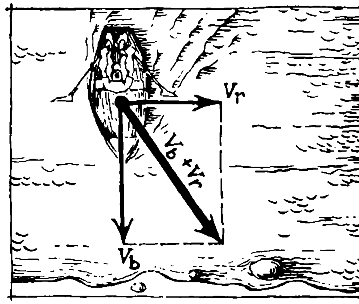

Figure 1.5

If both motions take place along a single line, then vector addition reduces to ordinary addition when both motions have the same direction, and to subtraction when they have opposite directions.

But what if the motions take place at an angle? Then we turn to geometrical addition.

If in crossing a swiftly flowing river you steer perpendicular to the current, you will be carried downstream. The boat participates in two motions: across the river and along the river. The total velocity of the boat is shown in Figure 1.5.

Another example. What does the motion of a stream of raindrops look like from the window of a train? You have no doubt observed rain from train windows. Even in windless weather it moves slantwise, as if a wind blowing towards the train from ahead were deflecting it (Figure 1.6).

If the weather is windless, a raindrop falls vertically downwards. But during the time the drop is falling near the window, the train has travelled a fair distance leaving the vertical line of fall behind; this is why the rain seems to be slanting.

If the velocity of the train is $\mathbf{v}_t$, and the velocity of the raindrop is $\mathbf{v}_r$, then the velocity of its fall relative

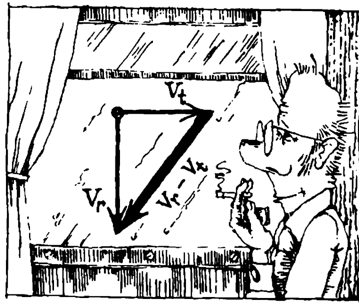

Figure 1.6

to a passenger of the train is obtained by the vector subtraction of $\mathbf{v}_t$ from $\mathbf{v}_r$.* The velocity triangle is shown in Figure 1.6. The direction of the slanting vector indicates the direction of the rain; now it is clear why we see the rain slanting. The length of the slantwise arrow yields the magnitude of this velocity in the chosen scale. The faster the train goes and the slower the raindrop falls, the more the stream of raindrops seems to slant.

Force Is a Vector

Force, just as velocity, is a vector quantity. For it always acts in a definite direction. Therefore, forces should also be added according to the rules which we have just discussed.

We often observe examples in real life which illustrate the vector addition of forces. A rope on which a package is hanging is shown in Figure 1.7. A person is pulling the package to one side with a string. The rope is being stretched by the action of two forces: the force of the

*Here and in what follows we shall use bold-face letters to denote vectors, i.e. characteristics for which not only magnitude but also direction is of significance.

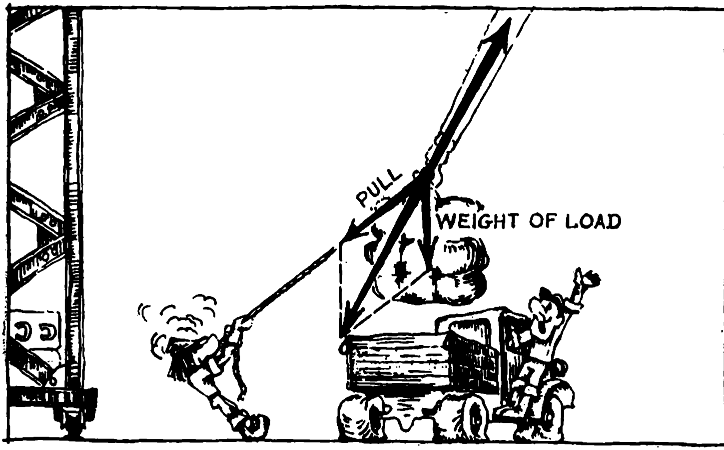

Figure 1.7

weight of the package and the force that the person exerts on it.

The rule of vector addition of forces allows us to determine the direction of the rope and compute the tension. The package is at rest; hence, the sum of the forces acting on it must be equal to zero. And we can also put it this way—the tension in the rope must be equal to the sum of the weight of the package and the force pulling it to one side with the aid of the string. The sum of these forces yields the diagonal of a parallelogram which will be directed along the rope (for otherwise it could not be "annihilated" by the tension in the rope). The length of this arrow will represent the tension. The two forces acting on the package could be replaced by such a force. The vector sum of forces is therefore sometimes called the resultant.

There very often arises a problem which is inverse to

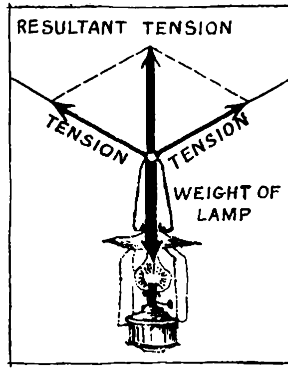

Figure 1.8

the addition of forces. A lamp is suspended on two ropes. In order to determine the tension in the ropes, we must decompose the weight of the lamp along these two directions.

From the end point of the resultant vector (Figure 1.8) we draw lines parallel to the ropes up to the points of intersection. The parallelogram of forces is constructed. Measuring the lengths of the sides of the parallelogram, we find (in the same scale in which the weight is represented) the magnitude of the tension in the rope.

Such a construction is called a decomposition of force. Every number can be represented in an infinite number of ways as the sum of two or several numbers; the same thing can also be done with a force vector: any force can be decomposed into two forces—sides of a parallelogram—one of which can always be chosen arbitrarily. It is also clear that to each vector there can be attached an arbitrary polygon.

It is often convenient to decompose a force into two mutually perpendicular forces—one along a direction of interest to us and the other perpendicular to this direction.

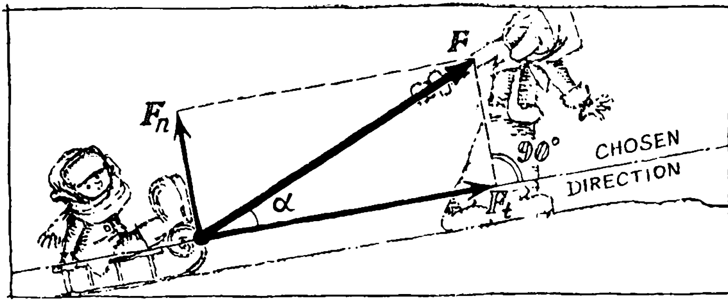

Figure 1.9

They are called the tangential and normal (perpendicular) components of force.

The component of force in a particular direction, constructed by a decomposition along the sides of a rectangle, is also called the projection of the force in this direction.

It is clear that in Figure 1.9

$$F^2 = F_t^2 + F_n^2$$

where F_t and F_n are the projections of the force in the chosen direction and normal to it.

Those who know some trigonometry will establish without difficulty that

$$F_t = F \cos \alpha$$

where α is the angle between the force vector and the direction onto which it is projected.

A very curious example of the decomposition of forces is given by the motion of a sailboat. How does it manage to sail against the wind? If you ever watched a sailboat doing this, you might have noticed that it zigzagged. Sailors call such a motion tacking.

Of course, it is impossible to sail directly against

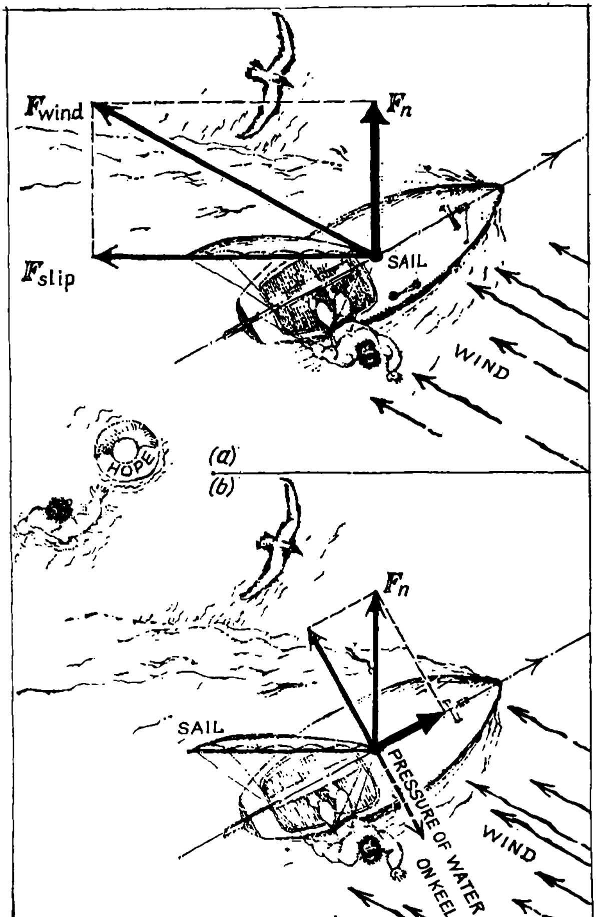
F_{wind}
F_n
F_{slip}
SAIL
WIND
HOPE
(a)
(b)
F_n
SAIL
PRESSURE OF WATER ON KEEL
WIND

Figure 1.10

the wind, but why is it possible to sail against the wind at all, if only at an angle?

The possibility of beating against the wind is based on two circumstances. The wind always pushes the sail at right angles to the latter's plane. Look at Figure 1.10*a*: the force of wind is decomposed into two components—one of them F_{slip} makes the air slip past the sail and, hence, does not act on the sail, and the other—the normal component—exerts pressure on the sail.

But why does the boat move not in the direction of the wind but roughly in the direction of the bow? This is explained by the fact that a movement of a boat across its keel line would meet with a very strong resistance on the part of the water. Therefore, in order for a boat to move forward, it is necessary that the force pressing on the sail should have a forward component along the keel line. This aspect is illustrated in Figure 1.10*b*.

In order to find the force which drives the boat forward, we must decompose the force of the wind a second time. We have to decompose the normal component along and across the keel line. It is just the tangential component that drives the boat at an angle towards the wind, and the normal component is balanced by the pressure of the water exerted on the keel. The sail is set in such a way that its plane bisects the angle between the direction of the path of the boat and that of the wind.

Inclined Plane

It is more difficult to overcome a steep rise than a gradual one. It is easier to roll a body up an inclined plane than to lift it vertically. Why is this so, and how much easier is it? The law of the addition of forces permits us to gain an understanding of these matters.

Figure 1.11 depicts a waggon on wheels which is held on an inclined plane by the tension in a string. Besides

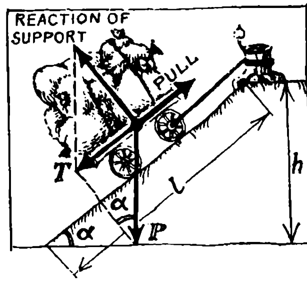

Figure 1.11

this pull, two other forces are acting on the waggon—its weight and the force of the reaction of the support which always acts along the normal to a surface, regardless of whether the surface of the support is horizontal or inclined.

As has already been said, if a body rests on a support, the latter counteracts the pressure or, as we say, creates the *reaction force*.

We want to know to what degree it is easier to pull a waggon up along an inclined plane than to lift it vertically.

We decompose the forces in such a way that one component is directed along, and the other perpendicular to, the surface on which the body is moving. In order for the body to be at rest on the inclined plane, the tension in the string must balance only the tangential component. As for the second component, it is balanced by the reaction of the support.

We can find the force we are interested in, i.e. the tension T in the rope, either by means of a geometrical construction or with the aid of trigonometry. The geometrical construction consists in dropping a perpendicular from the end point of the weight vector P to the plane.

One can find two similar triangles in the figure. The ratio of the length l of the inclined plane to its height

h is equal to the ratio of the corresponding sides of the force triangle. Thus,

$$\frac{T}{P} = \frac{h}{l}$$

The less the plane is inclined (the smaller the value of h/l), the easier it will be, of course, to pull the body upwards.

And now, for those who are acquainted with trigonometry: since the angle between the normal component of the weight and the weight vector is equal to the angle of inclination α of the plane (these are angles with mutually perpendicular sides), we have

$$\frac{T}{P} = \sin\alpha \quad \text{and} \quad T = P\sin\alpha$$

Therefore, it is $1/\sin\alpha$ times easier to wheel a waggon up a plane with the angle of inclination α than to lift it vertically.

It is helpful to memorize the values of the trigonometric functions for angles of 30, 45 and 60° Knowing these numbers for the sine ($\sin 30° = 1/2$; $\sin 45° = \sqrt{2}/2$; $\sin 60° = \sqrt{3}/2$), we get a good idea of the amount of force saved by moving up an inclined plane.

It is evident from our formulas that for a 30° angle of inclination, the force we exert will be half the weight of the body: $T = P/2$. For angles of 45 and 60°, we have to pull the rope with forces equal to about 0.7 and 0.9 of the weight of the waggon. As we see, such steeply inclined planes do not make our task much easier.

2. Laws of Motion

Various Points of View About Motion

The valise is standing in the baggage rack. At the same time, it is moving together with the train. The house is standing on the Earth, but it is also moving together with it. It is possible to say about one and the same body: it is moving in a straight line, it is at rest, it is rotating. And all these statements will be true, but from different points of view.

Not only the graph of the motion but also its properties can be entirely different if regarded from different points of view.

Recall what happens to objects on a ship which is being rocked by the sea. How they misbehave! The ash-tray on the table overturned and dove headlong under the bed. The water splashes in the bottle, and the lamp vibrates like a pendulum. Without any visible cause, some objects begin moving and others stop. An observer on such a ship might say that the basic law of motion is that at any moment an unfastened object can start travelling in any direction with an arbitrary speed.

This example shows that among the various points of view on motion there are those which are really awkward.

But what point of view is the most "reasonable"?

If suddenly, for no reason whatsoever, the lamp on the table were to bend over, or the paper-weight were to jump, then at first you would think that it was only your imagination. If these miracles were repeated, you would urgently start looking for the cause which drove these bodies out of the state of rest.

It is therefore perfectly natural to regard the point of view on motion, according to which bodies at rest do not budge without the action of a force, as a rational one. Such a point of view seems quite natural: a body is at rest—hence, the sum of the forces acting on it is equal to zero; it moved—this happened under the action of a force.

This point of view presupposes the presence of an observer. However, it is not the observer himself who is of interest to us, but his location. Therefore, instead of "point of view on motion", we shall say "frame of reference in which the motion is regarded", or simply "frame of reference".

For us, inhabitants of the Earth, an important frame of reference is the Earth. However, bodies moving on the Earth, say, a ship or a train, can also frequently serve as frames of reference.

Let us now return to the "point of view" on motion which we called rational. This frame of reference has a name—it is called *inertial*.

We shall see a bit later where this term comes from.

Consequently, the properties of an inertial frame of reference are as follows: bodies in a state of rest with respect to such a frame of reference do not feel the action of forces. Therefore, not a single motion in such a frame of reference is begun without the action of a force. The simplicity and convenience of such a frame of reference are obvious. It would pay to study motion in them.

The fact that the frame of reference associated with the Earth does not differ greatly from an inertial one is extremely important. We can therefore begin our investigation of the basic regularities of motion considering them from the point of view of the Earth. Nevertheless, we must bear in mind that, strictly speaking, everything that will be said in the next section deals with an inertial frame of reference.

The Law of Inertia

There can be no quarrel—an inertial frame of reference is convenient and has invaluable advantages.

But is such a frame of reference unique or do there, perhaps, exist many inertial frames of reference? The Ancient Greeks, for example, took the former point of view. In their writings we find many naive reflections on the causes of motion. These ideas find their completion in Aristotle. In the opinion of this philosopher, the natural state of a body is rest—of course, with respect to the Earth. Every displacement of a body with respect to the Earth must have a cause—a force. But if there is nothing causing a body to keep moving, it must halt, return to its natural state. And this is what rest with respect to the Earth is. From this point of view, the Earth is the unique inertial frame of reference.

We are indebted to the great Italian Galileo Galilei (1564-1642) for discovering the truth and disproving this false but congenial to naive psychology opinion.

Let us think over the Aristotelian explanation of motion and search familiar phenomena for confirmation or refutation of the idea that rest is the natural state of bodies on the Earth.

Imagine that we are in an airplane taking off from an airport at dawn. The Sun has not yet warmed up the air, so there are no "air-pockets", which cause many passengers unpleasantness. The airplane is moving smoothly, imperceptibly. If you don't look out of the window, you won't even notice that you're flying. A book is lying on an empty seat; an apple is at rest on a table. All objects inside the airplane are motionless. Is this how things should be if Aristotle were right? Of course not. As a matter of fact, according to Aristotle, the natural state of a body is rest on the Earth. But then why are all the objects not piled up at the rear wall of the airplane trying to lag behind its motion, "wanting" to return to the state of "true" rest? What makes the apple lying on the table, hardly touching the surface of the table, move with an enormous speed of several hundred kilometres an hour?

What is the correct answer to the question of the cause of motion? Let us first take up the question of why moving bodies come to a stop. For example, why does a ball rolling along the Earth's surface come to a stop? In order to give a correct answer, we should consider in which cases a ball comes to a stop quickly, and in which cases slowly. We don't need any special experiments for this. We know perfectly well from our practical experience that the smoother the surface on which a ball is moving, the farther it will roll. From these and similar experiences, there arises the natural idea of the force of friction as a hindrance to motion, as the cause for the slowing down of an object which is rolling or slipping along the Earth. Friction can be decreased in various ways. The more we work on the destruction of every kind of resistance to motion (for example, the smoother we construct our roads, the better we lubricate our engines and the more we perfect our ball bearings), the greater the distance a moving body will cover freely without being acted on by any external force.

The following question arises: What would happen if there were no resistance, if the force of friction were absent? Obviously, in such a case a motion would continue infinitely, with a constant speed and along one and the same straight line.

We have formulated the law of inertia in about the same form as it was first given by Galileo. Inertia is

Galileo Galilei (1564-1642)—a great Italian physicist and astronomer, the first to apply the experimental method of investigation in science. Galileo introduced the concept of inertia, established the relativity of motion, investigated the laws of free fall, of the motion of bodies on an inclined plane, and of the motion of an a brief designation for this ability of a body to move rectilinearly and uniformly without any cause, contrary to Aristotle. Inertia is an inalienable property of each particle in the Universe.

In what way can we check the validity of this remarkable law? As a matter of fact, it is impossible to create conditions under which no forces would be acting on a moving body. Even though this is true, we can, on the other hand, observe the opposite. In every case when a body changes the speed or direction of its motion, it is always possible to find a cause—a force responsible for this change.

A body acquires speed in falling to the Earth; the cause is the Earth's gravitation. A stone twirls on a string circumscribing a circle; the cause deflecting the stone from a rectilinear path is the tension in the string. If the string breaks, the stone will fly off in the same direction in which it was moving at the moment the string broke. An automobile running with the motor turned off slows down; the causes are air resistance, friction between the tires and the road, and imperfections in the ball bearings.

The law of inertia is the foundation on which the entire study of the motion of bodies rests.

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object thrown at an angle to the horizontal, used a pendulum for the measurement of time. For the first time in the history of mankind, he looked at the sky through a telescope, discovered many new stars, proved that the Milky Way consists of an enormous number of stars, discovered Jupiter's satellites, sunspots and the rotation of the Sun, investigated the structure of the Moon's surface. Galileo actively supported Copernicus' heliocentric system banned in those days by the Catholic church. Persecution by the Inquisition darkened the last ten years of the great scientist's life.

Motion Is Relative

The law of inertia leads to the derivation of the multiplicity of inertial frames of reference.

Not one but many frames of reference exclude "causeless" motions.

If one such frame of reference is found, we can immediately find another, moving (without rotation) uniformly and rectilinearly with respect to the first. Moreover, one inertial frame of reference is not the least bit better than the others, does not in any way differ from the others. It is in no way possible to find a best frame of reference among the multitude of inertial frames of reference. The laws of motion of bodies are identical in all inertial frames of reference: a body is brought into motion only under the action of forces, is slowed down by forces, and in the absence of any forces acting on it either remains at rest or moves uniformly and rectilinearly.

The impossibility of distinguishing some particular inertial frame of reference with respect to the others by means of any experiments whatsoever constitutes the essence of the *Galilean principle of relativity*—one of the most important laws of physics.

But even though the points of view of observers studying phenomena in two inertial frames of reference are fully equivalent, their judgements about one and the same fact will differ. For example, one of the observers will say that the seat on which he is sitting in a moving train is located at the same place in space all the time, but another observer standing on the platform will assert that this seat is moving from one place to another. Or one observer firing a rifle will say that the bullet flew out with a speed of 500 m/s, while another observer, if he is in a frame of reference which is moving in the same direction with a speed of 200 m/s, will say that the bullet is flying considerably slower, with a speed of 300 m/s.

Who of the two is right? Both. For the principle of the relativity of motion does not allow a preference to be given to any single inertial frame of reference.

It turns out that no unconditionally true (as is said, absolute) statements can be made about a region of space or the velocity of motion. The concepts of a region of space and the velocity of motion are relative. In speaking about such relative concepts, it is necessary to indicate which inertial frame of reference one has in mind.

Therefore, the absence of a single unique "correct" point of view on motion leads us to recognize the relativity of space. Space could have been called absolute only if we were able to find a body at rest in it—at rest from the point of view of all observers. But this is precisely what is impossible to do.

The relativity of space means that space may not be pictured as something into which bodies have been immersed.

The relativity of space was not recognized immediately by science. Even such a brilliant scientist as Newton regarded space as absolute, although he also understood that it would be impossible to prove this. This false point of view was widespread among a considerable number of physicists up to the end of the 19th century. The reasons for this are apparently of a psychological nature: we are simply very much accustomed to see the immovable "same places in space" around us.

We must now figure out what absolute judgements can be made about the character of motion.

If bodies move with respect to one frame of reference with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$, then their difference (vector, of course) $\mathbf{v}_1 - \mathbf{v}_2$ will be identical for any inertial observer, since both of the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ undergo the same change when the frame of reference is changed.

Thus, the vector difference between the velocities of two bodies is absolute. If so, the vector increment in the velocity of one and the same body for a definite interval of time is also absolute, i.e. its value is identical for all inertial observers.

The Point of View of a Celestial Observer

We decided to study motion from the point of view of an inertial frame of reference. Won't we then have to reject the services of the terrestrial observer? As a matter of fact, the Earth rotates about its axis and revolves around the Sun, as was proved by Nicolaus Copernicus (1473-1543). It may be difficult for the reader to feel now how revolutionary Copernicus' discovery was to realize that Giordano Bruno was burned at the stake, and Galileo suffered humiliation and exile for championing the truth of Copernicus' ideas.

What was it that Copernicus' genius accomplished? Why may we place the discovery of the Earth's rotation and revolution on one plane with the ideas of human justice for which progressive-minded people have been willing to give up their lives?

In his *Dialogue on the Two Chief Systems of the World* (the Ptolemaic and the Copernican), for whose writing he was persecuted by the Inquisition, Galileo gave the opponent of the Copernican system the name Simplicio, which means "simpleton".

In fact, from the point of view of a simple direct observer of the world, that which is not very aptly called "common sense", the Copernican system seems mad. How can the Earth rotate? As a matter of fact, I see it and it is stationary, but the Sun and the stars are really moving.

The attitude of theologians to Copernicus' discovery is shown by the following conclusion of the Assembly of Theologians (1616):

"The doctrine that the Sun is located at the centre of the world and is immovable is false and absurd, formally heretical and contrary to the Bible. More than that, the doctrine that the Earth does not lie at the centre of the world and moves, possessing in addition a daily rotation, is false and absurd from the philosophical point of view and at least erroneous from the theological one."

This conclusion, in which a lack of understanding of the laws of nature and a belief in the infallibility of religious dogmas are mixed up with a false "common sense", testifies better than anything else to the strength of Copernicus' spirit and mind, and those of his disciples having so resolutely broken with the "truths" of the 17th century.

But let us return to the question posed above.

If the velocity of an observer's motion changes or if he rotates, he must be deleted from the list of "correct" observers. But it is precisely under these conditions that an observer on the Earth is found. However, if the change in velocity or the observer's rotation during the time he is investigating a motion is small, such an observer may be conditionally regarded as "correct" Will this pertain to an observer on the Earth?

During a second the Earth will turn 1/240 of a degree, i.e. about 0.000 07 radian. This isn't very much. The Earth is therefore quite inertial with respect to a great many phenomena.

Nevertheless, one can no longer forget about the Earth's rotation when dealing with prolonged phenomena.

Under the dome of St. Isaac Cathedral in Leningrad hangs an enormous pendulum. If we start oscillating this pendulum, within a short time it will be possible to notice that the plane of its oscillation is slowly turning. After several hours, the plane of oscillation will turn through a noticeable angle. Such an experiment with this kind of pendulum was first performed by the French scientist Léon Foucault (1819-1868), and has born his name ever

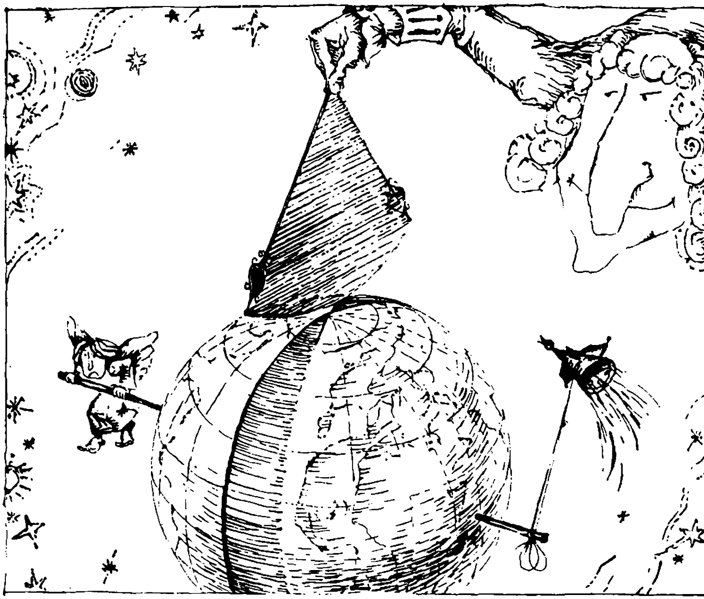

Figure 2.1

since. The Foucault experiment yields a visual demonstration of the Earth's rotation (Figure 2.1).

Thus, if the observed motion continues for a long time, we shall be forced to reject the services of the terrestrial observer and take a frame of reference associated with the Sun and the stars as our ba sis. Such a frame of reference was used by Copernicus assuming the Sun and the surrounding stars to be fixed. However, in reality Copernicus' frame of reference is not completely inertial.

The Universe consists of a great number of star-clusters—islands of the Universe, which are called galaxies. In the galaxy to which our solar system belongs, there are approximately one-hundred billion stars. The Sun is revolving around the centre of this galaxy with a period of about 180 million years and a speed of 250 km/s.

What error will be made by assuming a solar observer to be inertial?

For a comparison of the merits of terrestrial and solar observers, let us compute the angle through which the solar frame of reference turns during a second. If a complete revolution takes place every 180×10^6 years (6×10^{15} s), then in one second the solar frame of reference will turn through an angle of 6×10^{-14} degree or 10^{-15} radian. We may say that the solar observer is 100 billion times "better" than the terrestrial one.

Desiring an even closer approximation to an inertial frame of reference, astronomers take a frame of reference associated with several galaxies as a basis. Such a frame of reference is the most inertial of all possible kinds. It is impossible to find a better frame of reference.

Astronomers may be called star gazers in two senses: they observe stars and describe the motions of heavenly bodies from the point of view of the stars.

Acceleration and Force

In order to characterize the velocities that are not constant, physicists use the concept of acceleration.

The change in velocity during a unit of time is called *acceleration*. Instead of saying "the velocity of a body changed by a in 1 second," we say more briefly "the acceleration of a body is equal to a."

If we denote by v_1 the speed of a rectilinear motion at the first instant, and by v_2 at the next, the rule for calculating the acceleration a is expressed by the formula

$$a = \frac{v_2 - v_1}{t}$$

where t is the time during which the speed builds up.

Speed is measured in cm/s (or m/s, etc.), time in seconds. Hence, acceleration is measured in cm/s per second. A number of centimetres per second is divided by seconds. Thus, the unit of acceleration will be cm/s^2 (or m/s^2, etc.).

Of course, the acceleration can change during the course of a motion. However, we shall not complicate our treatment with this inessential fact. We shall implicitly assume that the velocity changes uniformly during the course of a motion. Such a motion is called *uniformly accelerated.*

What is acceleration of curvilinear motion?

Since velocity is a vector, a change (difference) in velocity is a vector, and so acceleration is also a vector. In order to find the acceleration vector, one must divide the vector difference between the velocities by the time. But we have already described how to construct a vector change in velocity.

The highway takes a turn. Let us note two nearby positions of a car and represent its velocities by vectors (Figure 2.2). Subtracting these vectors, we obtain a quantity which is by no means equal to zero; dividing it by the elapsed time, we find the acceleration vector. An acceleration took place even when the speed around the turn did not change. Curvilinear motion is always accelerated. Only uniform rectilinear motion is unaccelerated.

In speaking about the velocity of motion of a body, we always stipulated what our point of view was with regard to the motion. The velocity of a body is relative. From the point of view of one inertial frame of reference it can be great, and from the point of view of another inertial frame of reference it can be small. Don't we have to make the same kind of stipulations when speaking about acceleration? Of course not. Unlike velocity, acceleration is absolute. From the point of view of all imagin-

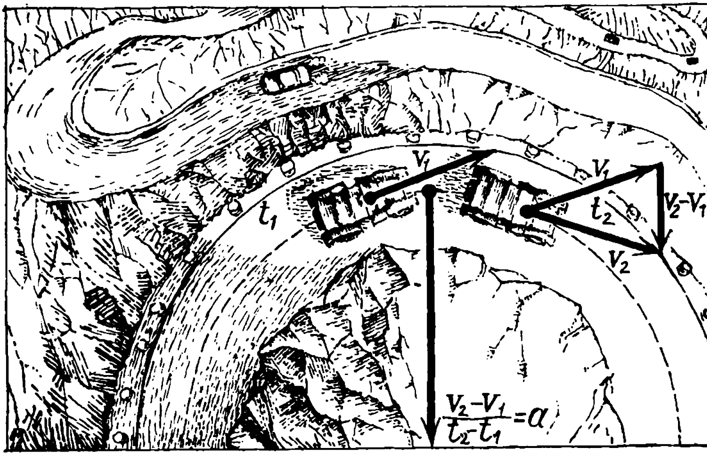

Figure 2.2

able inertial frames of reference acceleration will be identical. As a matter of fact, acceleration depends on the difference in the velocity of a body between the first and second instants of time, and this difference, as we already know, will be identical from all points of view, i.e. is absolute.

If no force is acting on a body, it can only move without acceleration. Conversely, the action of a force on a body accelerates it; moreover, the greater the force, the greater will be the acceleration. The faster we want to move a loaded waggon, the more we have to strain our muscles. As a rule, two forces act on a moving body: accelerating—the pull, and decelerating—the force of friction or air resistance.

The difference between these two forces, the so-called resultant force, may be directed along or against the motion. In the first case, the body speeds up its motion;

in the second case, it slows it down. If these oppositely acting forces are equal to each other, the body will move uniformly, just as though there were no forces acting on it.

But how is a force related to the acceleration it creates? The answer turns out to be very simple. The acceleration is proportional to the force:

$$a \propto F$$

(The symbol $\propto$ denotes "is proportional to".)

Another question still remains to be answered: How do the properties of a body influence its ability to accelerate its motion under the action of one or another force? For it is clear that one and the same force acting on different bodies will give them different accelerations.

We shall find the answer to the question we have posed in the remarkable fact that all bodies fall to the Earth with the same acceleration. This acceleration is denoted by the letter g. In the vicinity of Moscow $g = 981\ \text{cm/s}^2$.

Direct observation will not, at first sight, confirm the identity of acceleration for all bodies. The fact is that when a body is falling under ordinary conditions, besides gravity there is another, "hindering" force acting on it—air resistance. Philosophers of antiquity were quite confused by the difference in the way light and heavy bodies fall. A piece of iron falls quickly, but a feather glides through the air. A sheet of paper falls slowly to the ground, but if we roll it up, this same sheet will fall considerably faster. The fact that the atmosphere distorts the "true" picture of the motion of a body under the action of the Earth was already understood by the Ancient Greeks. However, Democritus thought that even if the air were deleted, heavy bodies would always fall faster than light ones. But air resistance can have the opposite effect, for example, a sheet of aluminium foil (all unrolled) will fall more slowly than a small ball made by crumpling a piece of paper.

Incidentally, metallic wire of such a thinness (several microns) is so manufactured now that it glides through the air like a feather.

Aristotle thought that all bodies should fall identically in a vacuum. However, he used this theoretical conclusion to make the following paradoxical deduction: "The falling of different bodies with the same speed is so absurd that the impossibility of the existence of vacuum is clear."

None of the scientists of antiquity or the Middle Ages guessed that it could be experimentally verified whether bodies fall to the Earth with different or the same accelerations. Only Galileo demonstrated by means of his remarkable experiments (he investigated the motion of balls down an inclined plane and the fall of bodies thrown from the top of the leaning tower of Pisa) that at any given point on the Earth all bodies fall with the same acceleration, regardless of their mass. At the present time such experiments are quite easily performed with the aid of a long tube out of which the air has been pumped. A feather and a stone fall identically in such a tube: only one force acts on the bodies, and that is weight; air resistance has been reduced to zero. In the absence of air resistance, the fall of any body is a uniformly accelerated motion.

Let us now return to the question posed above. How does the ability of a body to accelerate its motion under the action of a given force depend on its properties?

Galileo's law states that all bodies, regardless of their masses, fall with one and the same acceleration; hence, a mass of m kg under the action of a force of F kgf moves with an acceleration g.

Now suppose we are no longer talking about falling bodies, and a force of 1 kgf is acting on a mass of m kg.

Since acceleration is proportional to force, it will be m times less than g.

We have arrived at the conclusion that the acceleration a of a body for a given force (1 kgf in our example) is inversely proportional to its mass.

Uniting both conclusions, we may write:

$$a \propto \frac{F}{m}$$

i.e. for a constant mass the acceleration is directly proportional to the force, and for a constant force inversely proportional to the mass.

This law, relating acceleration to the mass of a body and the force acting on it, was discovered by the great English scientist Sir Isaac Newton (1643-1727), and bears his name.*

Acceleration is directly proportional to the acting force and inversely proportional to the mass of a body, and does not depend on any other properties of the body. It follows from Newton's law that it is precisely the mass which is the measure of the "inertness" of a body. For identical forces, it is more difficult to accelerate a body of greater mass. We see that the concept of mass, which we first knew as a "modest" quantity determined by weighing a body on a balance scale, has acquired a new deep meaning: the mass characterizes the dynamic properties of a body.

Newton's law may be written as follows:

$$kF = ma$$

where k is a constant coefficient. This coefficient depends on the chosen units.

*Newton himself showed that motion is subject to three laws. The law which we are now discussing appears on Newton's list as the second. He called the law of inertia the first law, and the law of action and reaction the third.

Instead of making use of the unit of force (kgf) we already have available, we shall act in a different manner. Just as physicists often try to do, we shall choose our unit of force in such a way that the coefficient of proportionality in Newton's law becomes equal to unity. Then Newton's law takes the following form:

$$F = ma$$

As we have already said, in physics it is customary to measure mass in grams, distance in centimetres, and time in seconds. The system of units based on these three fundamental quantities is called the cgs system.

Let us now choose, using the principle formulated above, the unit of force. A force will then obviously be equal to unity when it imparts the acceleration of 1 cm/s^2 to the mass of 1 g. Such a force received the name *dyne* (dyn) in this system.

According to Newton's law, $F = ma$, the force will be expressed in dynes if we multiply m g by a cm/s^2. One therefore makes use of the following notation:

$$1 \text{ dyn} = 1 \text{ g-cm/s}^2$$

The weight of a body is usually denoted by the letter P. The force P gives the body an acceleration g, and in dynes we obviously have

$$P = mg$$

But we already had a unit of force—the kilogram-force (kgf). We immediately find the relation between our new and old units from the last formula:

$$1 \text{ kgf} = 981\,000 \text{ dyn}$$

A dyne is a very small force. It is equal to about one milligram of weight.

Sir Isaac Newton (1643-1727)—a brilliant English physicist and mathematician, one of the greatest scientists in the history of mankind. Newton formulated the basic concepts and laws of mechanics, discovered the law of universal gravitation, creating by the same token a physical picture of the world with re-

We have already mentioned the system of units (SI). The name for the new unit of force, *newton* (N), is fully deserved. For such a choice of units, Newton's law will look as simple as possible; this new unit is defined as follows:

$$1\ \text{N} = 1\ \text{kg-m/s}^2$$

i.e. 1 N is the force necessary to impart the acceleration of 1 m/s^2 to the mass of 1 kg.

It is not difficult to relate this new unit to the dyne and the kilogram-force:

$$1\ \text{N} = 100\,000\ \text{dyn} = 0.102\ \text{kgf}$$

Rectilinear Motion with Constant Acceleration

Such motion arises, according to Newton's law, when the resultant force acting on a body, speeding it up or slowing it down, is constant.

Such conditions arise rather frequently, even though only approximately: a car moving with its motor cut off slows down under the action of the more or less constant

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mained inviolable until the beginning of the 20th century. He developed a theory of the motion of celestial bodies, explained the most important special features of the Moon's motion and gave an explanation for the tides. In optics, some remarkable discoveries facilitating the rapid growth of this branch of physics are due to Newton. Newton devised a powerful method of the mathematical investigation of nature; the honour of creating the differential and integral calculus belongs to him. This exerted an enormous influence on the entire subsequent development of physics and facilitated the introduction of mathematical methods of research.

force of friction: a weighty object falls from a height under the action of the constant force of gravity.

Knowing the magnitude of the resultant force, and also the mass of a body, we can find the magnitude of the acceleration according to the formula $a = F/m$. Since

$$a = \frac{v - v_0}{t}$$

where t is the time of the motion, v is the final speed, and v_0 is the initial speed, with the aid of this formula it is possible to answer a series of questions of, say, the following type: How long will it take a train to come to a halt if the decelerating force, the mass of the train and the initial speed are known? Or how much speed will a car gather if the power of the motor, the resistance, the mass of the car and the duration of acceleration are known?

We are often interested in knowing the distance covered by a body in a uniformly accelerated motion. If the motion is uniform, the distance covered is found by multiplying the speed of the motion by its time. If the motion is uniformly accelerated, the calculation of the distance covered is carried out as though the body were moving uniformly for the same time t with the speed equal to half the sum of the initial and final speeds:

$$s = \frac{1}{2}(v_0 + v)\,t$$

Thus, for uniformly accelerated (or decelerated) motion, the distance covered by a body is equal to the product of half the sum of the initial and final speeds by the time of the motion. The same distance would be covered during the same time in a uniform motion with speed $(v_0 + v)/2$. In this sense, one can say that $(v_0 + v)/2$ is the average speed of the uniformly accelerated motion.

It is helpful to compose a formula which would show the dependence of the distance covered on the acceleration. Substituting $v = v_0 + at$ in the last formula, we find:

$$s = v_0 t + \frac{1}{2} at^2$$

or, if the motion occurs without any initial speed,

$$s = \frac{1}{2} at^2$$

If a body travels 5 m in one second, then in two seconds it will travel (4 × 5) m, in three seconds (9 × 5) m, etc. The distance travelled grows in proportion to the square of the time.

A heavy body falls from a height in accordance with this law. The acceleration of free fall is equal to g, and our formula acquires the following form:

$$s = \frac{981}{2} t^2$$

if t is expressed in seconds and g in centimetres per second per second.

If a body could fall without hindrance for some 100 s, it would cover an enormous distance from the beginning of its fall—about 50 km. Moreover, only a mere 0.5 km would be covered in the first 10 s—this is what accelerated motion means.

But what speed will a body develop in falling from a given height? To answer this question we shall need formulas relating the covered distance to the acceleration and the speed. Substituting the time of the motion $t = (v - v_0)/a$ in $s = (1/2)(v_0 + v)t$, we obtain:

$$s = \frac{1}{2a}(v^2 - v_0^2)$$

or, if the initial speed is equal to zero,

$$s = \frac{v^2}{2a}, \quad v = \sqrt{2as}$$

Ten metres is the height of a small two- or three-storey house. Why is it dangerous to jump to the ground from the roof of such a house? A simple calculation shows that the speed of such a free fall would reach the value $v = \sqrt{2 \times 9.8 \times 10}$ m/s = 14 m/s ≈ 50 km/h, and this is, after all, the speed of a car within city limits.

Air resistance will not reduce this speed much.

The formulas we have singled out are employed for the most varied computations. Let us apply them in order to see how motions take place on the Moon.

In H. G. Wells' novel *The First Men in the Moon* we read about the surprises experienced by travellers in their fantastic trips. On the Moon, the acceleration of gravity is approximately six times less than terrestrial. If a falling body on the Earth covers 5 m in the first second, it will "float" down only 80 cm in all on the Moon (the acceleration there is about 1.6 m/s^2).

The formulas we have written out permit us to rapidly calculate the lunar "miracles".

A jump from a height of h m takes $t = \sqrt{2h/g}$ s. Since lunar acceleration is six times less than terrestrial, the jump will require $\sqrt{6} \approx 2.45$ times more time on the Moon. By how many times will the final speed of the jump be decreased ($v = \sqrt{2gh}$)?

One can jump safely from the roof of a three-storey house on the Moon. The height of a jump with the same initial speed will be increased by a factor of six ($h = v^2/2g$). A child will be able to jump higher than the record set on the Earth.

Path of a Bullet

People have been solving the problem of throwing an object as far as possible from time immemorial. A stone thrown by hand or shot from a sling, an arrow flown from

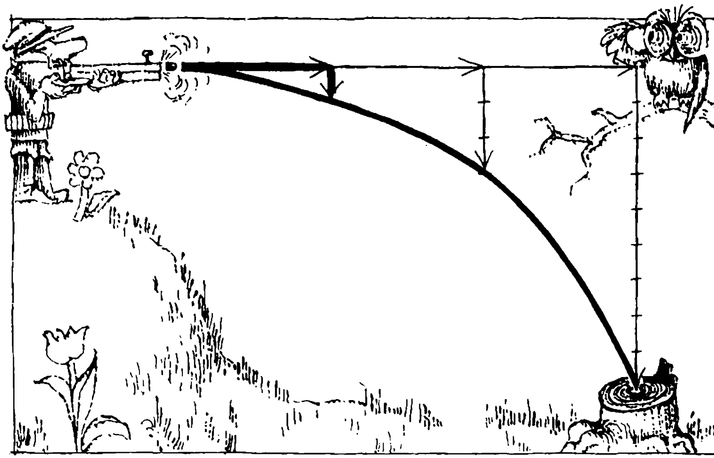


Figure 2.3

a bow, a rifle bullet, an artillery shell, a ballistic missile—here is a brief list of successes in this field.

The thrown object will move in a curved line called a parabola. It can be constructed without difficulty if we regard the motion of a thrown body as the sum of two motions—horizontal and vertical—taking place simultaneously and independently. The acceleration of free fall is vertical, and so a flying bullet moves horizontally by inertia with a constant velocity and simultaneously falls to the Earth vertically with a constant acceleration. But how can we add these two motions?

Let us begin with a simple case—when the initial velocity is horizontal (say, we are dealing with a shot from a rifle whose barrel is horizontal).

Take a sheet of graph paper and draw a vertical and a horizontal lines (Figure 2.3). Since the two motions are taking place independently, in t seconds the body is

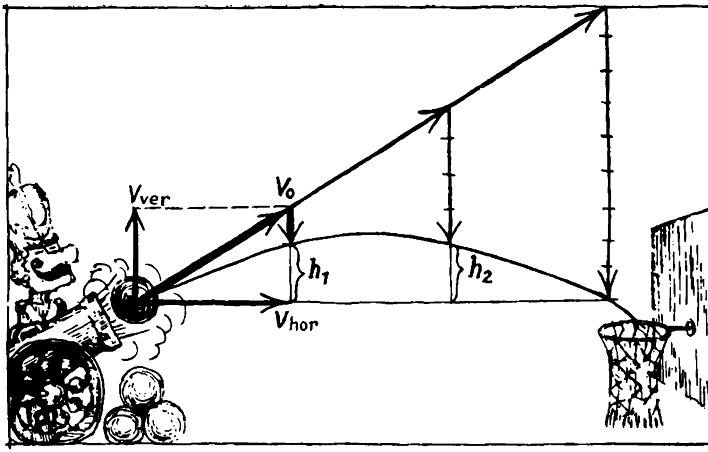


Figure 2.4

displaced by an interval of v_0t to the right and an interval of $gt^2/2$ downwards. Mark off the segment v_0t along the horizontal line, and from its end point, the vertical segment $gt^2/2$. The end point of the vertical segment represents the point where the body will be in t seconds.

This construction must be carried out for several points, i.e. for several instants of time. A smooth curve—the parabola representing the trajectory of the body—will pass through these points. The more frequently one lays off these points, the more accurately will the trajectory of the flight of the bullet be constructed.

A trajectory has been constructed in Figure 2.4 for the case when the initial velocity v_0 is directed at an angle.

The vector v_0 should first of all be decomposed into its vertical and horizontal components. On the horizontal line we mark off $v_{hor}t$—the distance through which the bullet will move horizontally in t seconds.

But the bullet simultaneously performs an upward motion. In t seconds it will rise to a height of $h = v_{\text{ver}}t - gt^2/2$. By means of this formula, substituting in it the instants of time of interest to us, we can compute the vertical displacements and mark them off on the vertical axis. The values of h will first increase (rise) and then decrease.

It now remains to mark the points of the trajectory on the graph, just as we did in the preceding example, and draw a smooth curve through them.

If the rifle barrel is held horizontally, the bullet will soon burrow into the ground; if the barrel is vertical, it will fall at the place where the shot was fired. Therefore, in order to shoot as far as possible, one must fix the barrel of the rifle at some angle to the horizontal. But at what angle?

Let us again employ the same device—decompose the initial velocity vector into its two components: a vertical vector equal to v_1, and a horizontal vector to v_2. The time between the moment the shot is fired until the moment the bullet reaches its highest point is equal to v_1/g. Note that the bullet will be falling downwards for the same length of time, i.e. the complete time of the flight of the bullet until it lands on the ground is $2v_1/g$.

Since the horizontal motion is uniform, the range of the flight is equal to

$$s = \frac{2v_1v_2}{g}$$

(we have ignored the height of the rifle above ground level in our calculation).

We have obtained a formula which shows that the range of the flight is proportional to the product of the velocity components. For what firing direction will this product be greatest? This question can be expressed by means of the geometrical rule of the addition of vectors. The velocities v_1 and v_2 form the sides of the velocity rectangle; a diagonal in it is the total velocity v. The product v_1v_2 is equal to the area of this rectangle.

Our question reduces to the following: Given the length of a diagonal, what sides must be taken for the area of the rectangle to be maximum? It is proved in geometry that this condition is satisfied by a square. Therefore, the range of the flight of the bullet will be greatest when $v_1 = v_2$, i.e. when the velocity rectangle reduces to a square. A diagonal of the velocity square forms an angle of 45° with the horizontal—this is precisely the angle at which the rifle must be held for the bullet to fly as far as possible.

If v is the total velocity of the bullet, then in the case of a square we have $v_1 = v_2 = v/\sqrt{2}$. The range-of-flight formula for this optimal case looks as follows: $s = v^2/g$, i.e. the range will be twice as great as the maximum height of a bullet fired upwards with the same initial speed.

The maximum height of a bullet fired at an angle of 45° will be $h = v_1^2/2g = v^2/4g$, i.e. four times less than the range of flight.

It should be admitted that the formulas we have been applying yield exact results only in the case, quite remote from practice, when air is absent. In many cases air resistance plays a decisive role and radically changes the entire picture.

Circular Motion

If a point moves around a circle, the motion is accelerated, if only because the velocity is changing its direction all the time. The speed may remain constant, and we shall confine our attention to precisely such a case.

We shall draw the velocity vectors at successive time intervals and transfer their initial points to a single

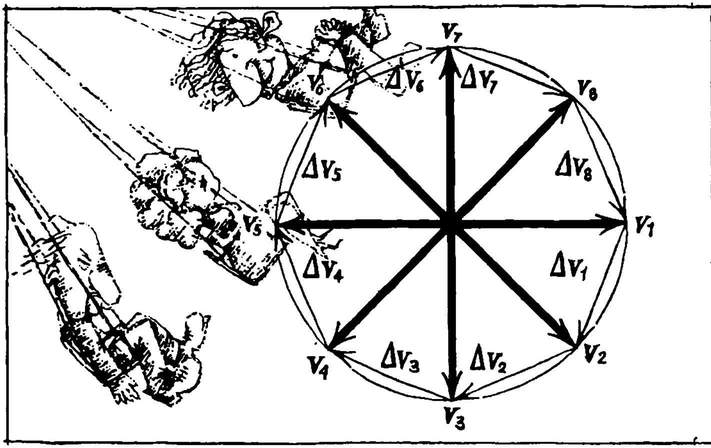


Figure 2.5

point. (We have the right to do this.) If a velocity vector is rotated through a small angle, the change in velocity, as we know, will be represented by the base of an isosceles triangle. Let us construct the changes in velocity during the course of a complete revolution of the body (Figure 2.5). The sum of the magnitudes of the changes in velocity during a complete revolution will be equal to the sum of the sides of the depicted polygon. In constructing each small triangle, we have implicitly assumed that the velocity vector changed by jumps, but its direction is actually changing continuously. It is perfectly clear that the smaller we take the vertex angles of the small triangles, the less will be our error. The smaller the sides of our polygon, the closer will they cling to the circle of radius v. Consequently, the exact value of the sum of the magnitudes of the changes in velocity during the course of the revolution of a point will be the circumference

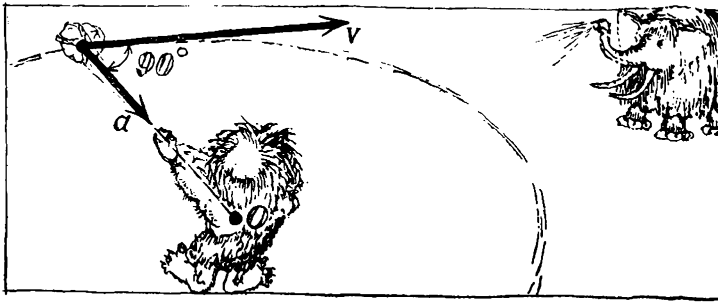


Figure 2.6

$2\pi v$ of the circle. The magnitude of the acceleration is found by dividing it by the time of a complete revolution T: $a = 2\pi v/T$.

The time of a complete revolution in motion around a circle of radius R can be expressed in the form $T = 2\pi R/v$. Substituting this expression in the preceding formula, we obtain the following for the acceleration: $a = v^2/R$.

For a constant radius of rotation, the acceleration is proportional to the square of the speed. For a given speed, the acceleration is inversely proportional to the radius.

This same reasoning shows us how the acceleration of a circular motion is directed at each given instant. The smaller the vertex angle of the isosceles triangles which we used for our proof, the nearer the angle between the increment in velocity and the velocity will be to 90°.

Therefore, the acceleration of a uniform circular motion is directed perpendicular to the velocity; and how are the velocity and acceleration directed relative to the trajectory? Since the velocity is tangent to the path, the acceleration is directed along the radius towards the centre of the circle. These relationships are clearly seen in Figure 2.6.

Try to twirl a stone on a string. You will clearly feel the need for muscular exertion in order to do this. And why is force necessary? After all, isn't the body moving uniformly? The whole point here is that it isn't! The body is moving with a constant speed, but the continuous change in the direction of the velocity makes this motion accelerated. Force is necessary in order to deflect the body from an inertial straight path. Force is needed in order to create the acceleration v^2/R, which we have just computed.

According to Newton's law, force in always pointed in the direction of the corresponding acceleration. Consequently, a body revolving around a circle with a constant speed should be subject to the action of a force directed along a radius towards the centre of the circle. The force acting on the stone exerted by the string is called *centripetal*; it is just this force that supplies the acceleration v^2/R. Hence, the magnitude of this force is mv^2/R.

The string pulls the stone; the stone pulls the string. In these two forces we recognize "an object and its mirror image"—forces of action and reaction. The force with which the stone acts on the string is frequently called *centrifugal*. The centrifugal force is, of course, equal to mv^2/R and directed along the radius out from the centre of the circle. The centrifugal force acts on the body counteracting the tendency of the revolving body to move rectilinearly.

What we have said applies also to the case when the role of the string is played by gravity. The Moon revolves around the Earth. What is it that retains our satellite? Why doesn't it go off, following the law of inertia, in an interplanetary trip? The Earth is holding on to the Moon with an "invisible string"—a *gravitational* force. This force is equal to mv^2/R, where v is the speed of the motion along the lunar orbit, and R is the distance to the

Moon. The centrifugal force in this case acts on the Earth, but, because of the Earth's great mass, it only slightly influences the character of our planet's motion.

Suppose that it is required to send an artificial Earth satellite into a circular orbit at a distance of 300 km from the Earth's surface. What should be the speed of such a satellite? At a distance of 300 km, the acceleration of free fall is somewhat less than on the surface of the Earth, and is equal to 8.9 m/s^2. The acceleration of a satellite moving in a circle is equal to v^2/R, where R is the distance from the centre of the circle (i.e. from the centre of the Earth)—about 6600 km = 6.6×10^6 m. On the other hand, this acceleration is equal to the acceleration of free fall, g. Consequently, $g = v^2/R$, from which we find the speed of the satellite's orbital motion:

$$v = \sqrt{gR} = \sqrt{8.9 \times 6.6 \times 10^6} = 7700 \text{ m/s} = 7.7 \text{ km/s}$$

The minimum speed necessary for a body thrown horizontally to become an Earth satellite is called the orbital velocity. It is clear from the example we have given that this speed is close to 8 km/s.

Life at *g* Zero

Above we found a "reasonable point of view" on motion. True, the "reasonable" points of view, which we called inertial frames of reference, turned out to be infinite in number.

Now, armed with a knowledge of the laws of motion, we can become interested in what motion looks like from an "unreasonable" point of view. Our interest in how inhabitants of non-inertial frames of reference live is by no means idle, if only because we ourselves are dwelling in such a system.

Let us imagine that having grabbed our measuring instruments we settled down in an interplanetary spaceship and went travelling in the starry world.

Time flies quickly. The Sun already resembles a little star. The engine has been cut off and the ship is far away from gravitating bodies.

Let us now see what's going on in our flying laboratory. Why does the thermometer that slid off its nail float in the air and not fall to the floor? In what a strange position deviating from the "vertical" has the pendulum hanging on the wall got stuck! We immediately find the solution: after all, the ship is not on the Earth but in interplanetary space. The objects have lost their weight.

Having feasted our eyes on this extraordinary scene, we decide to change our course. We turn on the jet engine by pressing a button, and suddenly ... the objects surrounding us seemed to come to life. All bodies which hadn't been made fast were brought into motion. The thermometer fell down, the pendulum began oscillating and gradually coming to rest assumed a vertical position, the pillow obediently sagged under the weight of the valise lying on it. Let us take a look at the instruments which indicate the direction in which our ship started accelerating. Upwards, of course! The instruments show that we chose a motion with an acceleration of 9.8 m/s^2, not very great considering the possibilities of our ship. Our sensations are quite ordinary; we feel the way we did on Earth. But why so? As before, we are unimaginably far from gravitational masses, there is no gravity but objects have acquired weight.

Let us drop a marble and measure the acceleration with which it falls to the floor of the spaceship. It turns out that the acceleration is equal to 9.8 m/s^2. This is the number we have just read on the instruments measuring the acceleration of the rocket. The ship is moving upwards with the same acceleration with which the bodies in our flying laboratory are falling downwards.

But what is "up" or "down" in a flying ship? How simple things were when we lived on the Earth. There the sky was up and the Earth was down. And here? Our up has one unquestionable property—it is the direction of the acceleration of the rocket.

It isn't difficult to understand the meaning of our observations: no forces were acting on the marble we dropped. It moves by inertia, whereas the rocket moves with an acceleration relative to the marble. To us who are inside the rocket it seems that the marble is falling in the direction opposite to that of the acceleration of the rocket. Naturally, the acceleration of this fall is equal in magnitude to the true acceleration of the rocket. It is also clear that all bodies in the rocket will "fall" with the same acceleration.

We may draw an interesting conclusion from all that has been said. Bodies start "weighing" when the rocket accelerates. Moreover, the "gravitational force" has a direction opposite to that of the acceleration of the rocket, and the acceleration of free "fall" is equal in magnitude to that of the motion of the jet ship. And what is most remarkable is the fact that in practice we are unable to distinguish the accelerated motion of a frame of reference from the corresponding gravitational force.* If we were inside a spaceship with closed windows, we could not tell whether we were at rest on the Earth or moving with an acceleration of 9.8 m/s^2. This indistinguishability of an acceleration from the action of a gravitational force is called in physics the *equivalence principle*.

*Only in practice. There is a difference in principle. Gravitational forces on the Earth are directed along radii towards the Earth's centre. This means that the directions of acceleration at two different points form an angle. In a rocket moving with an acceleration, the directions of weight are strictly parallel at all points. Acceleration also changes with height on the Earth; this effect is absent in an accelerating rocket.

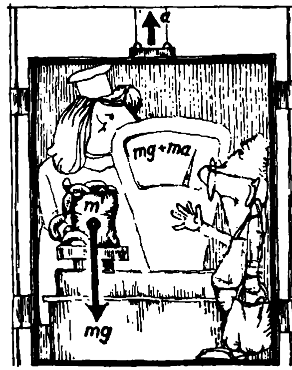


Figure 2.7

This principle, as we shall now see illustrated by a series of examples, permits one to quickly solve many problems by adding to real forces the fictitious gravitational force existing in an accelerating frame of reference.

The elevator can serve as our first example. Let us take along a spring balance with weights and go up in an elevator. We shall follow the behaviour of the pointer of the scale after placing a kilogram of vegetables on it (Figure 2.7). The ascent has begun; we see that the scale reading has increased, as though the weight weighed more than a kilogram. The equivalence principle will easily explain this fact. During the upward motion of the elevator with an acceleration a, there arises an additional gravitational force directed downwards. Since the acceleration of this force is equal to a, the additional weight is equal to ma. Hence, the scale shows a weight of $mg + ma$. The acceleration has ended, and the elevator is moving uniformly—the scale has returned to its initial position and shows a weight of 1 kg. We are getting close to the top floor, and the motion of the elevator is slowing down. What will now happen to the spring balance?

Well, of course, the load now weighs less than one kilogram. When the motion is slowing down, the acceleration vector points downwards. Therefore, an additional fictitious gravitational force is directed upwards, opposite to the direction of the Earth's gravitation. Now a is negative, and so the scale shows a quantity less than mg. After the elevator comes to a halt, the scale returns to its initial position. Let us begin the descent. The motion of the elevator speeds up; the acceleration vector is directed downwards; hence, an additional gravitational force is directed upwards. The load now weighs less than a kilogram. When the motion becomes uniform, the additional weight disappears, and towards the end of our trip on the elevator—when the downward motion is decelerating—the load will weigh more than a kilogram.

The unpleasant sensations experienced in rapidly accelerating and decelerating elevators are related to the change in weight under consideration.

If an elevator is falling with an acceleration, the bodies inside it seem to become lighter. The greater this acceleration, the greater will be the loss of weight. But what will happen when a frame of reference falls freely? The answer is clear: in this case, bodies stop pressing down on the scale—cease weighing: the Earth's gravitation will be balanced by the additional gravitational force existing in such a freely falling frame of reference. Being in such an "elevator", one can calmly place a ton on one's shoulders.

At the beginning of this section, we described life at g zero in an interplanetary spaceship which has left the sphere of gravitation. There is no weight in such a spaceship during uniform rectilinear motion, but the same thing also takes place during the free fall of a frame of reference. Hence, there is no need to leave the sphere of gravitation. Weight is absent in every interplanetary ship which is moving with its engine cut off. A free fall leads to the loss of weight in such systems. The equivalence principle brought us to the conclusion that a frame of reference moving rectilinearly and uniformly far from the action of gravitational forces is almost (see the footnote on p. 72) completely equivalent to a frame of reference falling freely under the action of its weight. In the first system there is no weight, and in the second the "downward weight" is balanced out by the "upward weight" We will not detect any difference between these systems.

Life at *g* zero begins in an artificial Earth satellite at the moment when the ship is orbited and begins moving without the aid of a rocket.

The first space traveller was the dog Laika, and soon afterwards a human being adapted to life at *g* zero in the cabin of the spaceship. The Soviet cosmonaut, Yuri Gagarin, was the first to do so.

Life in the cabin of a spaceship cannot be called ordinary. A great deal of inventiveness and ingenuity were needed in order to make objects so easily subordinated by gravity obedient. Is it possible, for example, to pour water from a bottle into a glass? For water pours "downwards" under the action of gravity. Is it possible to cook food if water cannot be heated on a stove? (Warm water will not mix with cold one.) How can one write with a pencil on paper if a slight push of the former against a table is enough to drive him aside? Neither a match nor a candle nor a gas burner will burn, since burned-up gases will not rise upwards (after all, there is no up!) to make room for oxygen. It was even necessary to think about how to guarantee a normal course for the natural processes occurring in the human organism, for these processes are "accustomed" to the Earth's gravitation.

Motion from an "Unreasonable" Point of View

Let us now take up the question of physical observations in an accelerating bus or streetcar. A peculiarity of this example distinguishing it from the preceding one consists in the following. In the example with the elevator, the additional weight and the Earth's gravitation were directed along a single line. In a decelerating or accelerating streetcar, the additional weight is directed at right angles to the Earth's gravitation. This induces distinctive, although customary, sensation in the passenger. If the streetcar increases its speed, there arises an additional force opposite in direction to that of motion. Let us add this force to that of the Earth's gravitation. The resultant force acting on a person in the car will be directed at an obtuse angle to the direction of the motion. Standing, as usual, face forward in the car, we sense that our "upwards" has moved. In order not to fall, we shall want to become "vertical"—as shown in Figure 2.8*a*. Our "vertical" is slanting. It is inclined at an acute angle to the direction of the motion. If a person stands at right angles to the motion without holding on to anything, he will be sure to fall backwards.

Finally, the motion of the streetcar becomes uniform, and we can stand calmly. However, we are drawing close to the next stop. The driver applies the brakes and our "vertical" is deviating. It is now directed, as can be seen from the drawing in Figure 2.8*b*, at an obtuse angle to the motion. In order not to fall, the passenger leans backwards. However, he won't remain long in such a position. The car comes to a halt, the deceleration disappears, and the "vertical" is now directed at right angles to the Earth. The position of one's body must again be changed. Check your sensations. Isn't it true that when the deceleration began you seemed to be

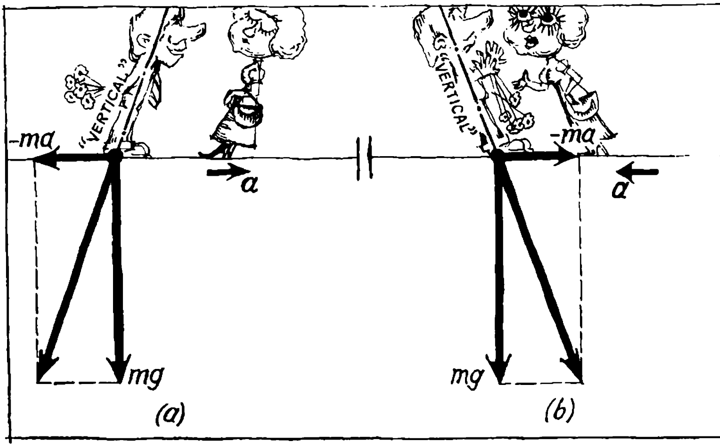


Figure 2.8

pushed from behind, and when the car came to a halt you seemed to be pushed in your chest.

Similar phenomena also occur when a streetcar moves around a curve. We know that motion around a circle, even with a constant speed, is accelerated. The faster the streetcar moves and the smaller the radius of curvature R, the greater the acceleration v^2/R. The acceleration of such a motion is directed along a radius towards the centre. But this is equivalent to the appearance of an additional force directed outwards from the centre. Therefore, an additional force of mv^2/R will be acting on a streetcar passenger during a turn throwing him out towards the external side of the curve. The radial force mv^2/R is called centrifugal. We have already met this force before, on p. 69 (true, considered from a somewhat different point of view).

The action of a centrifugal force during the turning of a streetcar or a bus can only lead to a slight unpleasantness. The force mv^2/R is not large in this case. However, during a speedy motion around a curve, the centrifugal force can become great enough to pose a threat to one's life. Pilots come across large values of mv^2/R when their airplanes "loop-the-loop" While the airplane is describing a circle, the centrifugal force acts on the pilot pinning him to his seat. The smaller the circumference of the loop, the greater the additional force with which the pilot is pinned to the seat. If this "weight" becomes large enough, a person can be "torn" because tissues of living organism possess limited strength and cannot withstand an arbitrary weight.

But how much weight can a person "put on" without seriously endangering his life? That depends on the duration of the overload. If it lasts a fraction of a second, a person is capable of withstanding an overload from $7g$ to $9g$. During ten seconds a pilot can withstand an overload from $3g$ to $5g$. Cosmonauts are interested in the kind of overload a person is able to bear for tens of minutes and even, perhaps, hours. In such cases, it is likely that the overload should be considerably lighter.

Let us compute the radii of a loop which an airplane flying at various speeds can describe without any danger to the pilot. We shall use the acceleration $v^2/R = 4g$. Then $R = v^2/4g$, and for a speed of 360 km/h = 100 m/s the radius of the loop is 250 m. But if the speed is four times greater, i.e. 1440 km/h (and such speeds have already been surpassed by modern jet airplanes), the radius of the loop should be increased by a factor of 16. The minimum radius of the loop becomes equal to 4 km.

Nor shall we leave a more modest form of transportation—the bicycle—without attention. Everyone has seen how a cyclist inclines while rounding a turn. Let us suggest to a cyclist that he should ride around a circle of radius R with speed v, i.e. move with an acceleration

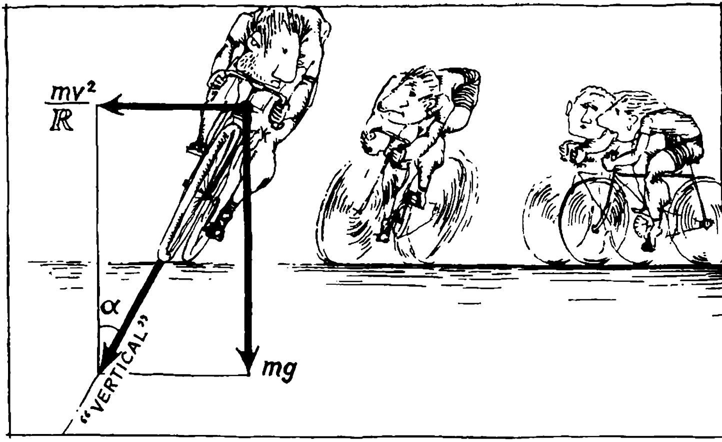


Figure 2.9

v^2/R directed towards the centre. Then besides the Earth's gravitation an additional centrifugal force directed horizontally outwards from the centre of the circle will act on the cyclist. These forces and their sum are shown in Figure 2.9. It is clear that the cyclist should hold himself "vertically", or else he will fall down. But ... his vertical does not coincide with that of the Earth. It can be seen from the figure that the vectors mv^2/R and mg are the legs of a right triangle. The ratio of the leg opposite angle α to the adjacent one is called the tangent of angle α in trigonometry. We have $\tan \alpha = v^2/Rg$; the mass has been cancelled out in full agreement with the equivalence principle. Hence, the cyclist's angle of inclination does not depend on his mass—both a stout and a thin riders must incline identically. The formula and the triangle drawn in the figure show the dependence of the incline on the speed of motion (it grows as the

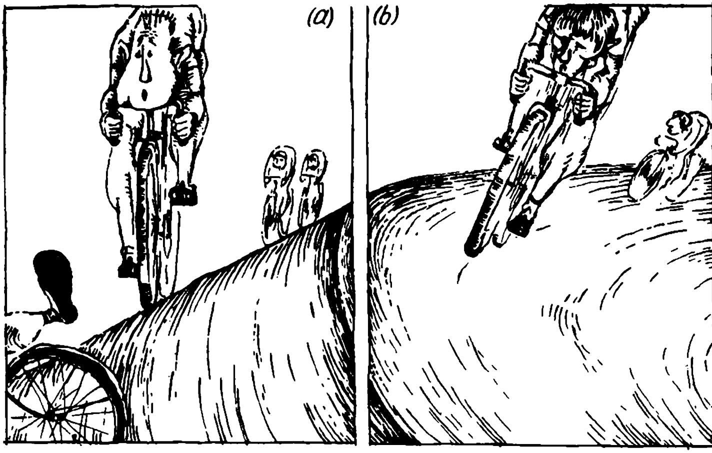


Figure 2.10

latter increases) and the radius of the circle (it increases as the latter decreases). We have explained why the vertical of the cyclist does not coincide with that of the Earth. What then will he feel? We must rotate Figure 2.9 in order to find it out. The road now looks like the slope of a mountain (Figure 2.10*a*), and it becomes clear to us that if the force of friction between the tires and the asphalt is insufficient (for example, when the road is wet), the bicycle may slip and a sharp turn may end with a fall into a ditch.

In order to forestall this, highways are built with sharp turns inclined, i.e. horizontal for a cyclist—as shown in Figure 2.10*b*. In this way, the tendency to slip can be greatly diminished, or even entirely eliminated. This is precisely how turns are constructed in bicycle tracks and superhighways.

Centrifugal Forces

Let us now deal with rotating systems. The motion of such a system is determined by the number of revolutions per second which it makes about an axis. It is also necessary, of course, to know the direction of the axis of rotation.

In order to better understand the peculiarities of life in rotating systems, let us consider the "wheel of laughs"—a well-known ride. Its construction is rather simple. A smooth disc, several metres in diameter, rotates rapidly. Those who so desire are invited to get on it and to try to keep their balance. Even people who know no physics quickly acquire the secret of success: one must go to the centre of the disc, since the farther one is from the centre, the more difficult it is to keep one's balance.

Such a disc is a non-inertial frame of reference with several special features. Every object attached to the disc moves around a circle of radius R with speed v, i.e. with acceleration v^2/R. As we already know, from the point of view of a non-inertial observer this implies the presence of an additional force mv^2/R directed along the radius outwards from the centre. This radial force will act at each point of the "devilish wheel" creating there a radial acceleration v^2/R. The magnitude of this acceleration will be identical for points lying on the same circle. And what about points on different circles? Don't rush to answer that according to the formula v^2/R the smaller the distance from the centre, the greater will be the acceleration. This isn't true because the speed of points farther from the centre of the wheel will be greater. In fact, if the wheel makes n revolutions per second, the path traversed by a point on the rim of the wheel in one second (the speed of this point) is $2\pi Rn$.

The speed of a point is directly proportional to its distance from the centre. We may now rewrite our formula for the acceleration:

$$a = 4\pi^2 n^2 R$$

Since the number of revolutions made in a second is the same for all points of the wheel, we arrive at the following result: the acceleration due to the force exerted by the "radial gravity" acting on a rotating wheel grows in proportion to the distance of a point from the centre of the wheel.

In this interesting non-inertial frame of reference the force of gravity is different on different circles. Therefore, the directions of the "verticals" will also be different for bodies located at different distances from the centre. The Earth's gravitational force is, of course, the same at all points of the wheel. But the vector characterizing the additional radial force becomes longer as the distance from the centre increases. Therefore, the diagonals of the rectangles deviate more and more from the vertical (normal to the Earth's surface).

If we imagine the successive sensations of a person slipping off the "wheel of laughs", from his point of view it can be said that the farther one gets from the centre, the more the disc "inclines" making it impossible to stay on it. To keep his place on the turntable, he must try to place his centre of gravity on a "vertical" inclined in such a way that the farther he is from the rotation axis, the greater the inclination angle (Figure 2.11).

However, could it be possible to invent a contraption analogous to an inclined highway for this inertial frame of reference? Of course it is, but the disc would have to be replaced by such a surface that the resultant gravitational force is perpendicular to it at each of its points. The form of such a surface can be computed. It is called a paraboloid. This name isn't accidental: every vertical cut of a paraboloid is a parabola—the curve along which

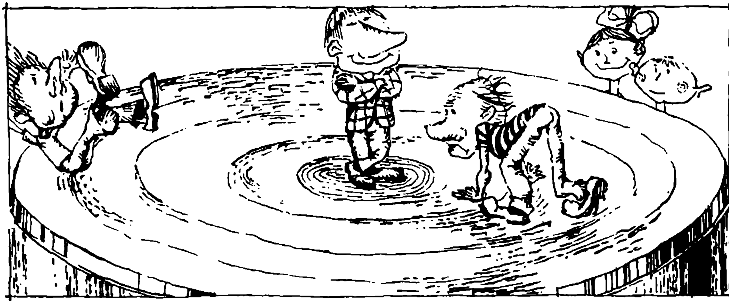


Figure 2.11

bodies fall. A paraboloid is obtained by rotating a parabola around its axis.

It is very easy to create such a surface by making a vessel containing water rotate rapidly. The surface of the rotating liquid is precisely a paraboloid. The water particles will stop moving just when the force pressing each particle to the surface is perpendicular to it. To every rotational velocity there corresponds a distinct paraboloid (Figure 2.12).

It is possible to demonstrate this property by making a solid paraboloid. A small ball placed at any point of a paraboloid rotating with a definite velocity will remain at rest. This means that the force acting on it will be perpendicular to the surface. In other words, a rotating paraboloid behaves as a flat surface. One can walk along such a surface and feel stable, just as on the Earth. However, the direction of the vertical will change during the walk.

Centrifugal phenomena are widely employed in technology. For example, the construction of a centrifuge is based on the use of these phenomena.

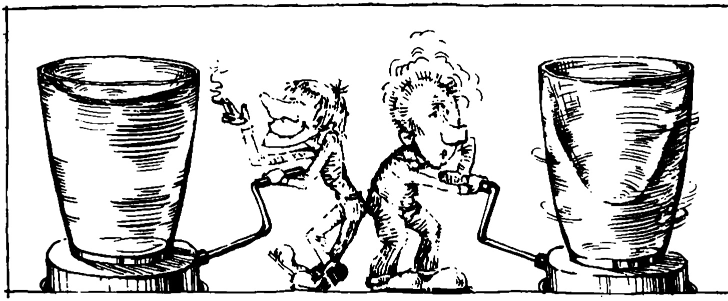


Figure 2.12

A centrifuge is a drum which rotates rapidly around its axis. What will happen if various objects are thrown into such a drum filled to the brim with water?

Let us drop a metal ball into the water—it will go to the bottom but not along our vertical; in moving away from the axis of rotation all the time it will come to a halt at the side. Now let us throw a cork ball into the drum—it, on the contrary, will immediately begin moving towards the axis of rotation and settle there.

If the drum of this model of a centrifuge has a large diameter, we shall notice that the acceleration increases sharply as the ball moves away from the centre.

The phenomena which take place do not puzzle us at all. There is an additional radial force within the centrifuge. If the centrifuge is rotating rapidly enough, its "bottom" is the lateral surface of the drum. The metal ball "sinks" in the water, but the cork ball "floats" The farther a body "falling" in the water is from the axis of rotation, the "heavier" it becomes.

In sufficiently perfected centrifuges, the rotational velocity can be raised to 60 000 rpm, i.e. 10^3 rps. At a distance of 10 cm from the axis of rotation, the acceleration due to the radial gravitational force will be approximately equal to

$$40 \times 10^6 \times 0.1 = 4 \times 10^6 \text{ m/s}^2$$

i.e. 400 000 times greater than terrestrial acceleration.

It is clear that the Earth's gravitation may be neglected for such machines; we really have the right to regard the lateral surface of the drum as the "bottom" in a centrifuge.

The fields of application of a centrifuge become clear from what we have said. If we want to separate the heavy particles in a mixture from the light ones, it is always advisable to apply a centrifuge. Everybody knows the expression: "The muddy liquid has settled." If dirty water stands long enough, the sediment (usually heavier than the water) will settle to the bottom. However, the process of settling may take months, but with the aid of a good centrifuge it is possible to clean up the water instantly.

Centrifuges rotating with velocities of tens of thousands of revolutions per minute are capable of separating the finest particles of sediment not only from water but also from viscous fluids.

Centrifuges are applied in the chemical industry for separating crystals from the solution out of which they grew, for dehydrating salts and for cleaning varnishes; they are used in the food industry for separating syrup from sugar.

The centrifuges which are applied in separating solid or liquid components from a large number of fluids are called separators. Their main application is the processing of milk. Milk separators whirl with velocities of 2000-6000 rpm; the diameters of their drums are as large as 5 m.

Centrifugal casting is widely applied in metallurgy. Even at velocities of 300-500 rpm the liquid metal flowing

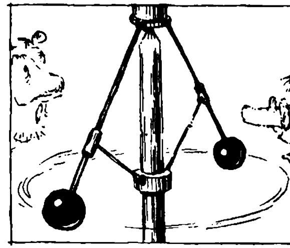


Figure 2.13

into the rotating cast is pressed against its outer surface with a considerable force. Metal pipes cast by this method are denser, more uniform and without blisters or cracks.

Here is another application of centrifugal force. A simple instrument that serves as a governor of the number of revolutions of the rotating parts of a machine is depicted in Figure 2.13. This instrument is called a centrifugal governor. As the velocity of rotation increases, the centrifugal force grows, and the small balls of the governor move farther away from its axis. The rods attached to the balls are deflected, and when the deflection reaches a definite level computed by an engineer, some electrical contacts may be broken, and in the case of a steam engine, for example, valves may be opened letting out excess steam. This will decrease the velocity of rotation and return the rods to their normal position.

Here is an interesting experiment. Place a small cardboard disc on the axis of an electric motor. Switch on the electricity and bring a piece of wood in contact with the whirling disc. A fairly thick beam can be sawed in half as easily as by a steel saw.

An attempt to saw wood by means of cardboard can only evince a smile if one employs it as a hand saw. Why then does the rotating cardboard cut wood? The cardboard particles on the boundary of the disc experience an enormous centrifugal force. The lateral forces which might alter the plane of the cardboard are insignificant in comparison with the centrifugal ones. By keeping its plane fixed, the cardboard disc acquires the ability of gnawing into the wood.

The centrifugal force arising as a result of the Earth's rotation leads to the differences in the weight of a body at various latitudes that we spoke of above.

A body weighs less at the equator than at a pole for two reasons. Bodies lying on the Earth's surface are at different distances from the Earth's axis depending on the latitude of their locations. Of course, this distance grows in passing from a pole to the equator. Moreover, a body located at a pole is on the axis of rotation, so the centrifugal acceleration is

$$a = 4\pi^2 n^2 R = 0$$

(the distance from the axis of rotation $R = 0$). At the equator, on the contrary, this acceleration is maximum. The centrifugal force reduces the gravitational force. Therefore, the pressure exerted by a body on a scale (the weight of the body) is minimum at the equator.

If the Earth had a precisely spherical form, then a kilogram weight carried from a pole to the equator would lose 3.5 g in weight. You can easily find this number if you use the expression $4\pi^2 n^2 Rm$ and substitute $n = 1$ revolution per day, $R = 6300$ km, and $m = 1000$ g. Only don't forget to convert the units of measurements to seconds and centimetres.

However, a kilogram weight will actually lose 5.3 g, and not 3.5 g. This is the case because the Earth is an oblate sphere called an ellipsoid in geometry. The distance from a pole to the centre of the Earth is about 1/300 less than a terrestrial radius extended to the equator.

This contraction of the Earth was caused by the very same centrifugal force. In fact, it is exerted on all the particles of the Earth. In remote times, the centrifugal force "moulded" our planet—gave it an oblate form.

Coriolis Forces

The peculiarities of the world of rotating systems are not exhausted by the existence of radial gravitational forces. We shall become acquainted with still another interesting effect whose theory was presented in 1835 by the Frenchman Gaspard Gustave de Coriolis (1792-1843).

Let us pose the following question: What does rectilinear motion look like from the point of view of a rotating laboratory? A design of such a laboratory is depicted in Figure 2.14. The rectilinear trajectory of some body is shown by means of a ray passing through the centre. We are considering the case when the path of the body passes through the centre of rotation of our laboratory. The disc on which the laboratory is standing rotates uniformly; five positions of the laboratory with respect to the rectilinear trajectory are shown in the figure. This is how the relative positions of the laboratory and the trajectory of the body look after one, two, three, etc., seconds. The laboratory, as you see, is rotating counterclockwise if looked upon from above.

Arrows corresponding to the segments through which the body passes during one, two, three, etc., seconds have been drawn on the line of its path. The body covers the same distance during each second, since we are dealing with uniform and rectilinear motion from the point of view of a fixed observer.

Imagine that the moving body is a freshly painted ball rolling along the disc. What kind of trace will remain on the disc? Our construction yields the answer to this question. The points which mark the ends of the arrows

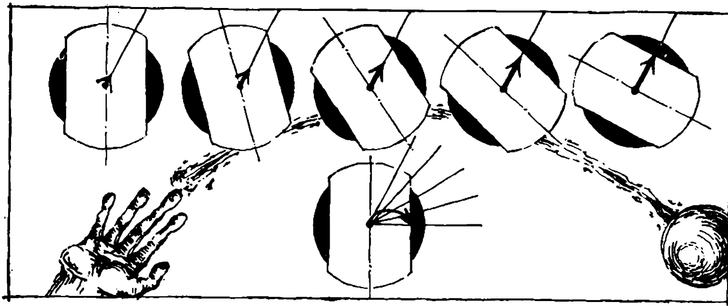


Figure 2.14

have been transferred from our five drawings to a single diagram. It remains to connect these points by a smooth curve. The result of our construction will not surprise us: rectilinear and uniform motion looks like curvilinear motion from the point of view of a rotating observer. The following rule attracts our attention: a moving body is deflected to the right of its path during the entire course of the motion. We now assume that the disc is rotating in the clockwise direction, and leave the repetition of our construction to the reader. It will show that, in this case, a moving body is deflected to the left of its path from the point of view of a rotating observer.

We know that a centrifugal force arises in rotating systems. However, its action cannot serve as the cause of the deformation of the path, for it is directed along the radius. Hence, besides the centrifugal force another additional force arises in rotating systems. It is called the *Coriolis force*.

Why is it that in the previous examples we did not come across the Coriolis force and managed superbly with only centrifugal? The reason is that until now we have not regarded motion from the point of view of a rotating observer, and a Coriolis force arises only in such a case. Only a centrifugal force is exerted on bodies which are stationary in rotating systems. A table in a rotating laboratory is screwed on to the floor—only a centrifugal force is exerted on it. But on a ball which has fallen from the table and rolled along the floor of the rotating laboratory besides a centrifugal force a Coriolis force is also exerted.

On what quantities does the magnitude of a Coriolis force depend? It can be calculated, but the computations are too complicated to be given here. We shall therefore present only the result of these computations.

Unlike a centrifugal force whose magnitude depends on the distance from the axis of rotation, a Coriolis force is independent of the position of a body. It is determined by the velocity vector (i.e. not only by its magnitude, but also by its direction with respect to the axis of rotation). If the body moves along the axis of rotation, the Coriolis force is equal to zero. The greater the angle between the velocity vector and the axis of rotation, the greater will be the Coriolis force; this force assumes its maximum value when the motion of the body is at right angles to the axis. As we know, it is always possible to decompose a velocity vector into any pair of its components and consider separately the two resulting motions in which the body is simultaneously involved.

If the velocity of a body is decomposed into components $v_{\parallel}$ and $v_{\perp}$—parallel and perpendicular to the axis of rotation—then the first motion will not be subject to the action of a Coriolis force. The magnitude of the Coriolis force F_C is determined by the component $v_{\perp}$ of the velocity. Computations lead to the formula

$$F_C = 4\pi n v_{\perp} m$$

Here m is the mass of the body, and n is the number of revolutions made by the rotating system in a unit of time. As can be seen from the formula, the faster the system rotates and the faster the body moves, the greater will be the Coriolis force.

Calculations also established the direction of a Coriolis force. This force is always perpendicular to the axis of rotation and the direction of the motion. Moreover, as has already been said above, the force is directed to the right of its path in a system rotating counterclockwise.

Many interesting phenomena occurring on the Earth are explained by the action of Coriolis forces. The Earth is a sphere, and not a disc. This makes the effect of Coriolis forces more complicated. These forces will not only influence motion along the Earth's surface but also the falling of bodies to the Earth.

Does a body fall exactly along a vertical? Not quite. Only at a pole does a body fall exactly along a vertical. Here the direction of the motion and the Earth's axis of rotation coincide, so there is no Coriolis force. The situation is different at the equator; here the direction of the motion forms right angles with the Earth's axis. If looked upon from the North Pole, the Earth's rotation will appear to be counterclockwise. Hence, a freely falling body should be deflected to the right of its path, i.e. to the East. The magnitude of this eastward deflection, the greatest at the equator, decreases to zero as the poles are approached.

Let us compute the magnitude of the deflection at the equator. Since a freely falling body moves with a uniform acceleration, the Coriolis force increases as the Earth is approached. We shall therefore restrict ourselves to an approximate computation. If the body falls from a height, say, of 80 m, its fall will last about 4 s according to the formula $t = \sqrt{2h/g}$. The average speed for the fall will be equal to 20 m/s.

This is the speed that we shall substitute in our formula for the Coriolis acceleration, $4\pi n v$. Let us convert the value $n = 1$ revolution in 24 hours to the number of revolutions per second; 24×3600 seconds are contained in 24 hours, so n is equal to 1/86 400 rps; consequently, the acceleration created by the Coriolis force is equal to $\pi/1080$ m/s^2. The distance covered during 4 s with such an acceleration is equal to $(1/2)(\pi/1080) \times 4^2 = 2.3$ cm. This is precisely the magnitude of the eastward deflection in our example. An exact computation, taking into account the non-uniformity of the fall, yields a close but somewhat different number.

While the deflection of a freely falling body is maximum at the equator and equal to zero at the poles, we shall see the opposite picture in the case of the deflection of a body moving in a horizontal plane under the action of a Coriolis force.

A horizontal site on the North or South Pole does not differ at all from the rotating disc with which we began our study of Coriolis forces. A body moving along such a site will be deflected to the right of its path by the Coriolis force at the North Pole, and to the left at the South Pole. Using the same formula for the Coriolis acceleration, the reader can calculate without difficulty that a bullet fired from a rifle with an initial speed of 500 m/s will be deflected from the target by 3.5 cm in a horizontal plane during one second (i.e. while it travels 500 m).

But why should the deflection in a horizontal plane at the equator be equal to zero? Without rigorous proofs, it is clear that this should be the case. At the North Pole a body is deflected to the right of its path, and at the South Pole to the left, hence, half-way between the poles, i.e. at the equator, the deflection will be equal to zero. Let us recall the experiment with the Foucault pendulum. A pendulum oscillating at a pole preserves the plane of its oscillations. The Earth in its rotation moves away from under the pendulum. This is how the stellar observer explains the Foucault experiment. But the observer

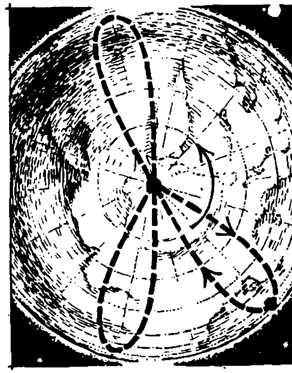


Figure 2.15

rotating together with the Earth explains this experiment by means of a Coriolis force. As a matter of fact, a Coriolis force is directed perpendicularly to the Earth's axis and perpendicularly to the direction of the motion of the pendulum; in other words, the force is perpendicular to the plane of the oscillation of the pendulum and will continually turn this plane. It can be arranged so that the end of the pendulum traces the trajectory of the motion. This trajectory is represented by the "rosette" shown in Figure 2.15. It can be seen from this figure that the "Earth" completes one quarter of a rotation during one and a half periods of the oscillation of the pendulum. The Foucault pendulum turns much more slowly. At a pole, the plane of oscillation of the pendulum will turn through one-fourth of a degree during one minute. At the North Pole the plane will be turned to the right of the path of the pendulum, and at the South Pole to the left.

The Coriolis effect will be somewhat less at Central European latitudes than at the equator. A bullet in the example we have just given will be deflected not by 3.5 cm but by 2.5 cm. The Foucault pendulum will

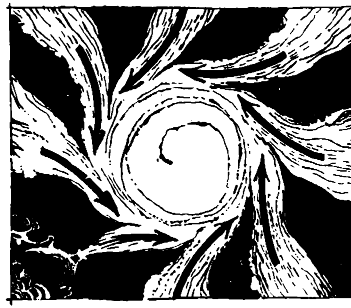


Figure 2.16

be turned by about one-sixth of a degree during one minute.

Must a gunner take the Coriolis force into account? Big Bertha used by the Germans to shell Paris during World War I was situated 110 km from the target. The Coriolis deflection is as much as 1600 m in such a case. This is no longer a small quantity. If a flying projectile is sent very far without taking the Coriolis force into account, it will be deflected significantly from its course. This effect is large not because the force is great (for a ten-ton projectile having a speed of 1000 km/h, the Coriolis force will be about 25 kgf) but because it is exerted continually for a long period of time.

Of course, the influence of wind on a rocket projectile may be no less significant. Flight corrections made by a pilot depend on the action of the wind, the Coriolis effect and imperfections in the airplane or flying bomb.

What specialists besides aviators and gunners should be aware of the Coriolis effect? Strange as it may seem, among such specialists are railroaders. Under the action of the Coriolis force, one of the rails of a railroad wears out on the inside noticeably more than the other. We know just which one: in the Northern Hemisphere it will be the right rail (relative to the motion of a train), and in the Southern Hemisphere the left one. Only the rail-

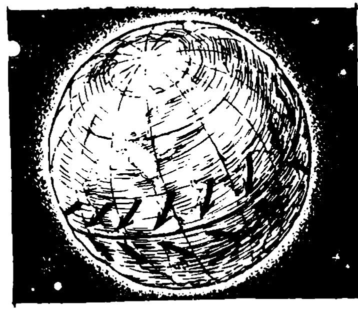


Figure 2.17

roaders in equatorial countries are saved from trouble in connection with this.

The washing away of right banks in the Northern Hemisphere is explained in exactly the same way as the wearing out of rails. The deviation of a river bed is to a large extent related to the action of the Coriolis force. It turns out that rivers in the Northern Hemisphere pass obstacles on the right.

It is known that streams of air flow into a low-pressure area. But why is such a wind called a cyclone? After all, the root of this word suggests a circular (cyclic) motion.

This is precisely the case—a circular motion of air masses arises in a low-pressure area (Figure 2.16). The cause lies in the action of the Coriolis force. In the Northern Hemisphere all air streams directed towards the low-pressure area are deflected to the right of their motion. Take a look at Figure 2.17—you see that this leads to a westward deflection of the winds blowing in both hemispheres from the tropics to the equator (trade-winds).

Why does such a small force play such a big role in the motion of air masses? This is explained by the insignificance of the frictional forces. Air is extremely mobile, and a small but constantly acting force can lead to important consequences.

3. Conservation Laws

Recoil

Even those who have not been at war know that when a gun is fired it jumps back abruptly. When a rifle is fired, recoil in the shoulder occurs. But it is possible to become acquainted with recoil without having recourse to firearms. Pour some water into a test tube, cork it up and suspend it horizontally on two threads (Figure 3.1). Now turn on a burner under the test tube, the water will begin boiling, and in a couple of minutes the cork will fly out in one direction, while the test tube will be deflected in the opposite direction.

The force which drove the cork out of the test tube is steam pressure. And the force deflecting the test tube is also steam pressure. Both motions arose under the action of one and the same force. The same thing also happens in shooting, only there the action is not that of steam but of gunpowder gas.

Recoil is an inevitable consequence of the principle of equality between an action and its reaction. If the steam acts on the cork, the cork also acts on the steam in the opposite direction, and the steam transmits this reaction to the test tube.

Perhaps the following objection occurs to you: Can one and the same force really lead to such dissimilar effects? The rifle moves backwards only slightly, but the bullet flies far away. We hope, however, that such an objection

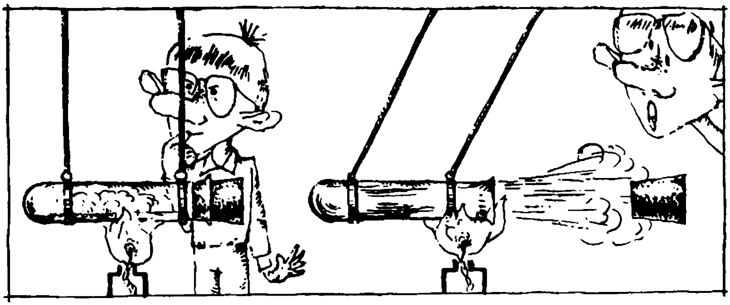


Figure 3.1

has not occurred to the reader. Identical forces certainly can lead to different effects: for the acceleration which a body receives (and this is precisely the effect of the action of the force) is inversely proportional to its mass. We must write out the acceleration of one of the bodies (shell, bullet, cork) in the form $a_1 = F/m_1$; the acceleration of the body experiencing recoil (gun, rifle, test tube) is then $a_2 = F/m_2$. Since the force is one and the same, we arrive at an important conclusion: the accelerations imparted by the interaction of two bodies participating in a "shot" will be inversely proportional to their masses:

$$\frac{a_1}{a_2} = \frac{m_2}{m_1}$$

This means that the acceleration imparted to the gun when it recoils will be as many times less than the acceleration of the shell as the gun weighs more than the shell.

The acceleration of the bullet, and also of the rifle during recoil, lasts as long as the bullet is moving through the muzzle. Let us denote this time by t. When this time has elapsed, the accelerated motion will become uniform.

For the sake of simplicity, we shall assume the acceleration to be constant. Then the speed with which the bullet flies out of the muzzle of the rifle is $v_1 = a_1 t$, and the speed of recoil is $v_2 = a_2 t$. Since the time during which the accelerations act is one and the same, then $v_1/v_2 = a_1/a_2$, and so

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}$$

The speeds with which the bodies fly apart after the interaction will be inversely proportional to their masses.

If we recall the vector nature of velocity, we can rewrite the last relation as follows: $m_1\mathbf{v}_1 = -m_2\mathbf{v}_2$; the minus sign indicates that the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ are oppositely directed.

Finally, let us rewrite our equation once again bringing the products of mass by velocity to one side:

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = 0$$

The Law of Conservation of Momentum

The product of the mass of a body by its velocity is called the *momentum* of the body (another name for it is *linear momentum*). Since velocity is a vector, momentum is also a vector quantity. Of course, the direction of the momentum coincides with that of the velocity of motion of the body.

With the aid of our new concept, Newton's law, $F = ma$, can be expressed differently. Since $a = (v_2 - v_1)/t$, we have $F = (mv_2 - mv_1)/t$, or $Ft = mv_2 - mv_1$. The product of the force by the duration of its action is equal to the change in the momentum of the body.

Let us return to recoil.

The result of our investigation of the recoil of a gun can now be formulated more concisely: the sum of the momenta of the gun and the shell will remain equal to zero after the firing. It is obvious that this was also the case before the firing, when the gun and the shell were in a state of rest.

The velocities occurring in the equation $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = 0$ are the velocities immediately after the firing. During the subsequent motion of the shell and the gun, the force of gravity and air resistance will begin acting on them, and the Earth will exert an additional frictional force on the gun. But if the shot were fired in a vacuum from a gun hanging in the void, the motion with the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ would continue arbitrarily long. The gun would move in one direction, and the shell in the opposite direction.

Guns mounted on a platform and firing while in motion are widely applied in current artillery practice. How should the equation we derived be changed in order that it be applicable to a shot fired from such a gun? We may write:

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = 0$$

where $\mathbf{u}_1$ and $\mathbf{u}_2$ are the velocities of the shell and the gun relative to the moving platform. If the velocity of the platform is $\mathbf{V}$, then the velocities of the shell and the gun relative to an observer who is at rest will be $\mathbf{v}_1 = \mathbf{u}_1 + \mathbf{V}$ and $\mathbf{v}_2 = \mathbf{u}_2 + \mathbf{V}$.

Substituting for $\mathbf{u}_1$ and $\mathbf{u}_2$ in our previous equation, we obtain:

$$(m_1 + m_2)\,\mathbf{V} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

In the right-hand side of the equation we have the sum of the momenta of the shell and the gun after the firing. And in the left-hand side? Before the firing, the gun and the shell with a total mass of $m_1 + m_2$ move together with the velocity $\mathbf{V}$. Therefore, in the left-hand side of the equation there is also the total momentum of the shell and the gun, but before the firing.

We have proved a very important law of nature, which is called the *law of conservation of momentum*. We proved it for two bodies, but it can easily be proved that the same result also holds for any number of bodies. What is the content of this law? The law of conservation of momentum asserts that the sum of the momenta of a number of interacting bodies does not change as a result of this interaction.

It is clear that the law of conservation of momentum will only be valid when no outside forces are exerted on the group of bodies under consideration. Such a group of bodies is called closed in physics.

A rifle and a bullet behave like a closed group of two bodies during a shooting in spite of the fact that they are subject to the Earth's gravitation. The weight of the bullet is small in comparison with the force exerted by gunpowder gases, and recoil occurs in accordance with one and the same laws, regardless of where the shot will be fired—on the Earth or in a rocket flying through interplanetary space.

The law of conservation of momentum allows us to easily solve various problems dealing with colliding bodies. Let us try to strike one clay ball with another—they will stick together and continue the motion together; if we shoot from a rifle at a wooden ball, it will roll together with the bullet stuck in it; a standing cart will roll if a person takes a running jump into it. All the examples we have given are very similar from the point of view of physics. The rule relating the velocities of the bodies involved in such kinds of collisions can be immediately obtained from the law of conservation of momentum.

The momenta of the bodies prior to their collision were $m_1\mathbf{v}_1$ and $m_2\mathbf{v}_2$, they united after the collision, and their total mass is equal to $m_1 + m_2$. Denoting the velocity of the united body by $\mathbf{V}$, we obtain:

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\,\mathbf{V}$$

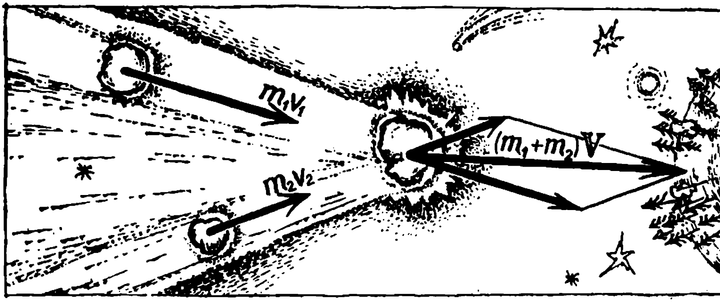


Figure 3.2

or

$$\mathbf{V} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$$

Let us recall the vector nature of the law of conservation of momentum. The momenta $m\mathbf{v}$ in the numerator of the formula must be added like vectors.

The "uniting hit" when bodies moving at an angle to each other meet is shown in Figure 3.2. In order to find the speed, we must divide the length of a diagonal of the parallelogram formed by the momentum vectors of the colliding bodies by the sum of their masses.

Jet Propulsion

A person moves by pushing off from the Earth; a boat sails because the rowers push against the water with their oars; a ship also pushes against the water, only not with oars but with propellers; a train moving on rails and an automobile also push off from the Earth—remember how hard it is for an automobile to get started on an icy road.

Thus, pushing off from a support seems to be a necessary

Figure 3.3

condition for motion; even an airplane moves by pushing the air with its propeller.

But is it really? Might there not be some intricate means of moving without pushing off from anything. If you ice-skate, you can easily convince yourself on the basis of your experience that such motion is quite possible. Pick up a heavy stick and get on the ice. Throw the stick forward—what will happen? You will glide backwards, although the thought of pushing against the ice with your foot didn't even cross your mind.

Recoil, which we have just studied, yields us the clue to carrying out motion without support, without pushing off. Recoil presents a possibility of accelerating motion even in a vacuum, where there really is absolutely nothing to push off from.

The recoil caused by a steam jet being driven out of a vessel (the reaction of the jet) was used back in Ancient Times for creating curious toys. An ancient steam turbine invented in the second century B.C. is pictured in Figure 3.3. A spherical cauldron was supported by a vertical axis. Escaping from the cauldron through elbow-shaped pipes, the steam pushed these pipes in the opposite direction, and the sphere rotated.

These days the use of jet propulsion has already gone far beyond the realm of the creation of toys and the collection of interesting observations. The twentieth century is sometimes called the century of atomic energy, but with no less reason one could call it the century of jet propulsion, since the far-reaching consequences of the use of powerful jet engines can scarcely be exaggerated. This is not only a revolution in aircraft construction but the beginning of mankind's contact with the Universe.

The principle of jet propulsion permits the creation of airplanes moving with a speed of several thousand kilometres per hour, flying missiles rising hundreds of kilometres above the Earth, artificial Earth satellites and cosmic rockets carrying out interplanetary flights.

A jet engine is a machine from which gases formed by the combustion of fuel are ejected with great force. The rocket moves in the direction opposite to that of the gas stream.

How strong is the thrust carrying the rocket off into space? We know that the force is equal to the change in momentum during a unit of time. According to our conservation law, the momentum of the rocket changes by the total momentum mv of the ejected gas.

This law of nature allows us to compute, for example, the relation between the force of the jet propulsion and the expenditure of fuel necessary for this. In doing so, one must assume a value for the speed of discharge of the combustion products. Let us take, say, the following values: the gases are ejected with a speed of 2000 m/s at the rate of 10 tons per second. Then the force in the jet propulsion will be about 2×10^{12} dyn, i.e. approximately 2000 tonf.

Let us determine the change in speed of a rocket moving in interplanetary space.

The momentum of the mass ΔM of gas ejected with speed u is equal to $u\,\Delta M$. The momentum of a rocket of mass M will increase by the amount $M\,\Delta V$. According to our conservation law, these two quantities must be equal to each other:

$$u\,\Delta M = M\,\Delta V, \quad \text{i.e.} \quad \Delta V = u\frac{\Delta M}{M}$$

However, if we wish to compute the speed of a rocket when the ejected mass is comparable to the mass of the rocket, the formula we have derived turns out to be useless. In fact, it assumes that the mass of the rocket is constant. However, the following important result remains valid: identical relative changes in mass lead to one and the same change in speed.

A reader acquainted with the basics of integral calculus will at once obtain the true formula. It has the form

$$V = u\ln\frac{M_{\text{in}}}{M} = 2.3u\log\frac{M_{\text{in}}}{M}$$

If you use a slide rule, you will find that when the mass of the rocket is cut in half, its speed will reach $0.7u$.

In order to raise the speed of the rocket to $3u$, it is necessary to burn up a mass $m = (19/20)\,M$. This means that only one-twentieth of the mass of the rocket can be preserved if we wish to raise its speed to $3u$, i.e. to 6-8 km/s.

In order to attain a speed of $7u$, the mass of the rocket must decrease by 1000 times during the speed-up.

These calculations warn us against striving to increase the mass of the fuel which can be put in the rocket. The more fuel we take, the more we must burn. For a given speed of gas ejection, it is very difficult to achieve an increase in the speed of the rocket.

The increase in the speed of gas ejection is the basic means of attaining high rocket speeds. In this respect, a significant role must be played by the application to rockets of engines running on a new atomic fuel.

For a constant speed of gas ejection, a gain in speed with the same mass of fuel is obtained by using multi-stage rockets. In a single-stage rocket, the mass of the fuel decreases, but the empty tanks keep moving with the rocket. An additional energy is required to accelerate the mass of the unnecessary fuel tanks. It would be expedient to throw away the fuel tanks whose fuel has been consumed. In modern multi-stage rockets, not only are the fuel tanks and piping thrown away but also the engines of the used stages.

Of course, it would be best to continuously throw away the unnecessary mass of the rocket. Such a construction does not yet exist. The take-off weight of a three-stage rocket can be made six times less than that of a single-stage rocket with the same "ceiling" A "continuous" rocket would be more profitable in this sense than a three-stage rocket by an additional 15%.

Motion Under the Action of Gravity

We shall roll a small cart down two very smooth inclined planes. Let us take two boards, one much shorter than the other, and place them on one and the same support. Then one inclined plane will be steep, and the other will be gently sloping. The tops of both boards—the starting places of the cart—will be at the same height. In which case do you suppose will the cart acquire the greater speed by rolling down its inclined plane? Many people will decide that it will be the one which rolls down the steeper board.

An experiment will show that they are wrong—in both cases the cart will acquire the same speed. While a body is moving along an inclined plane, it is subject to the action of a constant force, namely (Figure 3.4), the com-

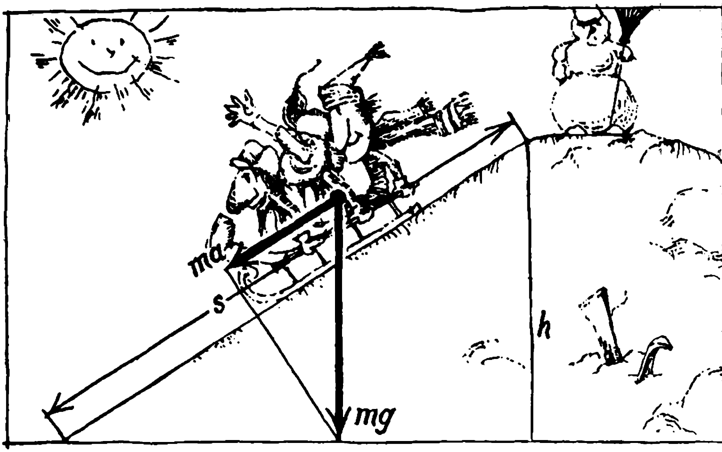


Figure 3.4

ponent of gravity directed along the line of its motion. The speed v which a body acquires moving with acceleration a along a path of length s is equal, as we know, to $\sqrt{2as}$.

What makes it evident that this magnitude does not depend on the angle of inclination of the plane? We see two triangles in Figure 3.4. One of them depicts the inclined plane. The small leg of this triangle denoted by h is the height from which the motion begins; the hypotenuse s is the path through which the body passes in its accelerated motion. The small force triangle with leg ma and hypotenuse mg is similar to the large one, since they are right triangles, and their angles as angles with mutually perpendicular sides are equal. Hence, the ratio of the legs should be equal to that of the hypotenuses, i.e.

$$\frac{h}{ma} = \frac{s}{mg}, \quad \text{or} \quad as = gh$$

We have proved that the product *as*, and hence the final speed of a body rolling down an inclined plane, is independent of the angle of inclination but depends only on the height from which the downward motion began. The speed $v = \sqrt{2gh}$ for all inclined planes subject to the sole condition that the motion began from one and the same height h. This speed turned out to be equal to the speed of free fall from height h.

Let us measure the speed of a body at two places on the inclined plane—at heights h_1 and h_2. Denote the speed of the body when it passes through the first point by v_1, and its speed when it passes through the second point by v_2.

If the initial height from which the motion began is h, the square of the speed of the body at the first point will be $v_1^2 = 2g(h - h_1)$, and at the second point $v_2^2 = 2g(h - h_2)$. Subtracting the former from the latter, we shall find out how the speeds of the body at the initial and end points of an arbitrary piece of an inclined plane are related to the heights of these points:

$$v_2^2 - v_1^2 = 2g(h_1 - h_2)$$

The difference between the squares of the speeds depends only on the difference in height. Note that the equation we have obtained is equally suitable for upward motion and downward motion. If the first height is less than the second (ascent), the second speed is less than the first.

This formula can be rewritten in the following way:

$$\frac{v_1^2}{2} + gh_1 = \frac{v_2^2}{2} + gh_2$$

We wish to emphasize by means of this formulation that the sum of half the square of the speed and g times the height is identical for all the points on the inclined plane. One may say that the quantity $(v^2/2) + gh$ is conserved during the motion.

What is most remarkable in the law we have found is that it is valid for frictionless motion on any hill and, in general, along any path consisting of alternating ascents and descents of various slopes. This follows from the fact that any path can be broken up into rectilinear portions. The smaller we take the segments, the closer will the broken line approximate the curve. Each straight line segment into which the curvilinear path has been broken up may be regarded as part of an inclined plane, and the rule we have found may be applied to it.

Therefore, the sum $(v^2/2) + gh$ is identical for all the points of the trajectory. Consequently, a change in the square of the speed does not depend on the form or length of the path along which a body moved but is determined solely by the difference in height of the initial and end points of het motion.

It may seem to the reader that our conclusion does not coincide with his daily experience: on a long, gently sloping path a body does not gather any speed at all, and eventually comes to a halt. This is the way things are, but we haven't taken the force of friction into account in our reasoning. The above formula is valid for motion within the Earth's gravitational field under the action of only the single force of gravity. If the frictional force is small, the derived law will be satisfied rather well. A sled with metal runners slides down smooth icy mountains with very little friction. It is possible to build long icy paths that begin with a steep descent on which a great speed is gathered and then twist up and down fantastically. The end of a trip on such a hill (when the sled stops by itself) would occur at a height equal to that of the start, provided that friction were entirely absent. But since it is impossible to avoid friction, the point at which the motion of the sled started will be higher than the place where it stops.

The law which asserts that the final speed of a motion

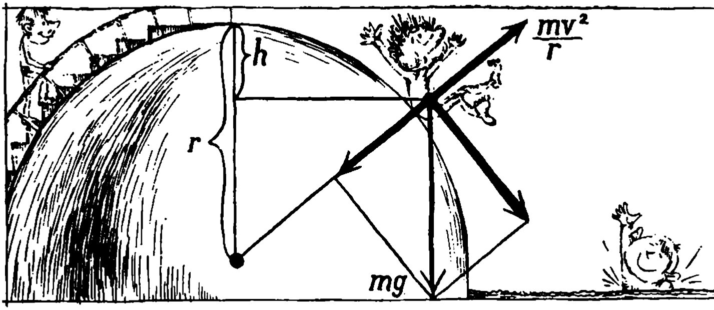


Figure 3.5

subject to the force of gravity is independent of the form of the path can be applied to the solution of various interesting problems.

"Looping-the-loop" in a vertical circle has been frequently presented at circuses as an exciting stunt. A cyclist or a cart with an acrobat in it is placed on a high platform. He then accelerates while descending. Now he is ascending. Look, he is in an upside-down position, then again a descent, and the loop has been looped. Let us consider a problem which a circus engineer must solve. At what height should the platform from which the descent begins be made, so that the acrobat might loop-the-loop within falling? We know a necessary condition: the centrifugal force pressing the acrobat against the cart must balance the oppositely directed gravitational force. Hence $mg \leqslant mv^2/r$, where r is the radius of the loop, and v is the speed of the motion at the top of the loop. In order that this speed be attained, it is necessary to begin the motion from a place which is a certain quantity h higher than the top of the loop. Since the initial speed of the acrobat is equal to zero, we have $v^2 = 2gh$ at the top of the loop. But, on the other hand, $v^2 \geqslant gr$. Hence, between the height h and the radius of the loop there is the relation $h \geqslant r/2$. The platform must be raised by at least half the radius of the loop above the top of the loop. Taking into account the inevitable frictional force, we shall, of course, have to choose our height with a margin of safety.

And here is another problem. Let us take a large, very smooth dome so that friction is minimum. Let us place a small object at the top and give it the opportunity of sliding down the dome by means of hardly noticeable push. Sooner or later the sliding body will get detached from the dome and start falling. We can easily answer the question as to just when the body breaks away from the surface of the dome: at the moment of the break the centrifugal force must equal the radial component of the weight (at this instant the body will cease pressing the dome, and this is precisely the moment of the break). Two similar triangles can be seen in Figure 3.5; the moment of the break is depicted. Let us form the ratio of a leg to the hypotenuse for the force triangle and set it equal to the corresponding ratio for the other triangle:

$$\frac{mv^2/r}{mg} = \frac{r-h}{r}$$

Here r is the radius of the spherical dome, and h is the difference in height between the start and finish of the sliding. Let us now make use of the independence of the final speed of the form of the path. Since the initial speed of a body is assumed equal to zero, we have $v^2 = 2gh$. Substituting this value in the above proportion and performing arithmetical transformations, we find $h = r/3$. Hence, the body will break away from the dome at a height which is one-third of a radius lower than the top of the dome.

The Law of Conservation of Mechanical Energy

We have convinced ourselves in the examples just considered how helpful it is to know a quantity not changing its numerical value (conserving it) throughout a motion.

So far we know such a quantity for one body only. But if several associated bodies are moving within a gravitational field? It is evident that we may not assume that the expression $(v^2/2) + gh$ remains constant for each of them, since each of the bodies is subject to the action of not only the force of gravity but also of the neighbouring bodies. Perhaps the sum of such expressions taken over the group of bodies under consideration is conserved?

We shall now show that this assumption is false. There exists a quantity conserved throughout the motion of many bodies; however, it is not equal to the sum

$$\left(\frac{v^2}{2} + gh\right)_{\text{body 1}} + \left(\frac{v^2}{2} + gh\right)_{\text{body 2}} +$$

but rather to the sum of such expressions multiplied by the masses of the corresponding bodies; in other words, the sum

$$m_1\left(\frac{v^2}{2} + gh\right)_1 + m_2\left(\frac{v^2}{2} + gh\right)_2 + \cdots$$

is conserved.

For the proof of this important law of mechanics, we turn to the following example.

Two loads are connected by a cord passing over a pulley, the large one of mass M, and the small one of mass m. The large load pulls the small one, and this group of two bodies will move with increasing speed.

The driving force is the difference in weight of these bodies, $Mg - mg$. Since the masses of both bodies participate in the accelerated motion, Newton's law for this case will be written out as follows:

$$(M-m)g=(M+m)a$$

Let us consider two instants during the motion and show that the sum of the expressions $(v^2/2)+gh$ multiplied by the corresponding masses really remains unchanged. Thus, it is required to prove the equality

$$m\left(\frac{v_2^2}{2}+gh_2\right)+M\left(\frac{V_2^2}{2}+gH_2\right)=$$
$$=m\left(\frac{v_1^2}{2}+gh_1\right)+M\left(\frac{V_1^2}{2}+gH_1\right)$$

Capital letters denote physical quantities characterizing the large load. The subscripts 1 and 2 refer here to the two instants which we are considering.

Since the loads are connected by a cord, $v_1=V_1$ and $v_2=V_2$. Using these simplifications and transferring all summands containing heights to the right-hand side, and summands with speeds to the left-hand side, we obtain:

$$\frac{m+M}{2}(v_2^2-v_1^2)=mgh_1+MgH_1-mgh_2-MgH_2=$$
$$=mg(h_1-h_2)+Mg(H_1-H_2)$$

The differences in height of the loads are, of course, equal (but opposite in sign since one load rises and the other falls). Therefore,

$$\frac{m+M}{2}(v_2^2-v_1^2)=g(M-m)s$$

where s is the distance covered.

We learned on p. 61 that the difference between the squares of the speeds at the initial and end points of a segment of length s of a path traversed with acceleration a

is equal to:

$$v_1^2 - v_2^2 = 2as$$

Substituting this expression in the preceding formula, we find:

$$(m + M)\, a = (M - m)\, g$$

But this is Newton's law, which we have written out above for our example. With this we have proved what was required: for two bodies the sum of the expressions $(v^2/2) + gh$ multiplied by the corresponding masses* remains constant during the motion, or, as one says, is conserved, i.e.

$$\left(\frac{mv^2}{2} + mgh\right) + \left(\frac{MV^2}{2} + MgH\right) = \text{const}$$

For the case of a single body, this formula reduces to the one proved earlier:

$$\frac{v^2}{2} + gh = \text{const}$$

Half the product of the mass by the square of the speed is called the *kinetic energy* K:

$$K = \frac{mv^2}{2}$$

The product of the weight of a body by its height is called the *potential energy* U of the gravitational attraction of the body to the Earth:

$$U = mgh$$

We have proved that during the motion of a two-body system (and it is possible to prove the same thing for a

*Of course, the expression $(v^2/2) + gh$ could equally well be multiplied by $2m$, or $m/2$, and, more generally, by an arbitrary factor. We agreed to act in the simplest manner, i.e. to multiply it simply by m.

system consisting of many bodies) the sum of the kinetic and potential energies of the bodies remains constant.

In other words, an increase in the kinetic energy of a group of bodies can only occur at the expense of a decrease in the potential energy of this system (and, of course, conversely).

The law just proved is called the *law of conservation of mechanical energy.*

The law of conservation of mechanical energy is a very important law of nature. We have not yet shown its significance in full measure. Later, when we have become acquainted with the motion of molecules, its universality and its applicability to all natural phenomena will be evident.

Work

If we push or pull a body meeting no hindrance to what we are doing, the result will be an acceleration of the body. The increase in kinetic energy taking place in this connection is called the *work* A performed by the force:

$$A = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$$

According to Newton's law, the acceleration of a body and hence also the increase in its kinetic energy, is determined by the vector sum of all the forces applied to it. Therefore, in the case of many forces, the formula $A = (mv_2^2/2) - (mv_1^2/2)$ expresses the work performed by the resultant force. Let us express the work A in terms of the magnitude of the force.

For the sake of simplicity, we shall restrict ourselves to the case when motion is possible only in one direction—we shall push (or pull) a cart of mass m, standing on rails (Figure 3.6).

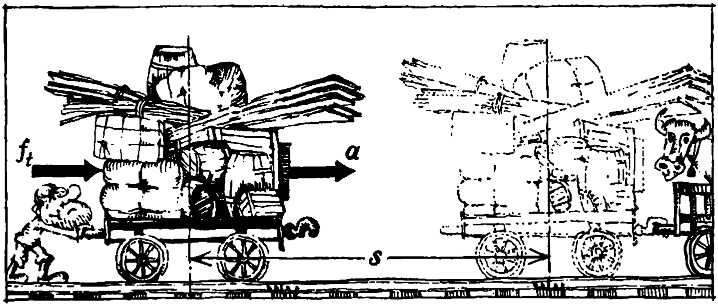


Figure 3.6

According to our general formula for uniformly accelerated motion, $v_1^2 - v_1^2 = 2as$. Therefore, the work performed by all the forces over a distance s is

$$A = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = mas$$

The product ma is equal to the component of the total force in the direction of the motion. Consequently, $A = f_t s$.

The work done by a force is measured by the product of the distance by the component of the force along the direction of the displacement.

This formula for the work is valid for forces of any origin and for motions along any trajectory.

Note that the work may be equal to zero even when forces act on a moving body.

For example, the work done by a Coriolis force is equal to zero, because such a force is perpendicular to the direction of the motion. It has no tangential component, so the work is equal to zero.

Any twist in the trajectory which is not accompanied by a change in speed requires no work, for the kinetic energy does not change under such conditions.

Can work be negative? Of course, if the force is directed at an obtuse angle to the motion, then it does not help but hinders the motion. The tangential component of the force in the direction of the motion will be negative. In this case we do say that the force performs negative work. The force of friction always slows down a motion, i.e. does negative work.

On the basis of the increase in kinetic energy, one can only judge the work done by the resultant force.

As for the work done by the individual forces, we should compute them as the products $f_t s$. An automobile is moving uniformly along a highway. There is no increase in kinetic energy, so the work done by the resultant force is equal to zero. But the work done by the motor is, of course, not equal to zero—it is equal to the product of the thrust of the motor by the distance covered, and is fully compensated by the negative work done by the force of friction and resistance.

Using the concept of "work", we can describe more briefly and clearly the interesting peculiarities of the gravitational force with which we have just become acquainted. If a body goes from one place to another under the action of gravity, its kinetic energy will change. This change in kinetic energy is equal to the work A. But we know from the law of conservation of energy that an increase in kinetic energy takes place at the expense of a decrease in potential one.

Therefore, the work done by gravity is equal to the decrease in potential energy:

$$A = U_1 - U_2$$

It is obvious that a loss (or gain) of potential energy, and hence an increase (or decrease) in kinetic energy, will be the same, regardless of the path along which a body moved. This implies that the work performed by gravity does not depend on the form of the path. If a body went from the first point to the second with an increase in kinetic energy, it will go from the second point to the first with a decrease in kinetic energy by exactly the same amount. Moreover, it makes no difference whether or not the form of the path "there" coincides with the form of the path "back". Hence, the work "there" and "back" will also be identical. And if the body takes a long trip with the initial and end points of its path coinciding, the work will be equal to zero.

Imagine a canal whose form is as fantastic as possible, through which a body slides without friction. Let us send it off on a trip from the highest point. The body rushes downwards gathering speed. At the expense of the kinetic energy so obtained, the body will surmount ascents and return finally to the station where it departed. With what speed? With the same, of course, with which it left the station. Its potential energy will return to its previous value. But if so, then its kinetic energy could neither have decreased nor increased. Hence, the work is equal to zero.

Not for all forces is the work done along a circular (physicists say: a closed) path equal to zero. There is no need to prove that the longer the path, the greater will be the work performed by friction, for example.

In What Units Work and Energy Are Measured

Since work is equal to the change in energy, then work and energy—potential as well as kinetic, of course—are measured in one and the same units. Work is equal to the product of a force by a distance. The work done by a force of one dyne over a distance of one centimetre is called the *erg*:

$$1 \text{ erg} = 1 \text{ dyn} \cdot 1 \text{ cm}$$

This is a very small work. Such a work is performed against gravity by a mosquito in order to fly from the thumb to the forefinger of someone's hand. A larger unit of work and energy used in physics is the *joule* (J). It is 10 million times as great as an erg:

$$1\ \text{J} = 10^7\ \text{ergs}$$

A unit of work which is quite often used is 1 *kilogram-force-metre* (1 kgf-m). This is the work which a force of 1 kgf performs in a displacement of 1 m. About this much work is done by a kilogram weight falling off a table to the floor.

As we know, a force of 1 kgf = 981 000 dyn, 1 m = = 100 cm. Hence, 1 kgf-m = 9.81×10^7 ergs = 9.81 J. Conversely, 1 J = 0.102 kgf-m.

The SI system of units requires that we drop the kilogram-force-metre as the unit of work and energy and use the joule instead, 1 J is the work done by a force of 1 N over a distance of 1 m. Knowing how easily force is defined in this case, one has no difficulty in understanding the reason for the advantages of the SI system of units.

Power and Efficiency of Machines

To estimate the potential of a machine to perform work and the consumption of energy, the concept of power was introduced. *Power* is work per unit time, or the time rate of doing work.

There are many different units of power. In the cgs system, the unit is the erg per second (erg/s). One erg per second, however, is very little power and, hence, not useful in practical life. A unit in much wider use is the joule per second, or *watt* (W): 1 W = 1 J/s = 10^7 erg/s. When this unit is not enough, it is multiplied by a thousand. The new unit is called a *kilowatt* (kW).

From the early days of technology we have inherited the unit of power called *horsepower* (hp). This name had a special meaning at that time. A person buying a 10-horsepower machine would conclude that it took the place of 10 horses, even if he knew nothing of units of power. Naturally, there are no two horses alike. The first person to introduce this unit of power apparently thought that the average horse can do 75 kgf-m of work per second. Thus, one horsepower was arbitrarily defined as 75 kgf-m/s. A heavy draught-horse can work at a rate greater than 1 hp; especially when starting. But the power of an average horse is about 0.5 hp. The relation of horsepower to kilowatt is 1 hp = 0.735 kW.

In everyday life and in technology we deal with a great variety of machines. The motor of the turntable of a record player has a power output of about 10 W, the engine of the Soviet car *Volga* 100 hp or 73 kW, and the engines of the Soviet passenger airliner "IL-18" 16 000 hp. A small electric power station used to supply a cooperative farm with electricity has a power output of about 100 kW, whereas the Krasnoyarsk hydroelectric plant on the Yenisei river in Siberia has a record power output of 5 million kilowatts.

The units of power we have elaborated on give us a clue to a unit of work or energy used exclusively for electricity, namely the kilowatt-hour (kW-h). A *kilowatt-hour* is the work produced by a source with the power output of one kilowatt in the course of one hour. From this new unit it is easy to transfer to the old ones: 1 kW-h = 3.6×10^6 J = 861 kcal = 367 000 kgf-m.

But with so many units of energy was there any need to introduce one more? Yes, there was. The idea of energy is used in a great variety of fields of physics, so for the sake of convenience physicists introduced a new unit for each field. The same happened with other units of measurement. Finally, there appeared a need for a unified system of units (the SI system) for all fields of physics. Some time will have to pass, however, before the "old" units will make way for the favoured one, the joule. The kilowatt-hour is thus not the last "outsider" that the reader will meet in his study of physics.

What are machines needed for? Obviously, to use sources of energy to do work: to lift loads, move other machines, or transport cargo or passengers. For any machine the amount of energy supplied to it and the output work done by it can be calculated. In all cases the work output is less than the work input: part of the energy is lost in the machine. The ratio of the work output for any machine to the work input is called *efficiency* and is usually expressed in per cent. For example, a machine whose efficiency is 90 per cent loses only 10 per cent of the input energy. On the other hand, an efficiency of 10 per cent means that the machine uses only 10 per cent of the input energy.

The efficiency of a machine that transforms mechanical energy into work can be made very high if the unavoidable friction is reduced. We can bring the efficiency closer to 100 per cent by improving lubrication, using better bearings, reducing the resistance of the medium in which the movement takes place, etc.

When mechanical energy is transformed into work, there is often an intermediate stage (as in hydroelectric plants), namely transmission of electric energy. Naturally, this stage introduces new losses. But these are small, so that even when this stage is present, the total loss in transforming mechanical energy into work can be brought down to a small percentage.

Energy Loss

The reader has probably noticed that while illustrating the law of conservation of mechanical energy we persistently repeat: "in the absence of friction, if there were no friction... ." But friction inevitably accompanies any motion. What is the significance of a law which doesn't take into account such an important practical circumstance? We shall put off answering this question and consider now some consequences of friction.

Frictional forces are directed against motion, and so perform negative work. This causes an unavoidable loss of mechanical energy.

Will this inevitable loss of mechanical energy lead to a cessation of the motion? It is not difficult to convince oneself that not every motion can be stopped by friction.

Imagine a closed system consisting of several interacting bodies. The law of conservation of momentum is valid, as we know, in relation to such a closed system. A closed system cannot change its momentum, so it moves rectilinearly and uniformly. Friction within such a system can change relative motions of parts of the system, but cannot affect the speed and direction of the motion of the entire system as a whole.

There exists still another law of nature, called the *law of conservation of angular momentum* (we shall make its acquaintance later), which does not permit friction to destroy the uniform rotation of an entire closed system.

Therefore, the presence of friction leads to the cessation of all motions within a closed system of bodies, not obstructing only the uniform rectilinear and the uniform rotational motion of this system as a whole.

If the Earth does slightly change the speed of its rotation, the cause of this is not the friction exerted by terrestrial bodies against one another, but the fact that the Earth is not an isolated system.

As for the motions of bodies on the Earth, they are all subject to friction and lose their mechanical energy. Therefore, such motion will always cease if not supported from without.

This is a law of nature. But if one succeeded in tricking nature? Then ... then one might be able to bring about perpetuum mobile, which is Latin for "perpetual motion"

Perpetuum Mobile

Bertold, a hero of Pushkin's *Scenes from the Days of Knighthood*, dreamed of bringing about perpetuum mobile. "What is perpetuum mobile?" asks his interlocutor. "It is perpetual motion," answers Bertold. "If I find perpetual motion, I see no bounds to human creativity. To make gold is a tempting problem, a discovery can be curious and profitable, but to find a solution to the problem of perpetuum mobile... ."

Perpetuum mobile, or a perpetual motion machine, is a machine working not only contrary to the law of loss of mechanical energy, but also in violation of the law of conservation of mechanical energy, which, as we now know, holds only under ideal unattainable conditions—in the absence of friction. A perpetual motion machine must, as soon as it is constructed, begin working "by itself", for example, turning a wheel or lifting up a load. This work should take place perpetually and continually, and the machine should require neither fuel nor human hands nor the energy of falling water—in short, nothing got from without.

The earliest reliable document known so far dealing with the "realization" of a perpetual motion machine goes back to the 13th century. It is a curious fact that after six centuries, in 1910, exactly the same "project" was presented for "consideration" in one of Moscow's scientific institutions.

The project for this perpetual motion machine is depicted in Figure 3.7. As the wheel rotates, the loads are thrown back and, according to the inventor, support the motion, since these loads, acting at a greater distance from

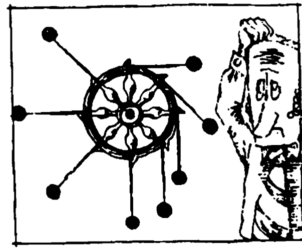


Figure 3.7

the axis, press down much harder than the others. Having constructed this by no means complicated "machine", the inventor convinces himself that after turning once or twice by inertia, the wheel comes to a halt. But this does not make him lose heart. An error has been committed: the levers should have been made longer, the protuberances must be changed in form. And the fruitless labour to which many self-made inventors have devoted their lives continues, but of course with the same success.

On the whole, there have not been many variants of proposed perpetual motion machines: various self-moving wheels not differing in principle from the one described; hydraulic machines, for example, the machine shown in Figure 3.8, which was invented in 1634; machines using siphons or capillary tubes (Figure 3.9), the loss of weight in water (Figure 3.10) or the attraction of iron bodies to magnets. It is by no means always possible to guess at the expense of what, according to the inventor, the perpetual motion should have occurred.

Even before the law of conservation of energy was established, we find the assertion of the impossibility of perpetuum mobile in an official declaration of the French Academy, made in 1775, when it decided not to accept any more projects for perpetual motion machines to be examined and tested.

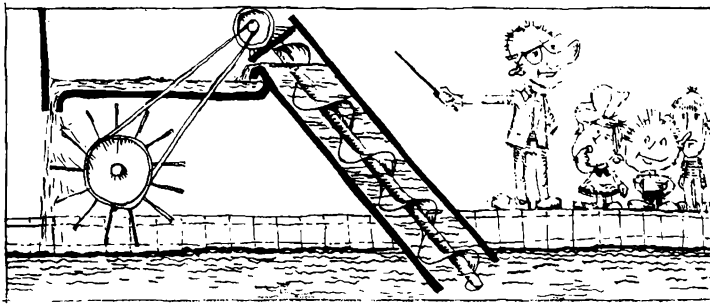


Figure 3.8

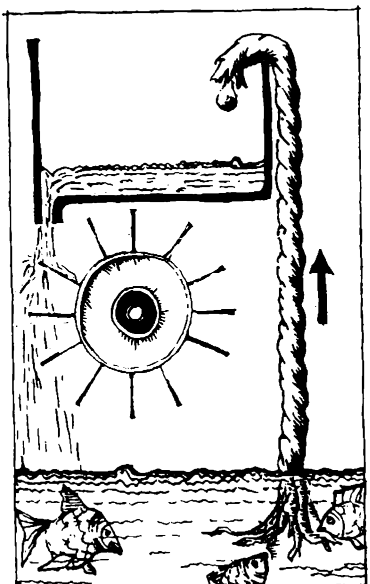


Figure 3.9

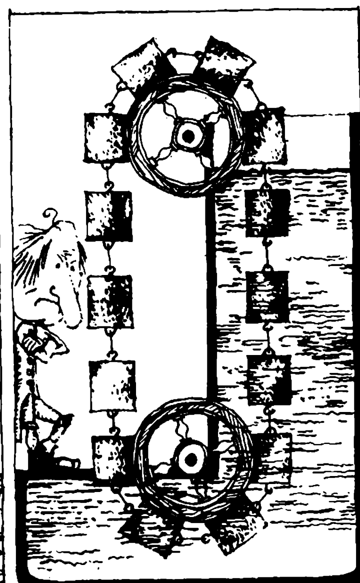


Figure 3.10

Many 17th and 18th century physicists had already assumed the axiom of the impossibility of perpetuum mobile as a basis of their proofs, in spite of the fact that the concept of energy and the law of conservation of energy entered science much later.

At the present time it is clear that inventors who try to create a perpetual motion machine not only come into contradiction with experiment, but also commit an error in elementary logic, for the impossibility of perpetuum mobile is a direct consequence of the laws of mechanics, which is what they proceed from in justifying their "inventions".

In spite of their complete fruitlessness, searches for perpetual motion machines probably played, nevertheless, some sort of useful role, since they led in the final analysis to the discovery of the law of conservation of energy.

Collisions

Momentum is conserved in every collision between two bodies. As for energy, it will necessarily decrease, as we have just explained, because of various kinds of friction.

However, if the colliding bodies are made of elastic material, say of ivory or steel, the energy loss will be insignificant. Such collisions, for which the sums of the kinetic energies before and after the collision are identical, are called *ideally elastic*.

A small loss of kinetic energy takes place even in collisions of the most elastic materials; it reaches, for example, 3-4% with ivory billiard balls.

The conservation of kinetic energy in elastic collisions permits us to solve a number of problems.

Consider, for example, a head-on collision between balls of different mass. The momentum equation has the form (we assume that the second ball has been stationary prior to the collision)

$$m_1v_1 = m_1u_1 + m_2u_2$$

and the energy equation

$$\frac{m_1v_1^2}{2} = \frac{m_1u_1^2}{2} + \frac{m_2u_2^2}{2}$$

where v_1 is the speed of the first ball before the collision, and u_1 and u_2 are the speeds of the balls after the collision.

Since the motion takes place along a straight line (the one passing through the centres of the balls—this is just what is meant by a head-on collision), the bold-face type denoting vectors has been replaced by italics.

From the first equation we have:

$$u_2 = \frac{m_1}{m_2}(v_1 - u_1)$$

Substituting this expression for u_2 in the energy equation, we obtain:

$$\frac{m_1}{2}(v_1^2 - u_1^2) = \frac{m_2}{2}\left[\frac{m_1}{m_2}(v_1 - u_1)\right]^2$$

One of the solutions of this equation is $u_1 = v_1$, which yields $u_2 = 0$. But this answer doesn't interest us, since the equalities $u_1 = v_1$ and $u_2 = 0$ imply that the balls did not collide at all. We therefore look for another solution of the equation. Dividing by $m_1(v_1 - u_1)$, we obtain:

$$\frac{1}{2}(v_1 + u_1) = \frac{1}{2}\frac{m_1}{m_2}(v_1 - u_1)$$

i.e.

$$m_2v_1 + m_2u_1 = m_1v_1 - m_1u_1$$

or

$$(m_1 - m_2)v_1 = (m_1 + m_2)u_1$$

which yields the following value for the speed of the first ball after the collision:

$$u_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

In a head-on collision with a stationary ball, the moving ball rebounds (negative u_1) if its mass is less. If m_1 is greater than m_2, both the balls continue the motion in the direction of the collision.

In case of an exact head-on collision during a game of billiards, one often observes the following scene: the driving ball comes to a sudden stop, and the target ball heads for a pocket. This is explained by the equation we have just found. The masses of the balls are equal, the equation yields $u_1 = 0$, and so $u_2 = v_1$. The colliding ball halts, and the second ball begins its motion with the former's previous speed. It is as though the balls have exchanged speeds.

Let us consider another example of a collision between bodies in accordance with the law of elastic collisions, namely an oblique collision between bodies of equal mass (Figure 3.11). The second body was stationary prior to the collision, so the laws of conservation of momentum and energy have the form:

$$m\mathbf{v}_1 = m\mathbf{u}_1 + m\mathbf{u}_2$$

$$\frac{mv_1^2}{2} = \frac{mu_1^2}{2} + \frac{mu_2^2}{2}$$

Cancelling the mass, we obtain:

$$\mathbf{v}_1 = \mathbf{u}_1 + \mathbf{u}_2$$

$$v_1^2 = u_1^2 + u_2^2$$

Vector $\mathbf{v}_1$ is the vector sum of $\mathbf{u}_1$ and $\mathbf{u}_2$, but this means that the lengths of the velocity vectors form a triangle.

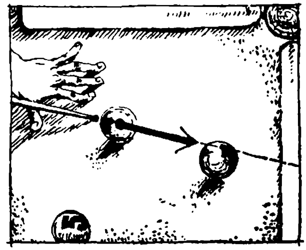


Figure 3.11

What kind of triangle is this? Recall the Pythagorean theorem. Our second equation is an expression of it. This means that the velocity triangle must be a right triangle with hypotenuse $\mathbf{v}_1$ and legs $\mathbf{u}_1$ and $\mathbf{u}_2$. Hence, $\mathbf{u}_1$ and $\mathbf{u}_2$ form right angles with each other. This interesting result shows that in any oblique elastic collision, bodies of equal mass fly apart at right angles.

4. Oscillations

Equilibrium

In certain cases it is very difficult to maintain an equilibrium—try to walk across a tightrope. At the same time, nobody rewards a person sitting in a rocking-chair with applause. But he is also maintaining his equilibrium.

What is the difference between these two examples? In which case is equilibrium maintained "by itself"?

The condition for equilibrium seems to be obvious. For a body not to be displaced from its position, the forces exerted on it must balance; in other words, the sum of these forces must be equal to zero. This condition is really necessary for the equilibrium of a body, but is it sufficient?

A side-view of a hill easily built out of cardboard paper is depicted in Figure 4.1. A ball will behave in different ways depending on the part of the hill where it is placed. A force which makes it roll down will be exerted on the ball at any point on the slope of the hill. This active force is gravity, or rather its projection on the tangent to the section of the hill passing through the point which is of interest to us. It is therefore clear that the more gentle the slope, the smaller the force acting on the ball.

We are interested above all in the points at which the force of gravity is completely balanced by the reaction of the support, and hence the resultant force acting on the ball is equal to zero. This condition will be fulfilled at the top of the hill and at its lowest points—the hollows.

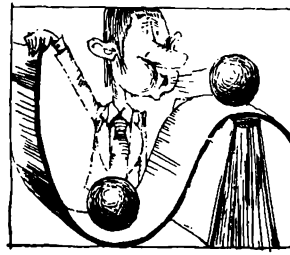


Figure 4.1

The tangents are horizontal at these points, and the resultant forces acting on the ball are equal to zero.

However, in spite of the fact that the resultant force is equal to zero at the top, we won't be able to put a ball there, but even if we could, we would immediately detect the accessory cause of our success—friction. A small push or a light puff will overcome the frictional force, and the ball will leave its place and roll down.

For a smooth ball on a smooth hill, only the low points of the hollows will be positions of equilibrium. If a push or an air stream displaces the ball from such a position, it will return there by itself.

A body in a hollow (a hole or a depression) is undoubtedly in equilibrium. If we deflect it from such a position, a force returns it back. The picture is different at the top of the hill: if a body has left such a position, the force exerted on it tends to take it further away rather than bring it back. Consequently, the resultant force equal to zero is a necessary but not a sufficient condition for stable equilibrium.

The equilibrium of a ball on a hill can also be regarded from another point of view. The hollows correspond to minima, and the top to maxima of potential energy. The law of conservation of energy prevents a change in positions for which the potential energy is minimum. Such a change would make the kinetic energy negative, which, however, is impossible. The situation is entirely different at the top. A departure from these points entails a decrease in potential energy, and hence not a decrease, but an increase in kinetic energy.

Thus, in a position of equilibrium, the potential energy must assume a minimum value with respect to its values at neighbouring points.

The deeper the hole, the greater will be the stability. Since we know the law of conservation of energy, we can immediately say under what conditions a body will roll out of a depression. For this it is necessary to impart to the body the kinetic energy which would be enough for raising it to the edge of the hole. The deeper the hole, the greater will be the kinetic energy needed for disturbing the stable equilibrium.

Simple Oscillations

If a ball lying in a depression is pushed, it will begin moving up the hill, gradually losing its kinetic energy. When it is completely lost, an instantaneous halt will occur and a downward motion will begin. Its potential energy will now be transformed into kinetic one. The ball will gain speed, rush past the equilibrium position by inertia and begin ascending again, only in the opposite direction. If the friction is insignificant, such an "upward-downward" motion can continue very long, while in the ideal case—in the absence of friction—it will continue indefinitely.

Therefore, motions near the position of stable equilibrium always have an oscillatory nature.

For studying oscillation, a pendulum is perhaps more suitable than a ball rolling back and forth in a hole, at least to the extent that it is easier to reduce the friction exerted on a pendulum to a minimum.

When a pendulum bob is deflected to its highest position, its speed and kinetic energy are equal to zero. Its potential energy is greatest at this moment. The bob goes down—the potential energy decreases and is transformed into kinetic one. Hence, the speed of the motion increases too. When the bob passes through its lowest position, its potential energy is least, and correspondingly the kinetic energy and speed are maximum. As the motion continues, the bob again rises. The speed now diminishes and the potential energy increases.

If we abstract from the friction losses, the bob will be deflected by the same distance to the right as it was originally deflected to the left. Its potential energy was transformed into kinetic one and then the same amount of "new" potential energy was created. We have described the first half of a single oscillation. The second half takes place in the same way, only the bob moves in the opposite direction.

Oscillatory motion is a repeating or, as one says, periodic motion. Returning to its starting point, the bob repeats its motion each time (if the changes resulting from friction are not taken into account) both with respect to its path and to its velocity and acceleration. The time spent on a single oscillation, i.e. in returning to the starting point, is identical for the first, second and all subsequent oscillations. This time—one of the most important characteristics of an oscillation is called the *period*; we shall denote it by T. After this time, the motion is repeated, i.e. after the time T, we shall always find a vibrating body at the same point in space and moving in the same direction. After a half-period, the displacement of the body and also the direction of the motion change sign. Since the period T is the time for one oscillation, the number n of oscillations in a unit of time will be equal to $1/T$.

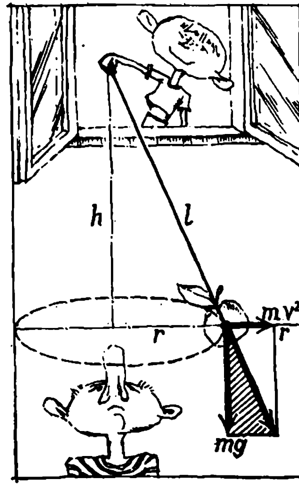


Figure 4.2

But what does the period of vibration of a body moving not far from the position of stable equilibrium depend on? In particular, what does the period of oscillation of a pendulum depend on? Galileo was the first to pose and solve this problem. We shall now derive Galileo's formula.

However, it is difficult to apply the laws of mechanics to non-uniformly accelerated motion in an elementary manner. Therefore, in order to bypass this difficulty, we shall not make the pendulum bob oscillate in a vertical plane, but have it describe a circle, remaining at a fixed height all the time. It isn't difficult to create such a motion—one merely has to remove the pendulum from its equilibrium position and give it an initial push with a properly chosen force in the direction exactly perpendicular to the radius of deflection.

Such a "conical pendulum" is depicted in Figure 4.2.

The bob of mass m moves around a circle. Hence, besides the force of gravity mg, a centrifugal force of mv^2/r, which we may also represent in the form $4\pi^2n^2rm$, is exerted on it. Here n is the number of revolutions per second. We may therefore also write out our expression for the centrifugal force as follows: $4\pi^2rm/T^2$. The resultant of these two forces pulls on the cord of the pendulum.

Two similar triangles—the force triangle and the distance triangle—are shaded in the figure. The ratios of the corresponding legs are equal; hence

$$\frac{mgT^2}{4\pi^2rm}=\frac{h}{r}, \quad \text{or} \quad T=2\pi\sqrt{\frac{h}{g}}$$

What factors does the period of oscillation of a pendulum depend on? If we perform experiments at one and the same location on the Earth (g doesn't change), the period of oscillation depends only on the difference in height between the point of suspension and the point where the bob is. The mass of the bob, as always in a gravitational field, does not affect the period of oscillation.

The following circumstance is of interest. We are investigating motion near the position of stable equilibrium. For small deflections, we may replace the difference in height h by the length l of the pendulum. It is easy to verify this. If the length of the pendulum is 1 m and the radius of deflection is 1 cm, then

$$h=\sqrt{10\,000-1}=99.995 \text{ cm}$$

The difference between h and l will reach 1% only when the deflection is 14 cm. Consequently, the period of the free oscillations of a pendulum for not too large a deflection from the equilibrium position is

$$T=2\pi\sqrt{\frac{l}{g}}$$

i.e. depends only on the length of the pendulum and the value of the acceleration of free fall at the place where the experiment is performed, but does not depend on the magnitude of the deflection of the pendulum from its equilibrium position.

The formula $T = 2\pi \sqrt{l/g}$ has been proved for a conical pendulum. What will it look like for a simple "plane" pendulum? It turns out that this formula retains its form. We shall not prove this rigorously, but call attention to the fact that the shadow cast onto a wall by the bob of a conical pendulum will oscillate almost like a plane pendulum: the shadow completes one oscillation during just the same time in which the bob describes a circle.

The use of small oscillations about an equilibrium position permits the measurement of time with very great accuracy.

According to legend, Galileo established the independence of the period of oscillation of a pendulum of its amplitude and mass while observing during services in a cathedral how two enormous chandeliers were swinging.

Therefore, the period of oscillation of a pendulum is proportional to the square root of its length. Thus, the period of oscillation of a metre-long pendulum is twice that of a 25-cm pendulum. It also follows from our formula for the period of oscillation of a pendulum that one and the same pendulum will not oscillate equally fast at different latitudes on the Earth. As we move closer to the equator, the acceleration of free fall decreases and the period of oscillation grows.

It is possible to measure a period of oscillation with a very great degree of accuracy. Therefore, experiments with pendulums enable us to measure the acceleration of free fall very accurately.

Displaying Oscillations

Let us attach a piece of soft lead to the bob of a pendulum and hang the pendulum over a sheet of paper in such a way that the lead touches the paper (Figure 4.3).

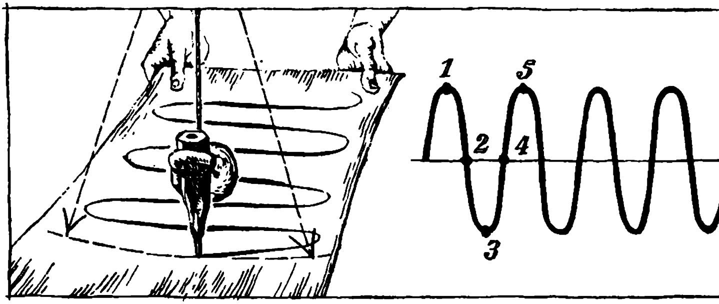


Figure 4.3

Now we deflect the pendulum slightly. The oscillating lead will trace a small line segment on the paper. At the midpoint of the oscillation, when the pendulum is passing through its equilibrium position, the pencil line will be thicker, since in this position the lead presses down harder on the paper. If we pull the sheet of paper in the direction perpendicular to the plane of the oscillation, the curve depicted in Figure 4.3 will be traced. It is not difficult to see that the wavelets so obtained will be dense if the paper is pulled slowly, and sparse if the sheet of paper moves with a considerable speed. In order for the curve to turn out accurate, it is necessary that the sheet of paper move uniformly.

In this manner we have in a sense "displayed" the oscillations.

The display is needed in order to say where the bob of the pendulum was located and where it was moving at one or another instant. Imagine that the paper moves with a speed of 1 cm/s from the time when the pendulum was as far as possible from, say, to the left of, the midpoint. This initial position corresponds to the point on our graph which has been marked with the number *1*.

After a quarter of the period the pendulum will pass through the midpoint. During this time the paper has moved $T/4$ centimetres (point *2* in the figure). The pendulum now moves to the right and the paper simultaneously crawls along. When the pendulum comes to its extreme right position, the paper will have moved $T/2$ centimetres (point *3* in the figure). The pendulum again moves towards the midpoint and arrives at its equilibrium position in $3T/4$ (point *4* in the diagram). Point *5* finishes a complete oscillation, after which the motion is repeated every T seconds or every T centimetres on our graph.

Thus, a vertical line on the graph is the scale of the displacement of a point from the equilibrium position, and the central horizontal line is the time scale.

The two quantities which characterize an oscillation in an exhaustive manner are easily found from such a graph. The period can be determined by the distance between two equivalent points, for example, between two neighbouring summits. The maximum displacement of a point from the equilibrium position can also be measured at once. This displacement is called the *amplitude* of the oscillation.

Displaying an oscillation permits us, moreover, to answer the question posed above: Where is an oscillating point at one or another instant? For example, where will an oscillating point be in 11 seconds if the period of oscillation is equal to 3 seconds and the motion began at the extreme left position? The oscillation begins from the very same point in every 3 seconds. Therefore, in 9 seconds the body will also be at the extreme left position.

Consequently, there is no need of a graph in which the curve is extended over several periods; a graph depicting the curve corresponding to one oscillation is quite enough. In 11 seconds the state of an oscillating point will be the same as in 2 seconds if the period is 3 seconds. Laying off 2 centimetres in our diagram (for we stipulated that the paper be pulled with a speed of 1 cm/s or, in other words, that the scale of our diagram be 1 second to 1 centimetre), we see that in 11 seconds the point will be on its way from the extreme right position to that of equilibrium. The magnitude of the displacement at this instant can be found from the figure.

It isn't necessary to turn to a graph in order to find the magnitude of the displacement of a point making small oscillations about its equilibrium position. Theory shows that in this case the curve depicting the dependence of the displacement on the time is a sinusoid. If we denote the displacement of a point by y, the amplitude by a, and the period of the oscillation by T, we can find the magnitude of the displacement at a time t after the beginning of the oscillation by means of the formula:

$$y = a \sin 2\pi \frac{t}{T}$$

An oscillation taking place in accordance with this law is called *harmonic*. The argument of the sine is equal to the product of 2π by t/T. The quantity $2\pi t/T$ is called the *phase*.

Having trigonometric tables at hand and knowing the period and amplitude, we can easily compute the magnitude of the displacement of a point, and figure out on the basis of the value of the phase in which direction it is moving.

It is not difficult to derive the formula for vibratory motion by considering the motion of the shadow cast on a wall by a bob moving around a circle (Figure 4.4).

We shall mark off the displacements of the shadow from its central position. At the extreme positions, the displacement y is equal to the radius a of the circle. This is the amplitude of the oscillation of the shadow.

If the bob has moved along the circle through an angle

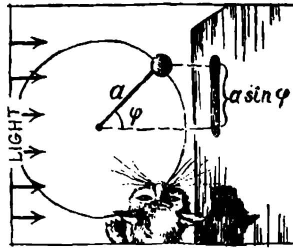


Figure 4.4

φ from the central position, its shadow will deviate from the midpoint by $a \sin \varphi$.

Let the period of the motion of the bob (which, of course, is also the period of oscillation of the shadow) be T; this means that the bob passes through 2π radians during the time T. We may form the proportion $\varphi/t = 2\pi/T$, where t is the time required for a revolution through an angle φ.

Consequently, $\varphi = 2\pi t/T$ and $y = a \sin 2\pi t/T$. This is precisely what we wished to prove.

The velocity of an oscillating point also changes according to a sinusoidal law. The same kind of reasoning about the movement of the shadow of a bob describing a circle will lead us to this conclusion. The velocity of the bob is a vector of constant length v_0. The velocity vector revolves together with the bob. Let us think of the velocity vector as a physical arrow capable of casting a shadow. At the extreme positions of the bob, the vector will lie along a ray of light and will not create a shadow. When the bob moves around the circle from an extreme position through an angle θ, the vector velocity will turn through the same angle and its projection will be equal to $v_0 \sin \theta$. But on the same basis as before, $\theta/t = 2\pi/T$, and so the instantaneous speed of the vibrating body

$$v = v_0 \sin \frac{2\pi}{T} t$$

Note that in the formula for determining the magnitude of the displacement, the time is equal to zero at the central position, and in the formula for the speed at the extreme positions. The displacement of a pendulum equals zero when the bob is at the central position, and the speed of oscillation is zero at the extreme positions.

There is a simple relation between the maximum speed v_0 of an oscillation and the maximum displacement (or amplitude): the bob describes a circle with a circumference $2\pi a$ during the period T of the oscillation. Therefore,

$$v_0 = \frac{2\pi a}{T} \quad \text{and} \quad v = \frac{2\pi a}{T} \sin \frac{2\pi}{T} t$$

Force and Potential Energy in Oscillations

During every oscillation about an equilibrium position, there is a force acting on the vibrating body "desiring" to return it to the equilibrium position. When a point is receding from its equilibrium position, the force decelerates its motion; when it is approaching this position, the force accelerates its motion.

Let us examine this force in the case of a pendulum (Figure 4.5). The bob of the pendulum is acted upon by the force of gravity and tension in the string. Let us decompose the force of gravity into two components—one directed along the string and the other perpendicular to it, along the tangent to the path. Only the tangential component of the gravitational force is of significance for the motion. It is precisely the restoring force in this case. As for the force directed along the string, it is balanced by the reaction on the part of the nail on which the pen-

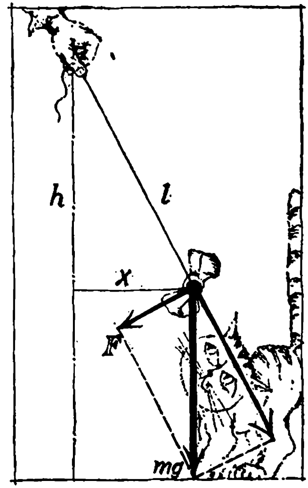


Figure 4.5

dulum is hanging, and it is only necessary to take it into account when we are interested in whether the string will withstand the weight of the vibrating body.

Denote the magnitude of the displacement of the bob by x. The motion takes place along an arc, but we have agreed to investigate oscillations near an equilibrium position. We therefore make no distinction between the magnitude of a displacement along the arc and the deviation of the bob from the vertical. Let us consider two similar triangles. The ratio of the corresponding legs is equal to the ratio of the hypotenuses, i.e.

$$\frac{F}{x} = \frac{mg}{l}, \quad \text{or} \quad F = \frac{mg}{l} x$$

The quantity mg/l does not change during the oscillation. If we denote this constant by k, then the restoring force F is given by the formula $F = kx$. We arrive at the following conclusion: the magnitude of the restoring force is directly proportional to that of the displacement of an oscillating point from its equilibrium position. The restoring force is maximum at the extreme positions of a vibrating body. When the body passes through the midpoint, the force vanishes and changes sign or, in other words, direction. While the body is displaced to the right, the force is directed to the left, and conversely.

The pendulum serves as the simplest example of an oscillating body. However, we are interested in the possibility of extending the formulas and laws which we find to arbitrary vibrations.

The period of oscillation of a pendulum was expressed in terms of its length. Such a formula applies only to a pendulum. But we can express the period of free oscillations in terms of the restoring force constant k. Since $k = mg/l$, we have $l/g = m/k$, and so

$$T = 2\pi \sqrt{\frac{m}{k}}$$

This formula extends to all cases of oscillations, since any free oscillation takes place under the action of a restoring force.

Let us now express the potential energy of a pendulum in terms of its displacement x from the equilibrium position. We may take the potential energy of the bob to be zero when it passes through the lowest point, and then the height of its ascent should be measured from this point. Denoting the difference in height between the point of suspension and the level of the deflected bob by h, we express the potential energy as follows: $U = mg(l - h)$ or, using the formula for the difference of squares,

$$U = mg \frac{l^2 - h^2}{l + h}$$

But, as can be seen from the figure, $l^2 - h^2 = x^2$, l and h differ very slightly and, therefore, $2l$ may be substituted for $l + h$. Thus, $U = mgx^2/2l$, or

$$U = \frac{kx^2}{2}$$

The potential energy of an oscillating body is proportional to the square of its displacement from the equilibrium position.

Let us check the correctness of the formula we have just derived. The loss of potential energy must be equal to the work performed by the restoring force. Consider two of the body's positions, x_2 and x_1. The difference in potential energy

$$-U_2 - U_1 = \frac{kx_2^2}{2} - \frac{kx_1^2}{2} = \frac{k}{2}(x_2^2 - x_1^2)$$

But a difference of squares may be written as the product of the sum by the difference. Hence,

$$U_2 - U_1 = \frac{k}{2}(x_2 + x_1)(x_2 - x_1) = \frac{kx_2 + kx_1}{2}(x_2 - x_1)$$

But $x_2 - x_1$ is the length of the path covered by the body, kx_1 and kx_2 are the magnitudes of the restoring force at the beginning and end of the motion, and $(kx_1 + kx_2)/2$ is equal to the average force.

Our formula led us to the correct result: the loss of potential energy is equal to the work performed.

Spring Vibrations

It is easy to make a ball oscillate by hanging it on a spring. Let us fasten one end of the spring and pull the ball (Figure 4.6). The spring will be in a stretched position as long as we pull the ball with our hand. If we let go, the spring will unstretch and the ball will begin moving towards its equilibrium position. Just as the pendulum, the spring will not come to a state of rest

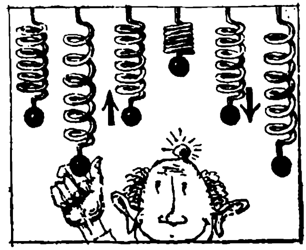


Figure 4.6

immediately. The equilibrium position will be passed by inertia, and the spring will begin compressing. The ball slows down, and at a certain instant it comes to a halt in order to start moving at once in the opposite direction. There arises an oscillation with the same typical features with which our study of the pendulum acquainted us.

In the absence of friction, the oscillation would continue indefinitely. In the presence of friction, the oscillations are damped; moreover, the greater the friction, the faster they are damped.

The roles of a spring and a pendulum are frequently analogous. Both one and the other serve to maintain constancy in the period of clocks. The precise movement of modern spring watches is ensured by the oscillatory motion of a small flywheel balance. It is set oscillating by a spring which winds and unwinds tens of thousands of times a day.

For the ball on a string, the role of the restoring force was played by the tangential component of gravity. For the ball on a spring, the restoring force is the elastic force of a contracted or stretched spring. Therefore, the magnitude of the elastic force is directly proportional to the displacement: $F = kx$.

The coefficient k has another meaning in this case. It is now the stiffness of the spring. A stiff spring is one which is difficult to stretch or contract. The coefficient k has precisely such a meaning. The following is clear from the formula: k is equal to the force necessary for the stretching or contraction of the spring by a unit of length.

Knowing the stiffness of the spring and the mass of the load hung on it, we find the period of free oscillation with the aid of the formula $T = 2\pi\sqrt{m/k}$. For example, a load of mass 10 g on a spring with a stiffness coefficient 10^5 dyn/cm (this is a rather stiff spring—a hundred-gram weight will stretch it by 1 cm) will oscillate with a period $T = 6.28 \times 10^{-2}$ s. During one second, 16 oscillations will take place.

The more flexible the spring, the slower will be the vibration. An increase in the mass of the load has the same effect.

Let us apply the law of conservation of energy to a ball on a spring.

We know that the sum of the kinetic and potential energies, $K + U$, for a pendulum does not vary:

$K + U$ is conserved

We know the values of K and U for a pendulum. The law of conservation of energy states that

$$\frac{mv^2}{2} + \frac{kx^2}{2} \text{ is conserved}$$

But the same thing is also true for a ball on a spring.

The deduction which we must inevitably make is quite interesting. Aside from the potential energy with which we became acquainted earlier, there also exists, therefore, a potential energy of a different kind. The former is called *gravitational potential energy*. If the spring were hanging horizontally, the gravitational potential energy would, of course, not change during the vibration. The new potential energy we discovered is called *elastic potential energy*. In our case it is equal to $kx^2/2$, i.e. it depends on the stiffness of the spring and is directly proportional to the square of the magnitude of contraction or stretching.

The total energy of the vibration, remaining constant, may be expressed in the following form: $E = ka^2/2$, or $E = mv_0^2/2$.

The quantities a and v_0 occurring in the last formulas are the maximum values which the displacement and speed take on during the vibration. (They are sometimes called the amplitude values of the displacement and speed.) The origin of these formulas is quite clear. In an extreme position, when $x = a$, the kinetic energy of vibration is equal to zero, and the total energy is equal to the potential energy. In the central position, the displacement of the point from the equilibrium position, and hence the potential energy, is equal to zero, the speed at this instant is maximum, $v = v_0$, and the total energy is equal to the kinetic energy.

The study of oscillations is an extensive branch of physics. One often has to deal with pendulums and springs. But this, of course, does not exhaust the list of bodies whose oscillations must be investigated. Mountings vibrate; bridges, parts of buildings, beams and high-voltage lines can begin vibrating. Sound is a vibration of the air.

We have listed several examples of mechanical vibrations. However, the concept of oscillation may refer not only to mechanical displacements of bodies or particles from an equilibrium position. We also come across oscillations in many electrical phenomena, moreover, these oscillations occur in accordance with laws closely resembling those which we have considered above. The study of oscillations permeates all branches of physics.

More Complex Oscillations

What has been said so far refers to oscillations near an equilibrium position, taking place under the action of a restoring force whose magnitude is directly proportional

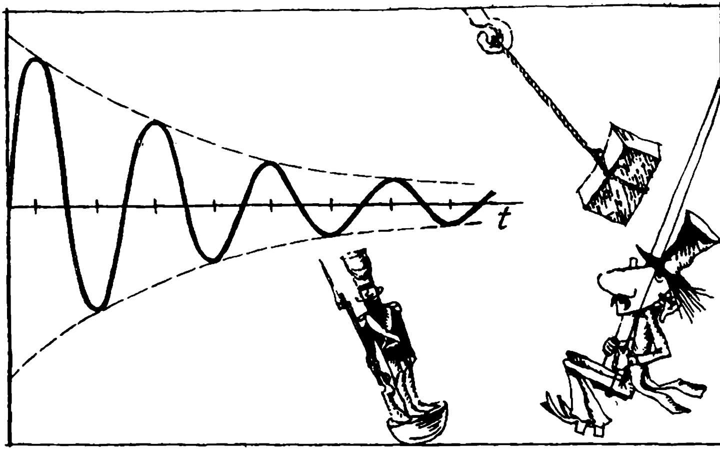


Figure 4.7

to the displacement of a point from its equilibrium position. Such motions occur in accordance with a sinusoidal law. They are called harmonic. The period of harmonic oscillations is independent of the amplitude.

Oscillations with a large amplitude are much more complex. Such oscillations do not occur in accordance with a sinusoidal law, and their display yields more complicated curves different for various oscillating systems. The period is no longer a characteristic property of the oscillation and depends on the amplitude.

Friction will significantly change any oscillations In the presence of friction, oscillations gradually damp. The greater the friction, the faster the damping occurs. Try making a pendulum immersed in water oscillate. It is unlikely that you will succeed in getting the pendulum to complete more than one or two oscillations. If a pendulum is immersed in a very viscous medium, there may fail to be any oscillation at all. The deflected pendulum will simply return to its equilibrium position. A typical graph for a damped oscillation is shown in Figure 4.7. The deviation from the equilibrium position has been plotted along the vertical axis, and the time along the horizontal one. The amplitude of a damped oscillation diminishes with each oscillation.

Resonance

A child is seated on a swing. His feet do not reach the ground. In order to swing him, one can, of course, raise the swing high up and then let it go. But this would be rather difficult and also quite unnecessary; it is enough to gently push the swing in time with the oscillations, and in a short time the child will be really swinging!

In order to swing a body, it is necessary to act in time with the oscillations. In other words, it is necessary to make one's pushes occur with the same period as that of the free oscillations of a body. In such cases one speaks of *resonance.*

Resonance, widespread in nature and technology, merits careful consideration.

You can observe a very amusing and peculiar occurrence of resonance if you construct the following apparatus. Extend a string horizontally and hang three pendulums on it (Figure 4.8), two short ones of identical length and a longer one. Now deflect and release one of the short pendulums. In a few seconds you will see how the other pendulum of the same length will gradually begin oscillating too. A few more seconds—and the second short pendulum will swing, so that it will no longer be possible to tell which of the two pendulums first began moving.

What is the reason for this? Pendulums of the same length have identical periods of free oscillations. The first pendulum swings the second. The oscillations are

Figure 4.8

transmitted from one to the other through the string connecting them. True, but there is yet another pendulum, of different length, hanging on the string. And what will happen to it? Nothing will happen to it. The period of this pendulum is different, and a short pendulum will not be able to swing it. The third pendulum will be present at this interesting energy "transfusion" from one pendulum to another, taking no part in it.

Each of us often comes across mechanical resonance phenomena. Perhaps you simply did not pay any attention to them, even though resonance is sometimes very bothersome. A streetcar passed by your window, and the dishes in the sideboard began jingling. What is the matter? Oscillations of the ground were transmitted to the building and simultaneously to the floor of your room, so your sideboard and the dishes in it started to vibrate. The oscillation was propagated so far and through so many objects. This happened as a result of resonance. The external oscillations were in resonance with the free oscillations of the bodies. Almost any rattling which we hear in a room, a factory or a car occurs because of resonance.

Resonance, as, incidentally, many phenomena, can be both useful and harmful.

A machine is standing on a mounting. Its moving parts move rhythmically, with a definite period. Imagine that this period coincides with that of free oscillation of the mounting. What will happen? The mounting will be soon vibrating, which could result in a breakdown.

The following fact is known. A company of soldiers was marching in step across a bridge in St. Petersburg. The bridge collapsed. An investigation into this matter was begun. It seemed that there were no grounds for anxiety over the fate of the bridge or the people: how many times had crowds gathered on this bridge, had heavy vehicles weighing much more than a company of soldiers slowly crossed it!

But a bridge sags by an insignificant amount under the action of a heavy weight. An incomparably greater sagging occurs when a bridge swings. The resonance amplitude of an oscillation can be thousands of times greater than the displacement caused by a stationary load of the same weight. This is precisely what the investigation showed—the period of free oscillation of the bridge coincided with that of an ordinary marching step.

Therefore, when a military subunit crosses a bridge, a command is given to break step. If people's movements are not coordinated, resonance will not set in and bridges will not swing. Incidentally, this tragedy is well remembered by engineers. In designing bridges, they try to make its period of free oscillation far from the period of a marching step.

Designers of mountings have similar problems. They try to make the mounting in such a way that its period of oscillation be as far as possible from that of the moving parts of a machine.

5. Motion of Solid Bodies

Torque

Try to get a heavy flywheel rotating by hand. Pull one of the spokes. You will find it difficult if you grasp it too near to the axle. Move your hand towards the rim, and things will become easier.

But what has changed? After all, the force is the same in both cases. The point of application of the force has changed.

In all that preceded, the question of where a force is applied did not arise, since the form and size of a body played no role in the problems under consideration. What we essentially did was to conceptually replace a body by a point.

The example with the rotation of a wheel shows that the question of the point of application of a force is far from idle when we are dealing with the rotation or revolution of a body.

In order to understand the role of the point of application of a force, let us compute the work which must be performed to turn a body through a certain angle. In this calculation, of course, it is assumed that all the particles of the body are rigidly bound to one another (we are ignoring at present the ability of a body to bend, contract and, in general, to change its form). Therefore, a force applied to one point of a body imparts kinetic energy to all its parts.

In computing this work, the role of the point of application of a force is clearly seen.

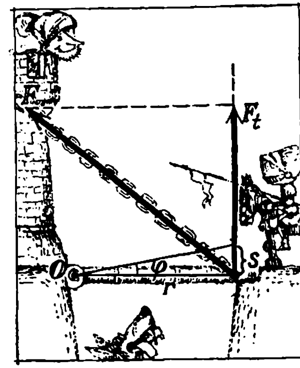


Figure 5.1

A body fastened to an axis is shown in Figure 5.1. When the body turns through a small angle φ, the point of application of a force moves along an arc—it is displaced by a distance s.

Projecting the force onto the direction of the motion, i.e. onto the tangent to the circle around which the point of application moves, we find a familiar expression for the work A:

$$A = F_t s$$

But the arc s may be represented as follows:

$$s = r\varphi$$

where r is the distance from the axis of rotation to the point of application of the force. Thus,

$$A = F_t r\varphi$$

Turning the body through one and the same angle in various ways, we may expend different amounts of work depending on where the force is applied.

If the angle is given, the work is determined by the product $F_t r$. This product is called the *moment of force*,

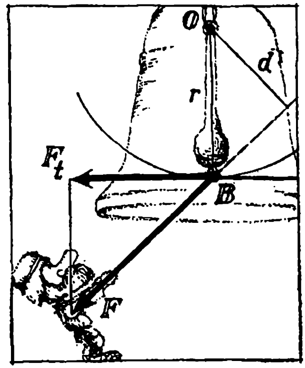


Figure 5.2

or the *torque*:

$$M = F_t r$$

Our formula for the torque can be given another form. Let O be the axis of rotation, and B the point of application of the force (Figure 5.2). The length of the perpendicular dropped from O to the direction of the force is denoted by d. The two triangles constructed in the figure are similar. Therefore,

$$\frac{F}{F_t} = \frac{r}{d}, \quad \text{or} \quad F_t r = Fd$$

The quantity d is called the *arm*, or the *lever arm*, of the force.

Our new formula $M = Fd$ reads as follows: the torque is equal to the product of the force by its lever arm.

If we displace the point of application of the force along its direction, then the lever arm d and with it the torque M will not be changed. Hence, it makes no difference just where the point of application lies on the line of action of the force.

With the aid of the new concept, the formula for the work can be written out more concisely.

$$A = M\varphi$$

i.e. the work is equal to the product of the torque by the angle of rotation.

Let two forces act on a body with moments M_1 and M_2. When the body is rotated through an angle φ, the work done will be $M_1\varphi + M_2\varphi = (M_1 + M_2)\varphi$. This equality shows that two forces with moments M_1 and M_2 rotate a body just as a single force with moment $M = M_1 + M_2$ would. Moments of force can help, as well as hinder, each other. If torques M_1 and M_2 tend to rotate a body in one and the same direction, we should regard them as magnitudes having the same sign. On the contrary, torques rotating a body in opposite directions have different signs.

As we know, the work done by all the forces acting on a body effects a change in its kinetic energy.

The rotation of a body slowed down or speeded up, hence, its kinetic energy changed. This can only take place in case the resultant torque is not equal to zero.

And what if the resultant torque is equal to zero? The answer is obvious—the kinetic energy does not change; consequently, the body either rotates uniformly by inertia or remains stationary

Thus, the equilibrium of a body capable of rotating requires the balancing of all the torques acting on it. If there are two such torques, the equilibrium requires that

$$M_1 + M_2 = 0$$

While we were interested in problems in which a body could be regarded as a point, the conditions for equilibrium were simple: in order for a body to remain stationary or move uniformly, stated Newton's law for such problems, it is necessary that the resultant force be equal to zero; the forces acting upwards must balance those directed downwards; the rightward force must compensate for the leftward one.

This law is also valid for our case. If a flywheel is stationary, the forces acting on it are balanced by the reaction of the axle around which the wheel can turn.

But these necessary conditions become insufficient. Besides the balancing of forces, the balancing of torques is also required. The balancing of moments of force is the second necessary condition for the rest or uniform rotation of a solid body.

Torques, if there are several of them, can be easily separated into two groups: some tend to rotate a body clockwise, and others counterclockwise. These are precisely the moments of force which must compensate for each other.

Lever

Can a person keep 100 tons from falling? Can one crush a piece of iron with one's hand? Can a child resist a strong man? Yes, they can.

Ask a strong man to turn a flywheel in the clockwise direction while holding a spoke near the axle. The torque will be small in this case: the force is great but the lever arm is short. If a child pulls the wheel in the opposite direction, holding a spoke near the rim, the torque may turn out to be large: the force is small but the lever arm is long. The condition for equilibrium will be

$$M_1 = M_2, \quad \text{or} \quad F_1 d_1 = F_2 d_2$$

Using the law of moments, a person can acquire fabulous strength.

The action of levers serves as the most striking example of this.

You want to lift an enormous stone with the aid of a crow-bar. This task will turn out to be possible for you to accomplish, even though the stone weighs several tons. The crow-bar is placed on a pivot and is the solid body of our problem. The pivot is the centre of rotation. Two torques act on the body: a hindering one from the weight of the stone, and a helping one from your hand. If the subscript 1 refers to the muscular force, and the subscript 2 to the weight of the stone, the possibility of lifting the stone can be expressed concisely: M_1 must be greater than M_2.

The stone can be supported above the ground provided that

$$M_1 = M_2, \quad \text{i.e.} \quad F_1d_1 = F_2d_2$$

If the short lever arm (from the pivot to the stone) is fifteen times smaller than the long one (from the pivot to the hand), then a person acting with his entire weight on the long end of the lever will support a one-ton stone in a raised position.

A crow-bar placed on a pivot is a rather widespread and the simplest example of a lever. A ten- to twenty-fold gain in force is usually achieved with the aid of a crow-bar. The length of a crow-bar is about 1.5 m, but it is usually difficult to place the pivot nearer than 10 cm from its bottom. Therefore, one of the lever arms will be from fifteen to twenty times as long as the other, and so this will also be the gain in force.

A chauffeur will easily raise an automobile weighing several tons with the aid of a jack. A jack is a lever, of the same type as a crow-bar, placed on a pivot. The points of application of the forces (the hand, the weight of the car) lie on opposite sides of pivot of the jack. Here the gain in force is about forty- to fifty-fold, which makes it possible to easily lift an enormous weight.

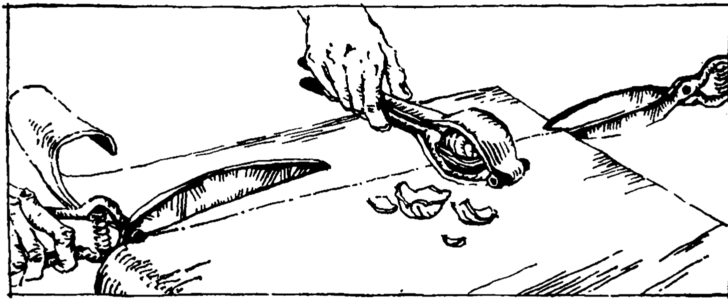


Figure 5.3

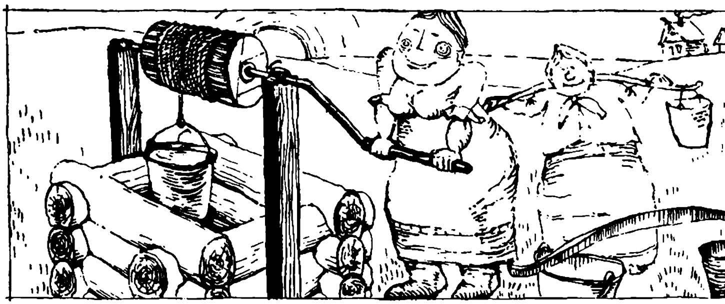


Figure 5.4

A pair of scissors, a nutcracker, pliers, pincers, nippers and many other instruments are all levers. You can easily find the centres of rotation (pivots) of the solid bodies depicted in Figure 5.3, as well as the points of application of the two forces—active and hindering.

In cutting tin-plate with a pair of scissors, one tries to open them as wide as possible. What is accomplished by this? One succeeds in slipping the piece of metal closer to the centre of rotation. The lever arm of the torque one is overcoming becomes shorter, and so the gain in force is greater. When moving a pair of scissors, or nippers, an adult ordinarily acts with a force of 40-50 kgf. One of the lever arms can be twenty times longer than the other. It turns out that we are able to cut into metal with a force of 1000 kgf. And this with the aid of such simple instruments.

The windlass is a variety of lever. With the aid of a windlass (Figure 5.4), water is taken out of a well in many villages.

Loss in Path

Instruments make a person strong, but it by no means follows from this that instruments permit one to expend a little work and obtain a lot. The law of conservation of energy convinces us that a gain in work, i.e. the creation of work out of "nothing", is impossible.

The work obtained cannot be greater than the work performed. On the contrary, the inevitable energy loss due to friction leads to the fact that the work obtained with the aid of an instrument will always be less than that performed. In the ideal case, these works can be equal.

Properly speaking, we are wasting time by explaining this obvious truth: for the rule of torques was derived from the condition of equality of the work performed by the active and overcome forces.

If the points of application of the forces moved distances s_1 and s_2, the condition of equality of the work assumes the following form:

$$F_1^t s_1 = F_2^t s_2$$

In overcoming some force F_2 along a path of length s_2 with the aid of a lever, we can make this by means of force F_1 much less than F_2. But the displacement s_1 of our hand must be as many times greater than s_2 as the muscular force F_1 is less than F_2.

This law is often expressed by the following brief sentence: the gain in force is equal to the loss in path.

The law of the lever was discovered by the greatest scientist of antiquity—Archimedes. Amazed at the strength of his proof, this remarkable scientist of antiquity wrote to King Hiero II of Syracuse: "If there were another world and I could go to it, I would move this one." A very long lever whose pivot is near the Earth would make it possible to solve such a problem.

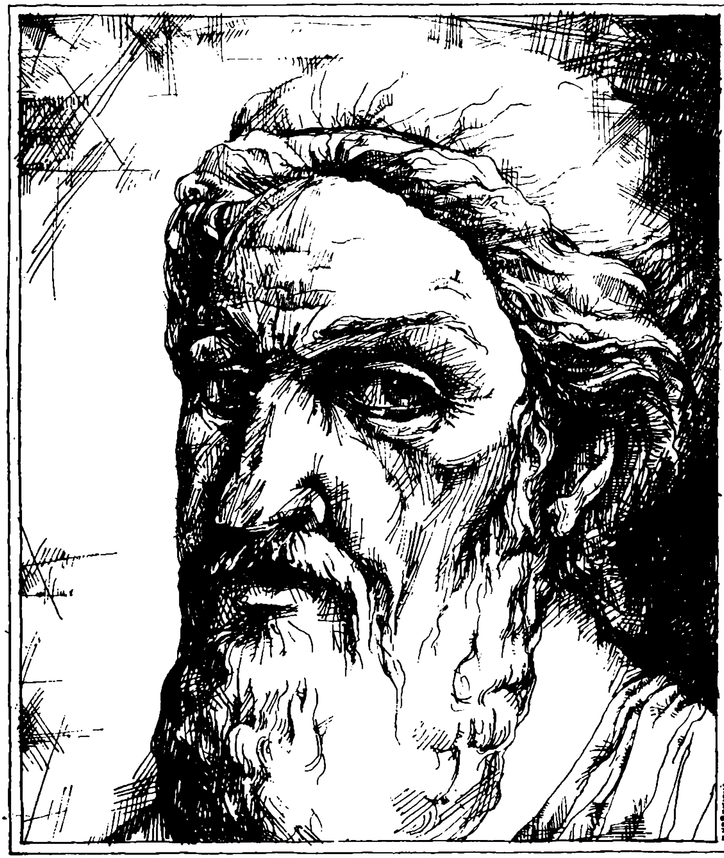
We shall not grieve with Archimedes over the absence of a fulcrum, which, as he thought, was all that he lacked to move the Earth.

Let us dream: take the strongest possible lever, place it on a pivot and "suspend a small sphere" of weight... 6×10^{24} kgf on its short end. This modest number shows how much the Earth "pressed into a small sphere" weighs. Now apply muscular force to the long end of the lever.

If the force exerted by Archimedes can be taken as 60 kgf, then in order to displace the "Earth nut" by 1 cm, Archimedes' hand would have to cover a distance $(6 \times 10^{24})/60 = 10^{23}$ times greater. But 10^{23} cm are 10^{18} km, which is three billion times greater than the diameter of the Earth's orbit!

This fantastic example clearly demonstrates the scale of the "loss in path" involved in the work of a lever.

Any of the examples considered by us above can be used to illustrate not only a gain in force but also a loss in path. The hand of the chauffeur jacking up his car covers a path which will be as many times longer than the height to which he raises it as his muscular force is less than the weight of the automobile. Moving a pair of scissors in order to cut a sheet of tin-plate, we perform work along a path which is as many times longer than the depth of the cut as our muscular force is less than the resistance of the tin-plate. The stone lifted by the crow-bar will rise to a height as many times less than that by which the



Archimedes (circa 287-212 B.C.)—the greatest mathematician, physicist and engineer of antiquity. Archimedes computed the volume and the surface area of a sphere and its parts, of a cylinder and of bodies formed by rotating an ellipse, hyperbola or parabola. He was the first to compute the ratio of the circumhand is lowered as the muscular force is less than the weight of the stone. This rule clarifies the principle of the screw's action. Imagine that we are screwing in a bolt, whose threading has a 1-mm screw pitch, with the aid of a wrench of length 30 cm. The bolt will advance 1 mm along its axis during a single turn, while our hand will cover a 2-m path during the same time. Our gain in force is two thousand-fold, and we either safely fasten the components together or move a heavy weight with a slight effort of our hand.

Other Very Simple Machines

A loss in path as payment for a gain in force is a general law not only for levers but also for all other devices and mechanisms used by man.

A tackle is widely used for lifting loads. This is what we call a system consisting of several movable pulleys joined to one or several fixed pulleys. The load in Figure 5.5 is suspended by six strings. It is clear that the weight is distributed among the strings, and so the tension in a string will be six times less than the weight. The lifting of a one-ton load will require an application of 1000/6 = 167 kgf. However, it is not difficult to see that in order to raise the load by 1 m, one must haul in 6 m

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ference of a circle to its diameter with a high degree of accuracy, showing that it satisfies the inequalities $3\frac{10}{71} < \pi < 3\frac{1}{7}$. In mechanics he established the laws of lever, the conditions governing floating bodies (Archimedes' principle), the composition of parallel forces. Archimedes invented the machine for pumping water (Archimedean screw, used in our times for transporting free-flowing or viscous cargo), systems of levers and blocks for raising heavy weights, and military engines successfully employed during the siege of his native city, Syracuse, by the Romans.

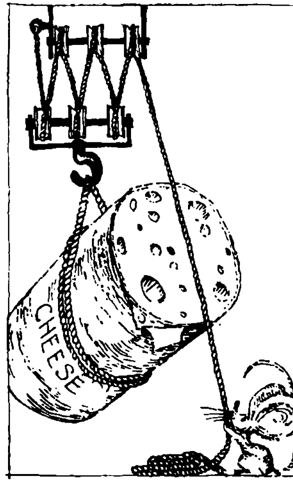

Figure 5.5

of string. For raising the load by 1 m, 1000 kgf-m of work are needed. We must supply this work in "some form"—a force of 1000/6 kgf must act along a 6-m path, a force of 10 kgf along a 100-m path, and a force of 1 kgf along a 1-km path.

An inclined plane, which we discussed on p. 37, is also a device permitting a gain in force at the expense of a loss in path.

A blow is a distinctive means of multiplying forces. A blow with a hammer, an axe, a ram and even a blow with a fist can create an enormous force. The secret of a strong blow isn't complicated. Driving a nail into an unyielding wall with a hammer, one must take a good wind-up. A big swing, i.e. a long path along which the force acts, generates a significant kinetic energy in the hammer. This energy is released along a small path. If the swing covers 1/2 m and the nail enters 1/2 cm into the wall, the force is intensified by a factor of 100. But if the wall is harder and the nail, after the same swing of the hand, enters 1/2 mm into the wall, the blow will be ten times as strong as in the former case. The nail does not enter a hard wall as deeply, and the same work is expended on a shorter path. It turns out that a hammer works like an automaton: it strikes harder where the wall is harder.

If a hammer of 1 kg is "speeded up", it will strike a nail with a force of 100 kgf. Also, in splitting logs with a heavy wood-chopper, we break the wood with a force of several thousands kgf's. Heavy forging hammers fall from small heights, of the order of a metre. Flattening a forged piece by 1-2 mm, a hammer of 1000 kg comes down on it with an enormous force, that of 10^6 kgf.

How to Add Parallel Forces Acting on a Solid Body

While solving mechanical problems in which a body was conceptually replaced by a point, the question of how to add forces was answered simply on the preceding pages. The parallelogram law of forces yielded an answer to this question, and if the forces were parallel, we added their magnitudes like numbers.

Now matters are more complicated. For the effect of a force on an object is characterized not only by its magnitude and direction but also by the point of its application, or—we have explained above that this is the same thing—its line of action.

To add forces means to replace them by a single force. This is by no means always possible.

The replacement of parallel forces by a single resultant is a problem which can always be solved (except in a special case, which will be discussed at the end of this section). Let us consider the addition of parallel forces. Of course, the sum of forces of 3 kgf and 5 kgf is equal to 8 kgf, provided that they have the same direction. The

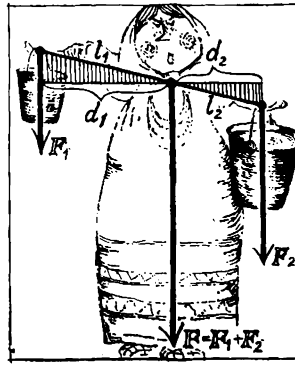

Figure 5.6

problem consists in finding the point of application (line of action) of the resultant force.

Two forces acting on a body are depicted in Figure 5.6. The resultant force F replaces the forces F_1 and F_2, but this means not only that $F = F_1 + F_2$; the action of F will be equivalent to that of F_1 and F_2 in case the torque produced by F is equal to the sum of the torques produced by F_1 and F_2.

We are looking for the line of action of the resultant force F. Of course, it is parallel to the forces F_1 and F_2, but how far is this line from F_1 and F_2?

A point lying on the segment joining the points of application of F_1 and F_2 is depicted in the figure as F's point of application. With respect to the chosen point, the moment of F is, of course, equal to zero. But then the sum of the moments of F_1 and F_2 with respect to this point should also be equal to zero, i.e. the torques produced by F_1 and F_2 opposite in sign will be equal in magnitude.

Denoting the lever arms of F_1 and F_2 by d_1 and d_2, we may write out this condition as follows:

$$F_1 d_1 = F_2 d_2, \quad \text{i.e.} \quad \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

It follows from the similarity of the shaded triangles that $d_2/d_1 = l_2/l_1$, i.e. the point of application of the resultant force divides the distance on the uniting segment between the added forces into parts, l_1 and l_2, which are inversely proportional to the forces.

Denote the distance between the points of application of F_1 and F_2 by l. It is obvious that $l = l_1 + l_2$.

Let us solve the following system of two equations in two variables

$$F_1 l_1 - F_2 l_2 = 0$$

$$l_1 + l_2 = l$$

We obtain

$$l_1 = \frac{F_2 l}{F_1 + F_2}, \quad l_2 = \frac{F_1 l}{F_1 + F_2}$$

By means of these formulas, we can find the point of application of the resultant force not only in the case when the forces have the same direction but also in the case of the forces with opposite directions (antiparallel forces, as we say). If the forces have different directions, they have opposite signs, and the resultant is equal to the difference $F_1 - F_2$ of the forces and not to their sum. Taking the smaller of the two forces, F_2, to be negative, we see by our formulas that l_1 becomes negative. This means that the point of application of F_1 lies not to the left (as before) but to the right of the point of application of the resultant force (Figure 5.7); moreover, as in the previous case,

$$\frac{F_1}{F_2} = \frac{l_2}{l_1}$$

An interesting result is obtained for equal antiparallel forces. Then $F_1 + F_2 = 0$. The formulas show that l_1 and l_2 will then become infinitely large. What is the physical meaning of this assertion? Since it is meaning-

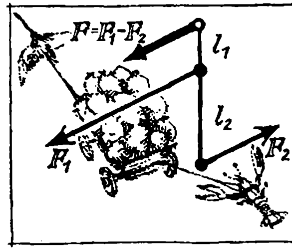

Figure 5.7

less to put the resultant at infinity, it is therefore impossible to replace equal antiparallel forces by a single force. Such a combination of forces is called a *couple*.

The action of a couple cannot be reduced to the action of one force. Any other pair of parallel or antiparallel forces can be balanced by a single force, but a couple cannot.

Of course, it would be false to say that the forces constituting a couple cancel each other. A couple has quite a significant effect—it rotates a body; the peculiarity of the action of a couple consists in the fact that it does not produce a translational motion.

In certain cases, the question may arise not of adding parallel forces but of decomposing a given force into two parallel ones.

Two persons carrying a heavy basket together on a pole are depicted in Figure 5.8. The weight of the basket is distributed between the two of them. If the load presses down on the centre of the pole, they both feel the same weight. If the distances from the point of appplication of the load to the hands which carry it are d_1 and d_2, the force F is decomposed into forces F_1 and F_2 according to the rule

$$\frac{F_1}{F_2}=\frac{d_2}{d_1}$$

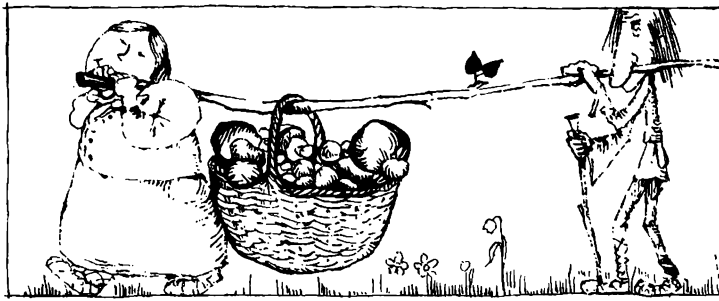

Figure 5.8

The stronger person should take hold of the pole nearer to the load.

Centre of Gravity

All particles of a body possess weight. Therefore, a solid body is subject to the action of an infinite number of gravitational forces. Moreover, all these forces are parallel. If so, it is possible to add them according to the rules which we have just considered and replace them by a single force. The point of application of the resultant force is called the *centre of gravity*. It is as if the weight of a body were concentrated at this point.

Let us suspend a body by one of its points. How will it then be situated? Since we may conceptually replace the body by one load concentrated at the centre of gravity, it is clear that in equilibrium this load will lie on the vertical passing through the pivot. In other words, in equilibrium the centre of gravity lies on the vertical passing through the pivot, and is at its lowest possible position.

One can place the centre of gravity on the vertical passing through the axis and above the pivot. It will be very difficult to do this and only because of the presence of friction. Such an equilibrium is *unstable*.

We have already discussed the condition for stable equilibrium—the potential energy must be minimum. This is precisely the case when the centre of gravity lies below the pivot. Any deflection raises the centre of gravity and, therefore, increases the potential energy. On the contrary, when the centre of gravity lies above the pivot, any puff removing the body from this position leads to a decrease in potential energy. Such a position is unstable.

Cut a figure out of cardboard. In order to find its centre of gravity, hang it up twice, attaching the suspending thread first to one and then to another point of the body. Attach the figure to an axis passing through its centre of gravity. Turn the figure to one position, a second, a third, ... We observe the complete neutrality of the body towards our operations. A special case of equilibrium is attained in any position. This is just what we call it—*neutral*.

The reason for this is clear—in any position of the figure, the material point replacing it is located at one and the same place.

In a number of cases, the centre of gravity can be found without any experiments or computations. It is clear, for example, that the centres of gravity of a sphere, circle, square and rectangle are located at the centres of these figures, since they are symmetrical. If we conceptually break up a symmetrical body into small parts, each of them will correspond to another located symmetrically on the other side of the centre. And for each pair of such particles, the centre of the figure will be the centre of gravity.

The centre of gravity of a triangle lies at the intersection of its medians. In fact, let us break up a triangle into narrow strips parallel to one of the sides. A median divides each of the strips in half. But the centre of grav-

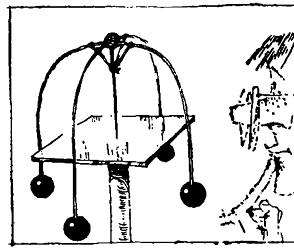

Figure 5.9

ity of a strip lies, of course, half-way along it, i.e. on the median. The centres of gravity of all the strips occur on the median, and when we add their weights, we arrive at the conclusion that the centre of gravity of the triangle lies somewhere on the median. But this argument is valid with respect to any of the medians. Therefore, the centre of gravity must lie at their intersection.

But perhaps you are not convinced that the three medians intersect in a single point. This is proved in geometry; but our argument also proves this interesting theorem. For a body cannot have several centres of gravity; but since the centre of gravity is one and lies on a median, no matter from which vertex we draw it, all the three medians therefore intersect in a single point. The formulation of a physical problem helped us prove a geometric theorem.

It is more difficult to find the centre of gravity of a homogeneous cone. It is only clear from considerations of symmetry that the centre of gravity lies on the axis. Computations show that it is located at the distance of a quarter of the height from the base.

The centre of gravity is not necessarily located inside a body. For example, the centre of gravity of a ring is located at its centre, i.e. outside the ring.

Can a pin be placed in a vertical position on a glass pedestal and stay stable?

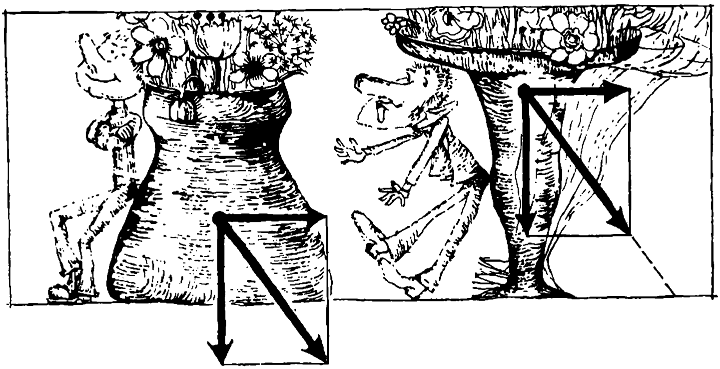

Figure 5.10

It is shown in Figure 5.9 how to do this. A small apparatus consisting of wires in the form of a double yoke with four small loads should be rigidly fastened to the pin. Since the loads are hanging lower down than the pivot, and the weight of the pin is small, the centre of gravity lies below the pivot. The position is stable.

So far we have been dealing with bodies possessing a point of support. What is the situation in those cases when a body is supported over an entire area element?

It is clear that in this case the location of the centre of gravity above the support does not at all imply that the equilibrium is unstable. How else could glasses stand on a table? It is necessary for stability that the line of action of the gravitational force drawn from the centre of gravity pass through the area of support. On the contrary, if the line of action passes outside the area of support, the body will fall.

Stability may differ greatly depending on how high above the support the centre of gravity is. Only a very clumsy person will overturn a glass of tea, but a flower

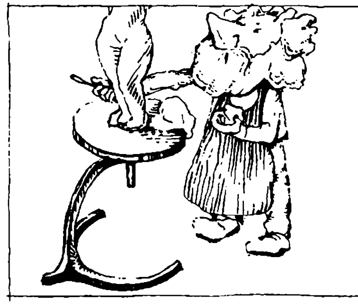

Figure 5.11

vase with a small base can be overturned by a careless touch. What is the point here?

Take a look at Figure 5.10. Identical horizontal forces are applied to the centres of gravity of two vases. The vase at the right will overturn, since the resultant force doesn't pass through its base but is directed to one side.

We have said that for a body to be stable, the force applied to it must pass through the area of support. But the area of support needed for equilibrium does not always correspond to the actual area of support. A body whose area of support has the form of a crescent is depicted in Figure 5.11. It is easy to see that the stability of the body will not change if the crescent is completed to a solid half-disc. Thus, the area of support determining the condition for equilibrium may be greater than the actual one.

In order to find the area of support for the tripod depicted in Figure 5.12, one must join its tips with straight-line segments.

Why is it so hard to walk a tightrope? Because the area of support has sharply decreased. It isn't easy to walk a tightrope, and skilful tightrope walkers aren't rewarded with applause for nothing. However, sometimes viewers make the mistake of acclaiming clever tricks simplifying the task as the epitome of artistry. The performer takes

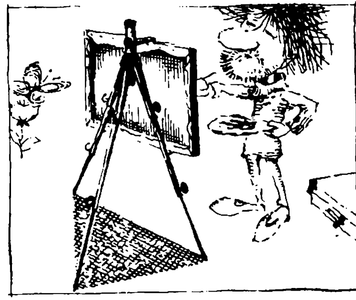

Figure 5.12

a heavily bent yoke with two pails of water; the pails turn out to be on the level of the tightrope. With a straight face, while the orchestra has ceased playing, the performer takes his walk along the tightrope. How complicated has the trick become, thinks the inexperienced viewer. As a matter of fact, the performer has simplified his task by lowering the centre of gravity.

Centre of Mass

It is entirely legitimate to ask the following question: Where is the centre of gravity of a group of bodies? If many people are on a raft, its stability will depend on the location of their (together with the raft's) centre of gravity.

The meaning of this concept remains the same. The centre of gravity is the point of application of the sum of the gravitational forces of all the bodies in the group under consideration.

We know the result of the computation for two bodies. If two bodies of weights F_1 and F_2 are located at a distance x from each other, their centre of gravity is situated at distances x_1 from the first body and x_2 from the second, where

$$x_1 + x_2 = x \quad \text{and} \quad \frac{F_1}{F_2} = \frac{x_2}{x_1}$$

Since weight may be represented as a product mg, the centre of gravity of the pair of bodies satisfies the condition

$$m_1 x_1 = m_2 x_2$$

i.e. lies at the point which divides the distance between the masses into segments inversely proportional to the masses.

Let us now recall the firing of a gun attached to a platform. The momenta of the gun and the shell are equal in magnitude and opposite in direction. The following equalities hold:

$$m_1 v_1 = m_2 v_2, \quad \text{or} \quad \frac{v_2}{v_1} = \frac{m_1}{m_2}$$

where the ratio of the speeds retains this value during the entire interaction. In the course of the motion arising as a result of the recoil, the gun and the shell are displaced with respect to their initial positions by distances x_1 and x_2 in opposite directions. The distances x_1 and x_2—the paths covered by the two bodies—increase, but for a constant ratio of speeds, they will also be in the same ratio to each other all the time:

$$\frac{x_2}{x_1} = \frac{m_1}{m_2}, \quad \text{or} \quad x_1 m_1 = x_2 m_2$$

Here x_1 and x_2 are the distances of the gun and the shell from their original positions. Comparing this formula with the formula determining the position of the centre of gravity, we observe their complete identity. It immediately follows from this that the centre of gravity of the gun and the shell remains at its original position all the time after the firing.

In other words, we have arrived at the very interesting result—the centre of gravity of the gun and the shell remains stationary after the firing.

Such a conclusion is always true: if the centre of gravity of two bodies was initially stationary, their interaction, regardless of its nature, cannot change the position of the centre of gravity. This is precisely why it is impossible to pick oneself up by the hair or pull oneself up to the Moon by the method of the French writer Cyrano de Bergerac, who proposed (jokingly, of course) to this end that one threw a magnet upwards while holding a piece of iron which would be attracted by the magnet.

A stationary centre of gravity is moving uniformly from the point of view of a different inertial frame of reference. Hence, a centre of gravity is either stationary or moving uniformly and rectilinearly.

What we have said about the centre of gravity of two bodies is also true for a group of many bodies. Of course, for an isolated group of bodies; this is always stipulated when we are applying the law of conservation of momentum.

Consequently, every group of interacting bodies has a point which is stationary or is moving uniformly, and this point is their centre of gravity.

To emphasize the new property of this point, we give it an additional name: the *centre of mass*. As a matter of fact, the question of, say, the weight of the solar system (and hence its centre of gravity) can have only a hypothetical meaning.

No matter how the bodies forming a closed group move, the centre of mass (gravity) will be stationary or, in another frame of reference, will move by inertia.

Angular Momentum

We shall now become acquainted with another mechanical concept, which permits us to formulate a new important law of motion. This concept is called *angular* *momentum*, or *moment of momentum*. The very names suggest that we are dealing with the quantity which somehow resembles a moment of force.

A moment of momentum, just as a moment of force, requires the indication of the point with respect to which the moment is defined. In order to define the angular momentum relative to some point, one must construct the momentum vector and drop a perpendicular from the point to its direction. The product of the momentum mv by the lever arm d is precisely the angular momentum, which we shall denote by the letter N:

$$N = mvd$$

If a body is moving freely, its velocity does not change, the lever arm with respect to any point also remains constant, since the motion takes place along a straight line. Consequently, the angular momentum also remains constant during such a motion.

Just as for the moment of force, we can also obtain a different formula for the moment of momentum. Draw a radius between the position of the body and the point with respect to which we are interested in the angular momentum (Figure 5.13). Construct also the projection of the velocity onto the direction perpendicular to the radius. It follows from the similar triangles constructed in the figure that $v/v_\perp = r/d$. Therefore, $vd = v_\perp r$, and the formula for angular momentum may also be written in the following form: $N = mv_\perp r$.

During free motion, as we have just said, angular momentum remains constant. Well, but if a force is acting on the body? Computations show that the change in angular momentum during a second is equal to the torque.

This law can be extended without difficulty to systems of bodies. If we add the changes in the angular momenta of all the bodies belonging to the system in a unit of time, their sum turns out to be equal to the sum of the torques

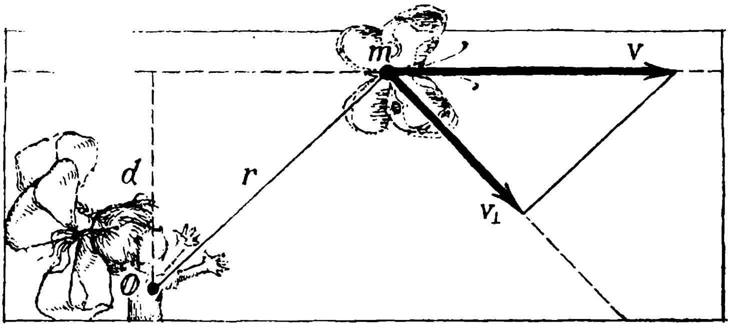

Figure 5.13

acting on the bodies. Consequently, the following statement is valid for a group of bodies: the change in the total moment of momentum in a unit of time is equal to the sum of the moments of all the forces.

Law of Conservation of Angular Momentum

If two stones are connected with a string and one of them is hurled, the other stone will fly after the first at the end of the stretched string. Each stone will pass the other, and this forward motion will be accompanied by a rotation. Let us forget about the gravitational field—assume that the throw was made in interstellar space.

The forces acting on the stones are equal in magnitude and directed towards each other along the string (for these are forces of action and reaction). But then the lever arms of both forces with respect to an arbitrary point will also be the same. Equal lever arms and equal but oppositely directed forces yield torques which are equal in magnitude and opposite in sign.

The resultant torque will be equal to zero. But it follows from this that the change in angular momentum will also equal zero, i.e. that the angular momentum of such a system remains constant.

We only needed the string connecting the stones for visualization. The *law of conservation of angular momentum* is valid for any pair of interacting bodies, no matter what the nature of this interaction.

Yes, and not only for a pair. If a closed system of bodies is being investigated, the forces acting between the bodies can always be divided up into an equal number of forces of action and reaction whose moments will cancel each other in pairs.

The law of conservation of total angular momentum is universal, it is valid for any closed system of bodies.

If a body is rotating about an axis, its angular momentum is

$$N = mvr$$

where m is the mass, v is the speed, and r is the distance from the axis. Expressing the speed in terms of the number n of revolutions per second, we have:

$$v = 2\pi nr \quad \text{and} \quad N = 2\pi mnr^2$$

i.e. the angular momentum is proportional to the square of the distance from the axis.

Sit down on a swivel stool. Pick up heavy weights, spread your arms wide apart and ask somebody to get you rotating slowly. Now press your arms to your chest rapidly—you will suddenly begin rotating faster. Arms out—the motion slows down, arms in—the motion speeds up. Until the stool stops turning because of friction, you will have time to change your rotational velocity several times.

Why does this happen?

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For a constant number of revolutions per second, the angular momentum would decrease in case the weights approached the axis. In order to "compensate" for this decrease, the rotational velocity increases.

Acrobats make good use of the law of conservation of angular momentum. How does an acrobat turn a somersault in mid-air? First of all, by pushing off from an elastic floor or his partner's hand. When pushing off, his body bends forward and his weight, together with the force of the push, creates an instantaneous torque. The force of the push creates a forward motion, and the torque causes a rotation. However, this rotation is slow, incapable of impressing the audience. The acrobat bends his knees. "Gathering his body" closer to the axis of rotation, the acrobat greatly increases the rotational velocity and quickly turns over. This is the mechanics of the somersault.

The movements of a ballerina performing a succession of rapid turns are based on this same principle. Ordinarily the initial angular momentum is imparted to the ballerina by her partner. At this instant the dancer's body is bent; a slow rotation begins, then a graceful and rapid movement—the ballerina straightens up. Now all points of her body are closer to the axis of rotation, and conservation of angular momentum leads to a sharp increase in speed.

Angular Momentum as a Vector

So far we have been dealing with the magnitude of angular momentum. But angular momentum has the properties of a vector.

Consider the rotation of a point with respect to some "centre" Two nearby positions of the point are depicted in Figure 5.14. The motion in which we are interested is characterized by the magnitude of its angular momentum

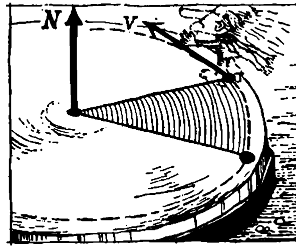


Figure 5.14

and the plane in which it takes place. The plane of the motion is shaded in the figure—it is the area swept out by the radius drawn from the "centre" to the moving point.

Information about the direction of the plane of the motion and about the magnitude of the angular momentum can be combined. The angular momentum vector, directed along the normal to the plane of motion and equal in magnitude to the absolute value of the angular momentum, serves for this purpose. However, this is still not all—one must take into account the direction of the motion in the plane: for a body can rotate about a centre in the clockwise as well as in the counterclockwise direction.

It is customary to draw an angular momentum vector in such a manner that we see the point rotating in the counterclockwise direction when we look at it facing the vector. This can also be said otherwise: the direction of the angular momentum vector is related to the direction of the rotation in the same way as the direction of a turning corkscrew is related to the direction of the motion of its handle.

Thus, if we know the angular momentum vector, we can determine the magnitude of the angular momentum, the position of the plane of motion in space, and the direction of the rotation with respect to the "centre".

If the motion takes place in one and the same plane, and the lever arm and speed change, the angular momentum vector preserves its direction in space, but changes in length. And in the case of an arbitrary motion, the angular momentum vector changes both in direction and in magnitude. It may seem that such a fusion into one concept of the direction of the plane of motion and the magnitude of an angular momentum serves only the purpose of saving words. In reality, however, when we are dealing with a system of bodies moving in more than one plane, we obtain the law of conservation of angular momentum only when we add moments of momentum as vectors. This circumstance shows that the attribution of a vector nature to angular momentum has a profound content.

Angular momentum is always defined with respect to some conditionally chosen "centre" It is only natural that this quantity depends, generally speaking, on the choice of this point. Nevertheless, it can be shown that if the system of bodies under consideration is stationary on the whole (its total momentum is equal to zero), its angular momentum vector is independent of our choice of "centre" This angular momentum may be called the internal angular momentum of the system of bodies.

The law of conservation of angular momentum vector is the third and last conservation law in mechanics. However, we are not being entirely precise when speaking of three conservation laws. In fact, momentum and angular momentum are vector quantities, and a law of conservation of a vector quantity implies that not only its magnitude remains constant but also its direction. To put it otherwise, the three components of a vector in three mutually perpendicular directions in space remain constant. Energy is a scalar quantity, momentum is a vector quantity, and angular momentum is also a vector quantity. It would therefore be more precise to say that seven conservation laws hold in mechanics.

Tops

Try to place a plate topside up on a thin stick and keep it in a position of equilibrium. Nothing will come of your efforts. However, this is a favourite trick of Chinese jugglers. They succeed in performing it with several sticks simultaneously. A juggler doesn't even attempt to maintain his thin sticks in a vertical position. It appears to be a miracle that the plates slightly supported by the ends of the horizontally inclined sticks do not fall and practically hang in the air.

If you have the opportunity of observing jugglers at work at close range, note the following significant detail: the juggler twists the plates in such a fashion that they rotate rapidly in their planes.

Juggling maces, rings or hats, the performer will in all cases impart a spin to them. Only then will the objects return to his hand in the same state in which they were put at the beginning.

What is the cause of such stability? It is related to the law of conservation of angular momentum. For when there is change in the direction of the axis of rotation, the direction of the angular momentum vector also changes. Just as a force is needed to change the direction of velocity, so a torque is needed to change the direction of rotation; the faster the body rotates, the greater the torque required.

The tendency of a rapidly rotating body to preserve the direction of its axis of rotation can be observed in many cases similar to those mentioned. Thus, a spinning top does not tip over even if its axis is inclined.

Try to overturn a spinning top with your hand. It proves to be not so easy to do this.

The stability of a rotating body is utilized in the artillery. You have probably heard that gun barrels are rifled. An outgoing projectile spins about its axis and, because

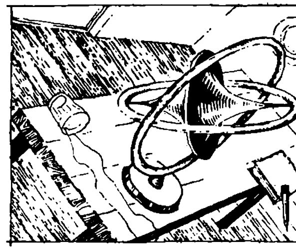


Figure 5.15

of this, does not "tumble" through the air. A rifled gun gives incomparably better aiming and greater range than an unrifled one.

It is necessary for a pilot or a sea navigator to always be aware of the location of the true terrestrial vertical relative to the position of the airplane or the ship at the given instant. The use of a plumb-line is unsuitable for this purpose, since it is deflected during an accelerated motion. A rapidly spinning top of special construction is therefore employed—it is called a gyrovertical. If we set its axis of rotation along a terrestrial vertical, it will then remain in this position, regardless of how the airplane changes its position in space.

But what does the top stand on? If it is located on a support which is turning together with the airplane, how can its axis of rotation preserve its direction?

An apparatus like the Cardan suspension (Figure 5.15) serves as the support. In this apparatus, with a minimum of friction at the pivots, a top can behave as though it were suspended in air.

With the aid of spinning tops, it is possible to automatically keep torpedoes and airplanes on a given course. This is done by means of mechanisms "watching" the deviation of the direction of the torpedo's axis from that of the top's axis.

Such an important instrument as the gyrocompass is based on the application of the spinning top. It can be proved that under the action of the Coriolis force and friction, the top's axis eventually settles down parallel to the Earth's axis, and so points to the North.

Gyrocompasses are widely applied in navigation. Their main part is an engine with a heavy flywheel which does up to 25 000 rpm.

In spite of a number of difficulties involved in the elimination of various hindrances, in particular those due to the pitching of a ship, gyrocompasses have an advantage over magnetic compasses. The drawback of the latter is the distortion of the readings because of the influence of iron objects and electrical appliances aboard the ship.

Flexible Shaft

Shafts are important parts of modern steam turbines. The manufacture of such shafts which are 10 m in length and 0.5 m in diameter is a complex technological problem. The shaft of a powerful turbine can withstand a load of about 200 t and rotate with a speed of 3000 rpm.

At first glance, it might seem that such a shaft should be exceptionally hard and durable. This, however, is not so. At tens of thousands of revolutions per minute, a rigidly fastened and unbendable shaft will inevitably break, no matter how strong it may be.

It isn't difficult to see why rigid shafts are unsuitable. No matter how precisely engineers work, they cannot avoid at least a slight asymmetry in the wheel of a turbine. Enormous centrifugal forces arise when such a wheel rotates; recall that their magnitudes are proportional to the square of the rotational speed. If they are not exactly balanced, the shaft will start "beating" against the ball bearings (for the unbalanced centrifugal forces "rotate" together with the machine), break them and smash the turbine.

At one time, this phenomenon created an unsurmountable obstacle to the increase in the rotational speed of a turbine. A way out of the situation was found at the last turn of the century. The flexible shaft was introduced into the technology of turbine construction.

In order to understand the idea behind this remarkable invention, we must compute the total effect of the centrifugal forces. But how can these forces be added? It turns out that the resultant of all the centrifugal forces acts at the centre of gravity of the shaft and has the same magnitude as if the entire mass of the wheel of the turbine were concentrated at the centre of gravity.

Let us denote the distance from the centre of gravity of the wheel of the turbine to its axis, distinct from zero because of a slight asymmetry in the wheel, by a. During rotation, centrifugal forces will act on the shaft which will bend. Denote the displacement of the shaft by l. Let us compute this magnitude. We know the formula for centrifugal force (see p. 81). This force is proportional to the distance from the centre of gravity to the axis, which is now $a + l$, and is equal to $4\pi^2 n^2 M (a + l)$, where n is the number of revolutions per minute, and M is the mass of the rotating parts. The centrifugal force is balanced by the elastic force, which is proportional to the magnitude of the displacement of the shaft and is equal to kl, where the coefficient k characterizes the rigidity of the shaft. Thus:

$$kl = 4\pi^2 n^2 M (a + l)$$

whence

$$l = \frac{a}{k/4\pi^2 n^2 M - 1}$$

Judging by this formula, fast rotations are no problem for a flexible shaft. For very large (even infinitely large) values of n, the deflection l of the shaft does not grow without bound. The value of $k/4\pi^2n^2M$ figuring in our last formula tends to zero, and the deflection l of the shaft becomes equal in magnitude to the asymmetry, but opposite in sign.

This computational result implies that, for fast rotations, the asymmetrical wheel, instead of smashing the shaft, bends it in such a way as to cancel the effect of asymmetry. The bending shaft centres the rotating parts, transfers the centre of gravity to the axis of rotation by means of its deformation, and thus nullifies the action of the centrifugal force.

The flexibility of the shaft is by no means a drawback; on the contrary, it is a necessary condition for stability. As a matter of fact, it is necessary for stability that the shaft bend by a distance of the order of a without breaking.

An attentive reader may have noticed an error in the reasoning employed. If we displace a shaft "centring" during fast rotations from the position of equilibrium we have found and consider only centrifugal and elastic forces, it is easy to see that this equilibrium is unstable. It turns out, however, that Coriolis forces save the situation and make this equilibrium quite stable.

A turbine starts turning slowly. At first, when n is very small, the fraction $k/4\pi^2n^2M$ will be great. As long as this fraction is greater than unity with increasing n, the deflection of the shaft will have the same sign as that of the original displacement of the centre of gravity of the wheel. Therefore, at the beginning of the motion the bending shaft does not centre the wheel, but, on the contrary, increases the total displacement of the centre of gravity by means of its deformation, and hence also the centrifugal force. To the degree that n increases (with the condition $k/4\pi^2n^2M > 1$ preserved), the displacement grows and, finally, the critical moment is reached. The denominator of our formula for the displacement l vanishes when $k/4\pi^2 n^2 M = 1$, and so the deflection of the shaft formally becomes infinitely large. The shaft will break at such a speed of rotation. In starting a turbine. this moment must be passed very quickly; it is necessary to slip by the critical number of revolutions per minute and pass over to a much faster motion of the turbine for which the phenomenon of self-centring described above will begin.

But what is this critical moment? We can rewrite its condition in the following form:

$$4\pi^2 \frac{M}{k} = \frac{1}{n^2}$$

Or, expressing the number of revolutions per minute in terms of the period of rotation by means of the relation $n = 1/T$ and extracting square roots, we can rewrite it as follows:

$$T = 2\pi \sqrt{\frac{M}{k}}$$

But what kind of quantity have we obtained in the right-hand side of the equality? Our formula looks rather familiar. Turning to p. 142, we see that the period of free vibration of the wheel on the shaft figures in our right-hand side. The period $2\pi \sqrt{M/k}$ is that with which the wheel of a turbine of mass M would vibrate on a shaft of rigidity k if we were to deflect the wheel to one side, so that it might vibrate by itself.

Thus, the dangerous instant is when the rotational period of the wheel of the turbine coincides with the period of free vibration of the system turbine-shaft. Resonance is responsible for the existence of a critical number of revolutions per minute.

6. Gravitation

What Holds the Earth Up!

In the distant past, people gave a simple answer to this question: the three whales. True, it remained unclear what was holding the whales up. However, this did not disturb our naive forefathers.

Correct ideas about the nature of the Earth's motion, the Earth's form and many regularities in the motion of the planets around the Sun had arisen long before an answer was given to the question of the causes for the motion of the planets.

And really, what "holds up" the Earth and the planets? Why do they move around the Sun along definite paths instead of flying away from it?

There was no answer to these questions for a long time, and the Church, struggling against the Copernican system of the Universe, used this to negate the fact of the Earth's motion.

We are obliged to the great English scientist Isaac Newton for his discovery of the true answers.

A well-known historical anecdote asserts that while sitting in an orchard under an apple-tree, thoughtfully observing how one apple after another fell to the ground because of gusts of wind, Newton arrived at the idea of the existence of gravitational forces between all bodies in the Universe.

As a result of Newton's discovery, it became clear that many apparently miscellaneous phenomena—the free fall of bodies to the Earth, the apparent motions of the

Moon and the Sun, the ocean tides, etc.—are manifestations of one and the same law of nature—the law of universal gravitation.

Between all bodies in the Universe, asserts this law, be they grains of sand, peas, stones or planets, forces of mutual attraction are exerted.

At first sight, this law seems false: we somehow haven't noticed that the objects surrounding us were attracted to each other. The Earth attracts all bodies to itself; no one will have any doubt about this. But perhaps this is a special property of the Earth? No, that isn't so. The attraction of two arbitrary objects is slight, and this is the only reason why it doesn't arrest our attention. Nevertheless, it can be detected by means of special experiments. But more about that later.

The presence of universal gravitation, and nothing else, explains the stability of the solar system and the motion of the planets and other celestial bodies.

The Moon is kept in orbit by terrestrial gravitational forces, and the Earth on its trajectory by solar gravitational forces.

The circular motion of celestial bodies occurs in the same way as the circular motion of a stone twirled on a string. The forces of universal gravitation are invisible "ropes" compelling celestial bodies to move along definite paths.

The assertion of the existence of universal gravitational forces didn't really mean much. Newton discovered the law of gravitation and showed what these forces depend on.

Law of Universal Gravitation

The first question which Newton asked himself was the following: How does the Moon's acceleration differ from that of an apple? To put it otherwise, what is the difference between the acceleration g which the Earth creates on its surface, i.e. at the distance r from its centre, and the acceleration created by the Earth at the distance R at which the Moon is located from the Earth?

In order to calculate this acceleration, v^2/R, it is necessary to know the speed of the Moon's motion and its distance from the Earth. Both these figures were known by Newton. The Moon's acceleration turned out to be approximately equal to 0.27 cm/s^2. This is about 3600 times less than the value of g, 980 cm/s^2.

Hence, the acceleration created by the Earth decreases as one recedes from the centre of the Earth. But how rapidly? The distance from the Earth to the Moon equals sixty terrestrial radii. But 3600 is the square of 60. Increasing the distance by a factor of 60, we decrease the acceleration by a factor of 60^2.

Newton concluded that the acceleration, and therefore also the gravitational force, is inversely proportional to the square of the distance. Further, we already know that the force exerted on a body in a gravitational field is proportional to its mass. Therefore, the first body attracts the second with the force proportional to the mass of the second body; the second body attracts the first with the force proportional to the mass of the first body.

We are dealing with identically equal forces—forces of action and reaction. Consequently, the mutual gravitational force must be proportional to the mass of the first body as well as to that of the second or, to put it otherwise, to the product of the masses.

Thus,

$$F = G\frac{Mm}{r^2}$$

This is precisely the *law of universal gravitation.* Newton assumed that this law will be valid for any pair of bodies.

This bold hypothesis is now completely proved. Therefore, the force of attraction between two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

But what is this G that entered the formula? This is the coefficient of proportionality. May we assume it to be equal to unity, as we have already repeatedly done? No, we may not: we have agreed to measure mass in grams, distance in centimetres, and force in dynes. The value of G is equal to the force of attraction between two masses of 1 g located at a distance of 1 cm from each other. We cannot assume that the force is equal to anything (in particular, to one dyne). The coefficient G must be measured.

In order to find G, we don't, of course, have to measure the forces of attraction between gram weights. We are interested in carrying out measurements on massive bodies—the force will be greater then.

If we determine the mass of two bodies, know the distance between them and measure the force of attraction, then G will be found by a simple calculation.

Such experiments were performed many times. They showed that the value of G is always one and the same, independent of the material of the attracting bodies and also of the properties of the medium in which they are situated. The quantity G is called the *gravitational constant*. It is equal to 6.67×10^{-8} $cm^3/g \cdot s^2$.

The diagram of one of the experiments on measuring G is shown in Figure 6.1. Two balls of identical mass are suspended from the ends of a beam of scales. One of them is situated above a lead plate, the other beneath it. By means of its attraction, the lead (100 tons of it are taken for the experiment) increases the weight of the ball on the right and decreases that of the ball on the left. The former outweighs the latter. The value of G is computed on the basis of the magnitude of the deflection of the beam.

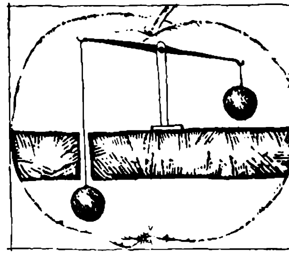


Figure 6.1

The difficulty in detecting gravitational forces between two objects is explained by the negligible value of G.

Two heavy 1000-kg loads pull each other with a negligible force equal in all to only 6.7 dyn, i.e. 0.007 gf, if these objects are situated, say, at a distance of 1 m from each other.

But how great are the forces of attraction between celestial bodies? Between the Moon and the Earth

$$F = 6.7 \times 10^{-8} \frac{6 \times 10^{27} \times 0.74 \times 10^{26}}{(38 \times 10^{9})^2} =$$

$$= 2 \times 10^{25} \text{ dyn} \approx 2 \times 10^{19} \text{ kgf}$$

between the Earth and the Sun

$$F = 6.7 \times 10^{-8} \frac{2 \times 10^{33} \times 6 \times 10^{27}}{(15 \times 10^{12})^2} =$$

$$= 3.6 \times 10^{27} \text{ dyn} \approx 3.6 \times 10^{21} \text{ kgf}$$

Weighing the Earth

Before beginning to make use of the law of universal gravitation, we must turn our attention to an important detail.

We have just calculated the force of attraction between two loads located at a distance of 1 m from each other. But if this distance were 1 cm? What would we then substitute in the formula—the distance between the surfaces of bodies or the distance between their centres of gravity or some other value?

The law of universal gravitation, $F = Gm_1m_2/r^2$, can be applied with complete rigour when such doubts do not arise. The distance between the bodies should be much greater than their dimensions; we should have the right to regard the bodies as points. But how should we apply the law to two nearby bodies? This is simple in principle: we must conceptually break up the bodies into small pieces, calculate the force F for each pair and then add (vectorially) all the forces.

In principle this is simple, but it is rather complicated in practice.

However, nature has helped us. Computations show that if the particles of a body interact with a force proportional to $1/r^2$, spherical bodies possess the property of attracting like points located at the centres of the spheres. For two nearby spheres, the formula $F = Gm_1m_2/r^2$ is exactly valid, just as for distant spheres, if r is the distance between their centres. We have already used this rule above in computing the acceleration on the Earth's surface.

We now have the right to apply the gravitational formula for computing the forces with which the Earth attracts bodies. We should take the distance from the centre of the Earth to the body as r.

Let M be the mass, and R the radius of the Earth. Then the force of attraction acting on a body of mass m at the Earth's surface

$$F = Gm\frac{M}{R^2}$$

But this is in fact the body's weight, which we always express as mg. Hence, the acceleration of free fall

$$g = G\frac{M}{R^2}$$

Now at last we can say how the Earth was weighed. The quantities g, G and R are known, so the Earth's mass can be computed from this formula. The Sun can also be weighed in the same manner.

But can we really call such a procedure weighing? Of course we can; indirect measurements play at least as great a role in physics as direct measurements.

Let us now solve a curious problem.

An essential role in the plans for creating world-wide television is played by the creation of a "24-hour satellite", i.e. one which will always be situated over one point on the Earth's surface. Will such a satellite experience a significant frictional force? This depends on how far from the Earth it will have to perform its rotation.

A 24-hour satellite should revolve with a period T equal to 24 hours. If r is the distance from the satellite to the centre of the Earth, then its speed $v = 2\pi r/T$ and its acceleration $v^2/r = 4\pi^2 r/T^2$. On the other hand, this acceleration whose source is the Earth's attraction is equal to $GM/r^2 = gR^2/r^2$. Equating our two expressions for the acceleration, we obtain:

$$g\frac{R^2}{r^2} = \frac{4\pi^2 r}{T^2}, \quad \text{i.e.} \quad r^3 = \frac{gR^2T^2}{4\pi^2}.$$

Substituting the rounded-off values of $g = 10$ m/s^2, $R = 6 \times 10^6$ m and $T = 9 \times 10^4$ s, we obtain: $r^3 = 7 \times 10^{22}$ m^3, i.e. $r \approx 4 \times 10^7$ m $= 40\,000$ km. There is no air friction at such a height, and a 24-hour satellite will not slow down its "motionless orbiting".

Measuring g in the Service of Prospecting

The topic is geological prospecting whose aim is to find deposits of useful minerals under the Earth without digging a pit or sinking a shaft.

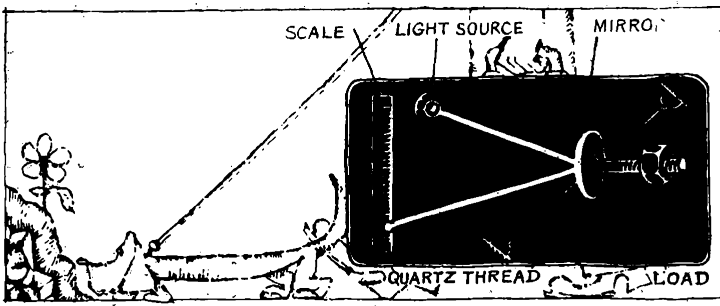


Figure 6.2

There exist several methods of determining the acceleration of free fall very accurately. It is possible to find g by simply weighing a standard weight on a spring balance. Geological balances should be extremely sensitive—their spring changes its tension when a load of less than a millionth of a gram is added. Quartz torsion balances yield excellent results. Their construction isn't complicated in principle. To a horizontally stretched quartz thread a lever is welded whose weight slightly twists the thread (Figure 6.2).

A pendulum is used for the same purposes. Not very long ago pendulum methods of measuring g were the only ones, and only in the last 10-20 years have the more convenient and precise balance methods begun to supplant them. In any case, measuring the period of oscillation of a pendulum one can find the value of g accurately enough from the formula $T = 2\pi\sqrt{l/g}$.

Measuring values of g at different places with the same apparatus, we can detect relative changes in the free fall up to one-millionth.

Measuring the value of g at some place on the Earth's surface, the experimenter ascertains: here the value is anomalous, it is so much less than the norm or such an amount greater than the norm.

But what is the norm for the value of g?

There are two natural changes which have long been observed and are well known to researchers in the value of the acceleration of free fall on the Earth's surface.

First of all, g decreases from a pole to the equator. This has been spoken of above. Let us only recall that such a change occurs as a result of two causes: firstly, the Earth isn't a sphere, and a body at a pole will be nearer to the centre of the Earth; secondly, the more a body advances towards the equator, the more will the force of gravity be weakened by the centrifugal force.

The second change in g is the decrease due to elevation. The greater the distance from the Earth's centre, the smaller will be the value of g in accordance with the formula $g = GM/(R + h)^2$, where R is the radius of the Earth, and h is the height above sea level.

Therefore, at one and the same latitude and at one and the same height above sea level, the acceleration of free fall should be identical.

Accurate measurements show that deviations from this norm—gravitational anomalies—are found quite often. The cause of an anomaly consists in the heterogeneity of the mass distribution near the place of measurement.

As we explained, the gravitational force due to a large body can be conceptually represented as the sum of forces emanating from the individual particles of the large body. The attraction of a pendulum to the Earth is the result of the action of all the particles of the Earth on it. But it is clear that the nearby particles make the greatest contribution to the resultant force, for the attraction is inversely proportional to the square of the distance.

If heavy masses are concentrated near the place of measurement, g will be greater than the norm; in the opposite case, g will be smaller than the norm.

If, for example, we measure g on a mountain and in an airplane flying over a sea at an altitude equal to the mountain's height, a greater value will be obtained in the former case. For example, the value of g is 0.292 cm/s^2 greater than the norm on Mount Etna in Italy. The value of g is also higher than the norm on isolated ocean islands. It is clear that in both cases the growth of g is explained by the concentration of additional masses at the place of measurement.

Not only the value of g but also the direction of the force of gravity can deviate from the norm. If a load is suspended on a thread, the stretched thread will indicate the vertical for the given place. This vertical may deviate from the norm. A normal vertical can be determined by the stars, since it has been calculated for any geographical point at what place in the sky the vertical to the "ideal" form of the Earth is "set" at a given instant of a day and year.

Imagine that you are performing experiments with a plumb-line at the foot of a large mountain. The load of the plumb-line is attracted by the Earth towards its centre, and by the mountain to one side. Under such conditions, the plumb-line must be deflected from the direction of a normal vertical (Figure 6.3). Since the Earth's mass is much greater than that of the mountain, such a deflection will not exceed several seconds of arc.

Plumb-line deflections sometimes yield strange results. For example, in Florence the influence of the Appenines leads not to an attraction, but to a repulsion of a plumb-line. The explanation can only be as follows: there are enormous empty spaces in mountains.

Measurements of the acceleration of free fall to the scale of continents and oceans yield remarkable results. Continents are considerably heavier than oceans; therefore, it would seem that the values of g over continents should

Figure 6.3

be greater than those over oceans. But in reality, the values of g measured along a single latitude over oceans and continents are identical, on the average. Again there is only one explanation: continents lie on lighter bed-rocks, and oceans on heavier ones. And as a matter of fact, where direct prospecting is possible, geologists ascertain that oceans lie on heavy basaltic bed-rocks, and continents on light granite ones.

But the following question immediately arises: Why do heavy and light bed-rocks compensate so exactly for the difference in weight between continents and oceans? Such a compensation cannot be a matter of chance; its cause must be rooted in the construction of the Earth's shell.

Geologists assume that it is as though the upper layers of the Earth's shell were floating on an underlying plastic (i.e. easily deformed like wet clay) mass. The pressure at depths of about 100 km should be identical everywhere, just as the pressure at the bottom of a vessel filled with water in which pieces of wood of various weights are floating is identical everywhere. Consequently, a column of matter with an area of 1 m^2 from the surface to a depth of 100 km should have the same weight under an ocean and under a continent.

This levelling of pressures (it is called isostasy) is just what leads to the situation where along a single latitude over oceans and continents the values of the acceleration of free fall g do not differ significantly.

Local gravitational anomalies serve us just as the magic wand, which banged on the ground where there was gold or silver, served little Mook in Hauf's fairy-tale.

One must look for heavy ore in those places where g is maximum. On the contrary, light salt deposits are discovered by finding localities with lowered values of g. It is possible to measure g with an accuracy up to a hundred-thousandth of 1 cm/s^2.

Prospecting with the aid of pendulums and superexact scales is called gravitational. It is of great practical value, in particular when looking for oil. The fact is that with gravitational prospecting, it is easy to discover underground salt domes. It so happens that often oil is found at those places too. Moreover, the oil lies at some depth, while the salt is nearer to the Earth's surface. Oil was discovered in Kazakhstan and in other places by gravitational prospecting.

Weight Underground

It remains for us to throw light on another interesting question. How will the force of gravity change if we go deep underground?

The weight of an object is the result of the tension in, so to say, invisible "threads" reaching out to this object from every piece of matter in the Earth. Weight is the resultant force, the result of the addition of the elementary forces exerted on the object by the Earth's particles. All these forces, even though directed at different angles, pull a body "down"—towards the centre of the Earth.

Figure 6.4

But what will be the weight of an object in an underground laboratory? Forces of attraction will be exerted on it both by the internal and external layers of the Earth.

Consider the gravitational forces exerted at a point lying inside the Earth by an external layer. If we break up this layer into thin shells, cut out in one of them a small square with side a_1 and draw lines from the vertices of the square through the point O (we are interested in the weight at those points), then on the opposite side of the shell we obtain a small square of a different size with side a_2 (Figure 6.4). The forces of attraction exerted at the point O by the two small squares are oppositely directed and proportional according to the law of gravitation to m_1/r_1^2 and m_2/r_2^2. But the masses of the squares, m_1 and m_2, are proportional to their areas. Therefore the gravitational forces are proportional to the expressions a_1^2/r_1^2 and a_2^2/r_2^2.

I suggest that the reader prove that these ratios are the same, that is, that the forces of attraction at point O acting from the two small squares balance.

Having broken up a thin shell into pairs of "opposite" similar squares, we established a remarkable fact: a thin homogeneous spherical shell does not act on a point within it. But this is true for all the thin shells into which we broke up the spherical layer lying above the underground point we are interested in.

Hence, the layer of the Earth lying above the body might just as well be absent. The action of its individual parts on the body is neutralized, and the resultant force of attraction exerted by the external layer is equal to zero.

Of course, throughout this reasoning we have assumed the Earth's density to be constant within each shell.

The result of our reasoning permits us to easily obtain a formula for the gravitational force exerted at any depth H under the Earth. A point situated at depth H only experiences the attraction exerted by the internal layers of the Earth. The formula for the acceleration due to gravity, $g = GM/R^2$, also applies to this case, where M and R are the mass and radius not of the entire Earth but of its "internal" part with respect to this point.

If the Earth had the same density in all its layers, the formula for g would assume the following form:

$$g = G\frac{\frac{4}{3}\pi\rho(R-H)^3}{(R-H)^2} = \frac{4}{3}\pi G\rho(R-H)$$

where ρ is the density, and R is the Earth's radius.

This implies that g would be directly proportional to $(R - H)$: the greater the depth H, the smaller would be g.

But as a matter of fact, the behaviour of g near the Earth's surface—we are able to observe it up to a depth of 5 km (below sea level)—does not obey this law at all. Experiments show that g, on the contrary, increases with depth within these layers. The lack of agreement between the experiments and our formula is explained by the fact that the difference in density at various depths was not taken into account.

The average density of the Earth is easily found by dividing its mass by its volume. This yields a value of 5.52. At the same time, the density of the surface bed-rocks is much smaller—it is equal to 2.75. The density of the Earth's layers increases with depth. Within the surface layers of the Earth, this effect dominates the ideal decrease which follows from the formula just derived, and so the value of g increases.

Gravitational Energy

We have already become acquainted with gravitational energy through a simple example. A body raised to height h above the Earth possesses potential energy mgh.

However, this formula may be used only when height h is much smaller than the Earth's radius.

Gravitational energy is an important quantity, and it would be interesting to obtain a formula for it which would apply to a body raised to an arbitrary height above the Earth and also, more generally, for two masses attracting each other in accordance with the universal law:

$$F = G\frac{m_1 m_2}{r^2}$$

Let us assume that the bodies approached each other somewhat under the action of their mutual attraction. The distance between them was r_1, but it became r_2. Moreover, the work $A = F(r_1 - r_2)$ is performed. The value of the force must be taken at some intermediate point. Thus,

$$A = G\frac{m_1 m_2}{r_{int}^2}(r_1 - r_2)$$

If r_1 and r_2 do not differ much, we may replace r_{int}^2 by the product $r_1 r_2$. We obtain:

$$A = G\frac{m_1 m_2}{r_2} - G\frac{m_1 m_2}{r_1}$$

This work is performed at the expense of the gravitational energy:

$$A = U_1 - U_2$$

where U_1 is the initial and U_2 the final value of the gravitational potential energy.

Comparing these two formulas, we find the following expression for the potential energy:

$$U = -G\frac{m_1 m_2}{r}$$

It resembles the formula for the gravitational force, but r is raised to the first power in the denominator.

According to this formula, the potential energy $U = 0$ for very large r's. This is reasonable, since the attraction will no longer be felt at such distances. But when the bodies approach each other, the potential energy should decrease. After all, the work takes place at its expense.

But in what direction can it decrease from zero? In the negative direction. Hence there is a minus sign in the formula. After all, —5 is less than zero, and —10 is less than —5.

If we are dealing with motion near the Earth's surface, we may replace the general expression for the gravitational force by mg. Then with greater accuracy we have $U_1 - U_2 = mgh$.

But on the Earth's surface, a body has potential energy $-GMm/R$, where R is the Earth's radius. Therefore, at height h above the Earth's surface,

$$U = -G\frac{Mm}{R} + mgh$$

When we first introduced the formula for potential energy, $U = mgh$, we agreed to measure height and energy from the Earth's surface. Using the formula $U = mgh$, we discard the constant term $-GMm/R$, regarding it as conditionally equal to zero. Since we are interested only in energy differences, for it is work which is an energy difference that is ordinarily measured, the presence of the constant term $-GMm/R$ in the potential energy formula does not play any role.

Gravitational energy determines the strength of the "chains" binding a body to the Earth. How can we break these "chains"? How can we ensure that a body thrown from the Earth will not return to the Earth? It is clear that to do this we must impart a large initial velocity to the body. But what is the minimum velocity that is required?

As a body (missile, rocket) thrown from the Earth increases its distance from the Earth, its potential energy will rise (the absolute value of U will fall); its kinetic energy will fall. If its kinetic energy becomes equal to zero prematurely, before we break the Earth's gravitational "chains", the missile that was thrown will fall back to the Earth.

It is necessary for the body to conserve its kinetic energy until its potential energy practically vanishes. Before its departure, a missile had potential energy $-GMm/R$ (M and R are the mass and radius of the Earth). Therefore, the missile must be given the velocity which would make its total energy positive. A body with a negative total energy (the magnitude of its potential energy is greater than that of its kinetic energy) will not get beyond the bounds of gravity.

Hence, we arrive at the simple condition. In order for a body of mass m to break away from the Earth, it must, as has been already said, overcome the gravitational potential energy

$$G\frac{Mm}{R}$$

For this, the speed of the missile should be increased to the value of the escape velocity from the Earth, v_2, which is easily computed by equating its kinetic and potential energies:

$$\frac{mv_2^2}{2}=G\frac{Mm}{R}, \quad \text{i.e.} \quad v_2^2=2G\frac{M}{R}$$

or, since $g = GM/R^2$,

$$v_2^2=2gR$$

The value of v_2 computed by means of this formula is 11 km/s, of course, without taking air resistance into account. This speed is $\sqrt{2} = 1.41$ times as great as the orbital velocity v_1 $\sqrt{gR}$ of an artificial satellite whose orbit is near the Earth's surface, i.e. $v_2 = \sqrt{2}v_1$.

The mass of the Moon is 81 times as small as that of the Earth; the radius of the Moon is four times as small as that of the Earth. Consequently, the gravitational energy on the Moon is twenty times less than that on the Earth, and a speed of 2.5 km/s is sufficient to break away from the Moon.

Kinetic energy $mv_2^2/2$ is spent in order to break the gravitational "chains" to the planet—the take-off station. If we want the rocket which has overcome gravity to move with speed v, then additional energy $mv^2/2$ is needed for this. In such a case, when launching the rocket, it is necessary to impart it energy $mv_0^2/2 = m_2^2v/2 + mv^2/2$. Therefore, the three speeds in question are connected by the simple relation:

$$v_0^2=v_2^2+v^2$$

What should be the speed v_3 necessary for overcoming the gravitation of the Earth and the Sun—the minimum speed of a missile sent to distant stars? We denoted this speed by v_3 because it is called the escape velocity from the solar system.

First of all, let us determine the speed necessary for overcoming only the single attraction of the Sun.

As we have just shown, the speed needed to escape from the Earth's attraction by a missile sent on a flight is $\sqrt{2}$ times as great as the speed with which an Earth satellite is sent into orbit. Our reasoning is equally valid or the Sun, i.e. the speed needed to escape from the Sun is $\sqrt{2}$ times as great as the speed of a satellite of the Sun (i.e. the Earth). Since the speed of the Earth's motion around the Sun is about 30 km/s, the speed needed to escape from the sphere of the Sun's attraction is 42 km/s. This is a very great speed, but for sending a missile to distant stars, we must, of course, use the Earth's motion and launch the body in the direction in which the Earth is moving. We then need to add only 42 — 30 = 12 km/s.

Now we can finally compute the escape velocity from the solar system. This is the speed with which a rocket must be launched in order that, escaping from the Earth's attraction, it have a speed of 12 km/s. Using the formula just adduced, we obtain:

$$v_3^2 = 11^2 + 12^2$$

from which $v_3 = 16$ km/s.

Thus, having a speed of about 11 km/s, a body will leave the Earth, but such a missile will not go "far" away; the Earth let it go, but the Sun will not free it. It will turn into a satellite of the Sun.

It turns out that the speed necessary for interstellar travel is only one and a half times as great as the speed needed for travelling through the solar system within the Earth's orbit. True, as has been already said, every appreciable increase in the initial speed of a missile is accompanied by many technical difficulties (see p. 104f).

How Planets Move

The question as to how planets move can be answered briefly: obeying the law of gravitation. For the forces of gravitation are the only forces applied to planets.

Since the mass of the planets is much less than that of the Sun, the forces of interaction between the planets do not play a large role. Each of the planets moves almost the way the gravitational force of the Sun alone dictates, as though the other planets did not even exist.

The laws of planetary motion around the Sun follow from the law of universal gravitation.

Incidentally, this isn't the way things developed historically. The laws of planetary motion were discovered by the outstanding German astronomer Johannes Kepler (1571-1630), before Newton and without the aid of the law of gravitation, on the basis of an almost twenty-year processing of astronomical observations.

The paths or, as astronomers say, the orbits which planets describe around the Sun are very close to circles.

How is the period of revolution of a planet related to the radius of its orbit?

The gravitational force exerted on a planet by the Sun is equal to

$$F = G\frac{Mm}{r^2}$$

where M is the mass of the Sun, m is the mass of the planet, and r is the distance between them.

But F/m is, according to the basic law of mechanics, none other than the acceleration; moreover, it is centripetal:

$$\frac{F}{m} = \frac{v^2}{r}$$

The speed of the planet can be represented as the length $2\pi r$ of the circumference divided by the period of revolution T. Substituting $v = 2\pi r/T$ and the value of the force F in the acceleration formula, we obtain:

$$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}, \quad \text{i.e.} \quad T^2 = \frac{4\pi^2}{GM} r^3$$

The coefficient of proportionality preceding r^3 is the quantity depending only on the mass of the Sun; it is identical for any planet. Consequently, the following relation holds for two planets:

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

The ratio of the squares of the periods of revolution of planets turns out to be equal to the ratio of the cubes of their orbital radii. This interesting law was derived empirically by Kepler. The law of universal gravitation explained Kepler's observations.

A circular motion of one celestial body around another is only one of the possibilities.

The trajectories of one body revolving around another due to gravitational forces can be very different. However, as shown by calculations and as Kepler had observed before any calculations were made, they all belong to one and the same class of curves, called ellipses.

If we tie a thread to two pins stuck in a sheet of drawing paper, stretch the thread with the point of a pencil and move the pencil in such a way that the thread remains stretched, a closed curve will eventually be drawn on the paper—this is an ellipse (Figure 6.5). The points where the pins are stuck will be the foci of the ellipse.

Ellipses can have various forms. If the thread is taken much longer than the distance between the pins, then the ellipse will be very similar to a circle. If, on the contrary, the length of the thread barely exceeds the distance between the pins, then an elongated ellipse—almost a stick—will be obtained.

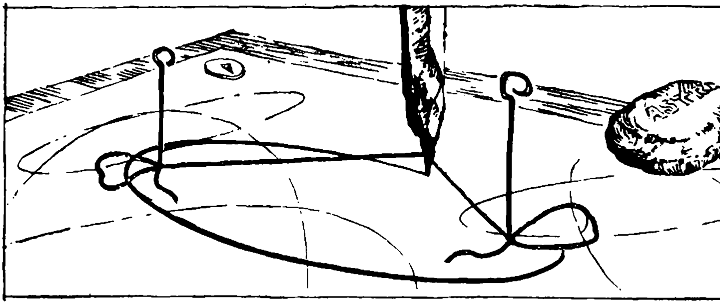


Figure 6.5

A planet describes an ellipse at one of whose foci is the Sun.

But what kind of ellipses do planets describe? It turns out that they are very close to circles.

The path of the planet nearest to the Sun—Mercury—differs most from a circle. But even in this case, the longest diameter of the ellipse is only 2% greater than the shortest one. The situation is different with the orbits of artificial satellites. Take a look at Figure 6.6. You can't distinguish the orbit of Mars from a circle.

However, since the Sun is located at one of the foci of the ellipse and not at its centre, the distance of a planet from the Sun changes more noticeably. Let us draw a line through the two foci of an ellipse. This line intersects the ellipse at two places. The point nearest to the Sun is called the perihelion, the farthest from the Sun the aphelion. Mercury, when located at the perihelion, is 1.5 times closer to the Sun than at the aphelion.

The major planets describe ellipses around the Sun which are close to circles. However, there are celestial bodies which move around the Sun in greatly flattened ellipses. Among them are comets. Their orbits are not

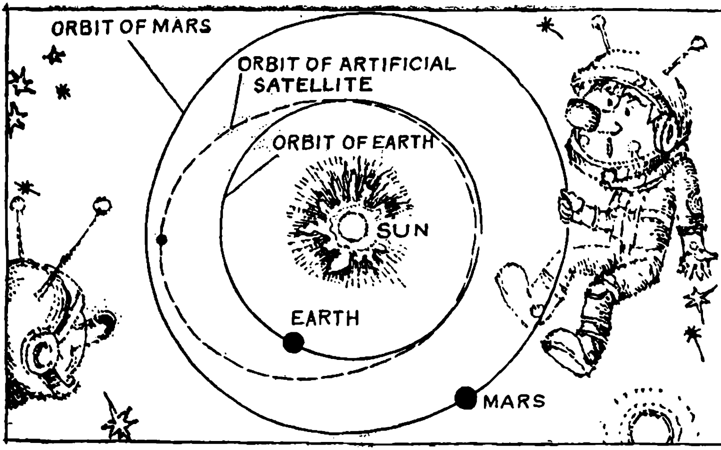


Figure 6.6

at all comparable with respect to elongation to those of the planets. With regard to the celestial bodies moving in ellipses it can be said that they belong to the solar family. However, casual newcomers also drop in at our system.

There have been observed comets describing curves around the Sun whose forms suggest the following conclusion: the comet will not return; it does not belong to the family of the solar system. The "open" curves described by comets are called hyperbolas.

Such comets move especially fast when passing near the Sun. This is understandable, since the total energy of a comet is constant and, when approaching the Sun, it has the minimum potential energy. Hence, its kinetic energy is maximum at this time. Of course, such an effect takes place for all the planets and for our Earth. However, this effect is slight, since the difference in the potential energies at the aphelion and perihelion is small.

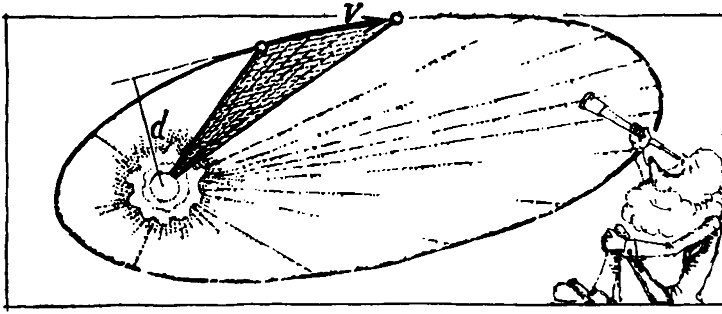


Figure 6.7

An interesting law f planetary motion follows from the law of conservation of angular momentum.

Two positions of a planet are depicted in Figure 6.7. From the Sun, i.e. from a focus of the ellipse, the two radii are drawn to the two positions of the planet, and the sector so formed is shaded. We are to determine the area swept out by a radius in a unit of time. For a small angle, the sector swept out by a radius in a second may be replaced by a triangle. The base of the triangle is the speed v (the distance covered in a second), while the altitude of the triangle is equal to the lever arm d of the velocity. Therefore, the area of the triangle is $vd/2$.

The constancy of the quantity mvd during the motion follows from the law of conservation of angular momentum. But if mvd is constant, so is the area of the triangle $vd/2$. We can draw sectors for any interval of time—they will turn out identical in area. The speed of a planet changes, but the so-called areal velocity remains constant.

Not all stars are surrounded by planetary systems. There are quite a few double stars in the sky. Two enormous celestial bodies revolve around each other.

The Sun's enormous mass makes it the centre of the

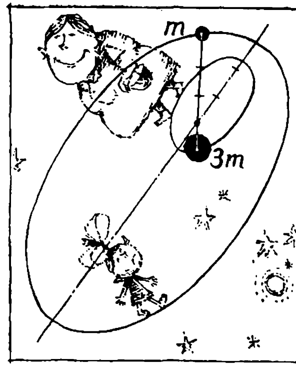


Figure 6.8

family. In double stars, both celestial bodies have masses of the same order of magnitude. In this case, we may not assume that one of the two stars is stationary. But how does the motion proceed in this case? We know that each closed system has one stationary (or uniformly moving) point—its centre of mass. Both the stars revolve around this point. Moreover, they describe similar ellipses, which follows from the condition written on p. 173, $m_1/m_2 = r_2/r_1$. The ellipse for one star is as many times greater than that for the other as the mass of one star is less than that of the other (Figure 6.8). In the case of equal masses, both the stars will describe identical trajectories around the centre of mass.

The planets of the solar system are in ideal conditions: they are not subject to friction.

The small, artificial celestial bodies created by people—satellites—are not in such an ideal position: frictional forces, however insignificant they may be at first, but none the less perceptible, interfere decisively in their motion.

The total energy of a planet remains constant. The total energy of a satellite falls slightly with every revolution. At first sight, it would seem that friction will slow down the motion of a satellite. In reality, the opposite occurs.

First of all, recall that the speed of a satellite is equal to $\sqrt{gR}$ or $\sqrt{GM/R}$, where R is its distance from the centre of the Earth, and M its mass.

The total energy of a satellite is equal to

$$E = -G\frac{Mm}{R} + \frac{mv^2}{2}$$

Substituting the value of the speed of the satellite, we find the expression $GMm/2R$ for the kinetic energy. We find that the magnitude of the kinetic energy is half as great as that of the potential one, while the total energy is equal to

$$E = -\frac{G}{2}\frac{Mm}{R}$$

In the presence of friction, the total energy falls, i.e. (since it is negative) its magnitude grows; the distance R starts decreasing: the satellite descends. What happens to the energy summands in this connection? The potential energy decreases (grows in its magnitude), the kinetic energy increases.

Nevertheless, the net change is negative, since the potential energy decreases twice as fast as the kinetic energy increases. Friction leads to a growth in the speed of a satellite and not to a reduction.

It is now clear why a large launch vehicle outflies a small satellite. The friction acting on a large rocket is greater.

Interplanetary Travel

We have already witnessed many trips to the Moon. Automatic space probes and manned craft have landed on its surface and then returned. Space probes travelled to Mars and Venus. And soon the other planets will also be visited and automatic stations and people will return from their surface.

We now know the main laws governing interplanetary travel, namely the principle of rocket motion and the method of calculating the different speeds that a body requires to orbit a celestial body and to escape its gravitational pull.

Let us take the trip to the Moon as an example. For this we must aim the rocket at a point on the Moon's orbit. The Moon must arrive at this point at the same time as the rocket. The rocket may follow various trajectories, even a straight one. But it is essential that it attain the Earth's escape velocity. We must also bear in mind that different trajectories require different amounts of fuel since fuel consumption depends on acceleration. Another factor is that flight time greatly depends on initial velocity. If this is minimum, the trip will take about five days, but if the velocity is increased by 0.5 km/s, flight time decreases to 24 hours.

It may seem that to get to the Moon the rocket must only reach the region of the Moon's attraction with zero velocity. After that it will simply fall onto the Moon. But such reasoning is erroneous, since when the rocket has a zero velocity with respect to the Earth, its velocity with respect to the Moon is the velocity of the Moon on its orbit around the Earth but oppositely directed.

Figure 6.9 shows the trajectory of a rocket launched at point A and the path of the Moon. We can imagine that the region of the Moon's attraction moves along the same path (in this region the only force that acts on the rocket is the Moon's gravitational pull). When the rocket enters this region at point B, the Moon is at point C and has a velocity v_M equal to 1.02 km/s. If at B the velocity of the rocket with respect to the Earth were zero, with respect to the Moon it would be $-v_M$. The rocket most certainly will miss the Moon.

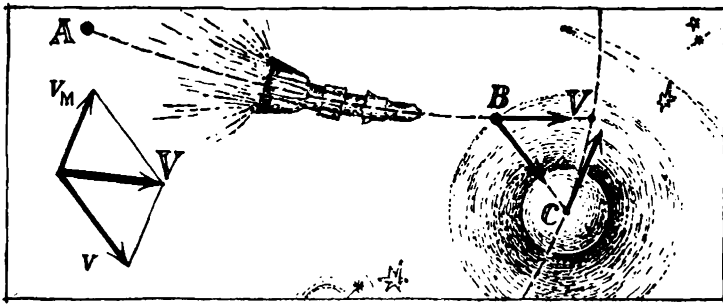


Figure 6.9

If we are observing the rocket from the Moon, we can be certain that it will meet the Moon at a right angle if its velocity is v. What, then, should its optimal trajectory and velocity be? The rocket must obviously hit point B not with a zero velocity but with the velocity V shown in Figure 6.9. For this we must simply use the velocity parallelogram shown in the same figure.

We still have some leeway. Velocity vector v does not have to be pointed at the very centre of the Moon. Besides, the gravitational pull of the Moon broadens the error range.

Calculations show, however, that there is very little elbowroom. The precision in initial velocity must be of the order of several metres per second, and the angle at which the rocket is launched must be set with an accuracy of one-tenth of a degree and the timing of the launching with an accuracy of several seconds.

So the rocket approaches the Moon with a non-zero velocity. Calculations show that this velocity, V, must be 0.8 km/s. The Moon's gravitational pull makes the velocity greater and the rocket will collide with the Moon at a velocity of 2.5 km/s. This is no good of course, since the rocket would disintegrate at impact. The only solution is to lower the speed of descent by using braking rockets. The process of cushioning touchdown requires a large supply of fuel. The formula on p. 104 shows that the rocket will "lose weight" by a factor of 2.7.

If we want the rocket to return to the Earth, it must have some fuel left. The Moon is a relatively small celestial body, only 3476 km across and with a mass of 7.34×10^{22} kg. We can easily see that its orbital velocity (i.e. the velocity required to maintain a satellite in an orbit around it) is 1680 m/s and its escape velocity is 2376 m/s, which means that to leave the Moon, the rocket must have a speed of about 2.5 km/s. With this minimum initial speed the rocket will return to the Earth after five days and will have the familiar speed of 11 km/s.

The path of reentry into the Earth's atmosphere must slope gently, since if there are astronauts inside the rocket the forces of acceleration must be kept to a minimum. But even if we are dealing with an automatic space probe, the probe must make several revolutions around the Earth so that the radius of its elliptical path decreases. Then the reentry vehicle does not get overheated and can safely return to the Earth.

Moon missions cost huge sums of money. If we assume that the return pay-load of a manned flight to the Moon is not less than 5 tons, then the total loaded weight at lift-off must be about 4.5 thousand tons. Experts believe that in the coming 20 years no more astronaut will visit the Moon or, for that matter, any other planet. New propulsion systems with greater exhaust velocities will have to be constructed. However, one cannot be sure of such predictions.

If There Were No Moon

We shall not discuss the sad consequences of the absence of the Moon for poets and lovers. The title of this section should be understood much more prosaically: how the Moon's presence affects terrestrial mechanics.

In our previous discussion of what forces act on a book lying on a table, we confidently stated: the Eearth's gravity and the reaction force. But, strictly speaking, a book lying on a table is also attracted by the Moon, the Sun and even the stars.

The Moon is our nearest neighbour. Let us forget about the Sun and the stars and consider how much the weight of a body on the Earth will change under the influence of the Moon.

The Earth and the Moon are in relative motion. With respect to the Moon the Earth as a whole (i.e. all points of the Earth) is moving with an acceleration Gm/r^2, where m is the mass of the Moon, and r is the distance from the centre of the Moon to that of the Earth.

Consider a body lying on the Earth's surface. We are interested in how much its weight will change owing to the Moon's action. Terrestrial weight is determined by acceleration with respect to the Earth. In other words, we are therefore interested in how much the acceleration with respect to the Earth of a body lying on the Earth's surface will be changed by the Moon's action.

The acceleration of the Earth with respect to the Moon is Gm/r^2; the acceleration with respect to the Moon of a body lying on the Earth's surface is Gm/r_1^2, where r_1 is the distance from the body to the centre of the Moon (Figure 6.10).

But we should find the additional acceleration of the body with respect to the Earth: it will be equal to the geometrical difference between the appropriate accelerations.

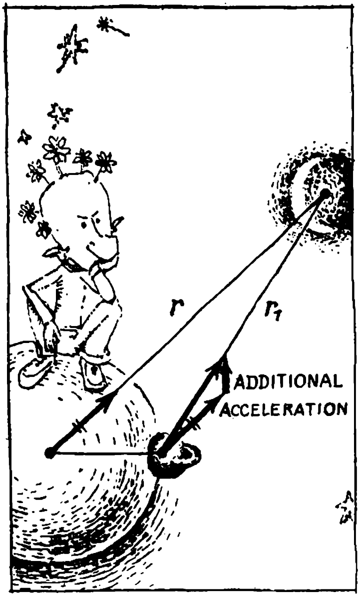


Figure 6.10

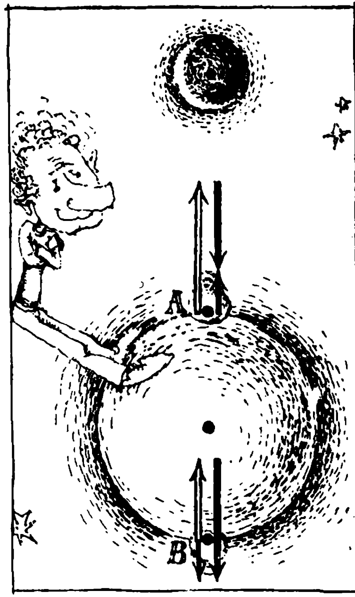


Figure 6.11

The value of Gm/r^2 is a constant number for the Earth, while the value of Gm/r_1^2 is different at various points on the Earth's surface. Hence, the geometrical difference of interest to us will differ at various places on the Earth.

What will the terrestrial weight be at the place nearest to the Moon, farthest from it and half-way along the Earth's surface?

To find the acceleration with respect to the centre of the Earth induced by the Moon on a body, i.e. the correction to the terrestrial g, it is necessary to subtract the constant value of Gm/r^2 from the value of Gm/r_1^2 at the indicated places on the Earth (light arrows in Figure 6.11).

Moreover, it should be remembered that the acceleration Gm/r^2—the acceleration of the Earth with respect to the Moon—is directed parallel to the line joining their centres. The subtraction of a vector is equivalent to the addition of the inverse vector. The vectors $-Gm/r^2$ are shown by means of bold-face arrows in the figure.

Adding the vectors depicted in the figure, we find what we are interested in: the change in the acceleration of free fall on the Earth's surface arising as a result of the influence of the Moon.

At the place nearest to the Moon, the resulting additional acceleration will be equal to

$$G\frac{m}{(r-R)^2}-G\frac{m}{r^2}$$

and directed towards the Moon. Earth's gravity diminishes; a body at point A becomes lighter than in the absence of the Moon.

Bearing in mind that R is much smaller than r, we are able to simplify the formula written above. Reducing to a common denominator, we obtain:

$$\frac{GmR\,(2r-R)}{r^2\,(r-R)^2}$$

Discarding from the parentheses the relatively small magnitude R subtracted from the much larger magnitudes r or $2r$, we obtain

$$\frac{2GmR}{r^3}$$

Let us now transfer to the antipode. At point B the acceleration of a body due to the Moon isn't greater, but less than the total acceleration of the Earth. But we are now at the farthest side of the Earth from the Moon. A decrease in the Moon's attraction at this side of the Earth leads to the same result as an increase in the attrac-

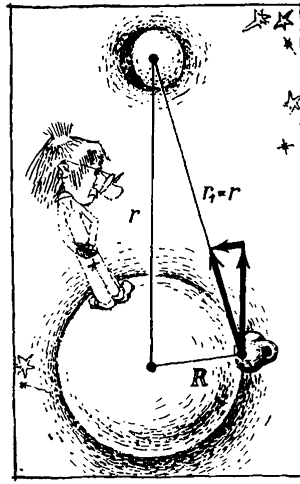


Figure 6.12

tion at point A—to a decrease in the acceleration of free fall. An unexpected result, isn't it? Here too a body becomes lighter under the action of the Moon. The difference

$$G\frac{m}{(r+R)^2}-G\frac{m}{r^2}\approx-\frac{2GmR}{r^3}$$

turns out to be the same in absolute value as at point A.

Things are different at the median line. Here the accelerations are directed at an angle to each other, and so the subtraction of the total acceleration Gm/r^2 of the Earth by the Moon and the acceleration Gm/r_1^2 of a body lying on the Earth by the Moon must be carried out geometrically (Figure 6.12). We shall depart insignificantly from the median line if we place the body on the Earth in such a way that r_1 and r are equal in magnitude. The vector difference between the accelerations is the base of an isosceles triangle. From the similarity of the triangles depicted in Figure 6.12, it is obvious that the required acceleration is as many times less than Gm/r^2 as R is less than r. Consequently, the required addition to g at the median line on the Earth's surface equals

$$\frac{GmR}{r^3}$$

in magnitude this is one-half of the weakening of the Earth's force of attraction at the extreme points. As for the direction of this additional acceleration, it again practically coincides, as can be seen from the figure, with the vertical at the given point on the Earth's surface. It is directed downwards, i.e. leads to an increase in weight.

Thus, the influence of the Moon on terrestrial mechanics consists in a change in weight of bodies located on the Earth's surface. Moreover, weight diminishes at the nearest and farthest points from the Moon, but grows on the median line, this change in weight in the latter case being half as great as in the former.

Of course, the reasoning carried out is valid for any planet, for the Sun or for a star.

It is not difficult to calculate that neither planets nor stars give even an insignificant fraction of the lunar acceleration.

It is very easy to compare the action of any celestial body with that of the Moon: we must divide the additional acceleration due to this body by the lunar acceleration:

$$\frac{GmR}{r^3} : \frac{Gm_M R}{r_M^3} = \frac{m}{m_M} \frac{r_M^3}{r^3}$$

This product will fail to be much less than unity only for the Sun. The Sun is much farther from us than the Moon, but the mass of the Moon is tens of millions of times less than that of the Sun.

Substituting numerical values, we find that under the influence of the Moon terrestrial weight is changed 2.17 times as much as under that of the Sun.

Let us now estimate by how much the weight of terrestrial bodies would be changed if the Moon were to leave its orbit around the Earth. Substituting numerical values in the expression $2GmR/r^3$, we find that the lunar acceleration is of the order of magnitude of 0.0001 cm/s^2, i.e. of one-ten-millionth of g.

Almost nothing, it would seem. Was it worthwhile to follow with strained attention the solution to a rather complicated mechanical problem for the sake of such an insignificant effect? Don't hurry with such a conclusion. This "insignificant" effect is the cause of powerful tidal waves. It creates 10^{15} J of kinetic energy daily, moving enormous masses of water. This energy equals that borne by all the Earth's rivers.

In fact, the percentagewise change in the quantity we computed is very small. A body which becomes lighter by such an "insignificant" amount will move a bit farther away from the centre of the Earth. But the radius of the Earth is 6 000 000 m, and an insignificant deviation will be measured in tens of centimetres.

Imagine that the Moon stopped its motion relative to the Earth and is shining somewhere over an ocean. Calculations show that the water level at this place would rise by 54 cm. Such a jump in the water level would also occur at the antipode. On the median line between these extreme points, the water level in the ocean would drop by 27 cm.

Thanks to the Earth's rotation about its axis, the "places" of rises and falls in the ocean are moving all the time. These are tides. During about six hours, a rise in the water level takes place and the water moves up the shore—this is high tide. Then low tide sets in; it also lasts six hours. Two high tides and two low tides occur every lunar day. The picture of tidal phenomena is greatly complicated by the friction of water particles, the form of the sea bottom and the contour of the shores.

For example, tides are impossible in the Caspian Sea simply because the entire surface of the sea is subject to the same conditions.

Tides are also absent from internal seas connected to an ocean by long and narrow straits, for example, the Black and Baltic seas.

Especially big tides occur in narrow bays, where a tidal wave coming in from the ocean rises steeply. For example, in the Gizhiginskaya Inlet on the Sea of Okhotsk, the height of waves attains several metres.

If the ocean shore is sufficiently flat (for example, in France), the rise of water during high tide can change the location of the boundary between land and sea by many kilometres.

Tidal phenomena hinder the Earth's rotation, for the motion of tidal waves is related to friction. Work must be expended to overcome this friction—it is called tidal. Therefore, the rotational energy, and with it the Earth's rotational speed about its axis, falls.

This phenomenon leads to the lengthening of the day, which was discussed on p. 13.

Tidal friction enables us to understand why one and the same side of the Moon always faces the Earth.

At one time, the Moon was probably in a liquid state. The rotation of this liquid sphere about the Earth was accompanied by strong tidal friction, which gradually slowed down the motion of the Moon. Finally, the Moon stopped rotating with respect to the Earth, the tides ceased and the Moon hid half of its surface from our sight.

7. Pressure

Hydraulic Press

A hydraulic press is an ancient machine, but it has retained its significance to the present day.

Take a look at Figure 7.1 depicting a hydraulic press. Two pistons—small and large—can move in a vessel with water. If we press one piston with our hand, the pressure is transmitted to the other piston—it will rise. Just as much water will rise above the initial position of the second piston as the first piston presses down into the vessel.

If the areas of the pistons are S_1 and S_2, and their displacements are l_1 and l_2, the equality of the volumes yields

$$S_1 l_1 = S_2 l_2, \quad \text{or} \quad \frac{l_1}{l_2} = \frac{S_2}{S_1}$$

We must discover the equilibrium condition for the pistons.

We shall find such a condition without difficulty, starting out from the fact that the work performed by the balancing forces should be equal to zero. Then during the displacement of the pistons the works done by the forces exerted on them should be equal (with opposite signs). Therefore,

$$F_1 l_1 = F_2 l_2, \quad \text{or} \quad \frac{F_2}{F_1} = \frac{l_1}{l_2}$$

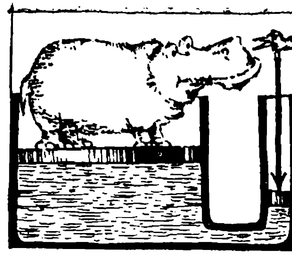


Figure 7.1

Comparing this with the preceding equality, we see that

$$\frac{F_2}{F_1}=\frac{S_2}{S_1}$$

This equation implies the possibility of an enormous multiplication of force. The piston transmitting pressure can have an area which is hundreds or thousands of times smaller. The force acting on the large piston will be just as many times greater compared to the muscular force.

With the aid of a hydraulic press, one can forge and punch metals, press the juice out of grapes and raise weights.

Of course, the gain in force will be accompanied by a loss in path. In order to compress a body by 1 cm with a press, one's hand would have to cover a path as many times greater as the forces F_2 and F_1 differ.

Physicists call the ratio of the force to the area, F/S, the *pressure* (it is denoted by the letter p). Instead of saying, "One kilogram-force acts on an area of 1 cm^2," we shall say more concisely, "The pressure p = 1 kgf/cm^2." This pressure is called the technical atmosphere (1 kgf/cm^2 = 1 at).

Instead of the relation $F_2/F_1 = S_2/S_1$, one can now write:

$$\frac{F_2}{S_2}=\frac{F_1}{S_1}, \quad \text{i.e.} \quad p_1=p_2$$

Thus, the pressures on both the pistons are the same.

Our reasoning does not depend on where the pistons are located or whether their surfaces are horizontal, vertical or inclined. And in general, it is not a matter of pistons. One may conceptually choose any two portions of a surface enclosing a liquid, and assert that the pressures on them are identical.

It turns out, therefore, that the pressure within a liquid is the same at all its points and in all the directions. In other words, an identical force is exerted on area elements of a definite size, irrespective of their orientation. This fact is called *Pascal's law*.

Hydrostatic Pressure

Pascal's law is valid for liquids and gases. However, it fails to take into account an important circumstance—the existence of weight.

Under terrestrial conditions, this should not be forgotten. Even water has weight. It is therefore obvious that two area elements situated at different depths under water will experience different pressures. But what will this difference be equal to? Let us conceptually single out within a liquid a right cylinder with horizontal bases. The water inside it presses on the surrounding water. The resultant force of this pressure is equal to the weight mg of the liquid in the cylinder (Figure 7.2). This resultant force is made up of the forces acting on the bases of the cylinder and on its lateral surface. But the forces acting on opposite sides of the lateral surface are equal in magnitude and opposite in direction. Therefore, the sum of all the forces acting on the lateral surface is equal to zero. Hence, the weight mg will be equal to the difference in force, $F_2 - F_1$. If the height of the cylinder is h, the area of its base is S, and the density of the liquid is ρ, we may write ρghS instead of mg.

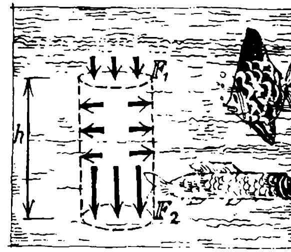


Figure 7.2

The difference in force is equal to this quantity. In order to obtain the difference in pressure, we must divide the weight by the area S. The difference in pressure turns out equal to ρgh.

In accordance with Pascal's law, the pressure on differently oriented area elements located at the same depth will be identical. Hence, at two points of a liquid situated one above the other at height h the difference in pressure will equal the weight of a column of the liquid whose cross-sectional area is equal to unity and whose height is h:

$$p_2 - p_1 = \rho gh$$

A pressure exerted by water caused by its weight is called *hydrostatic*.

Under terrestrial conditions, air most often presses down on the free surface of a liquid. The pressure exerted by air is called *atmospheric*. The pressure at a depth is composed of atmospheric and hydrostatic pressures.

In order to compute the force due to water pressure, it is only necessary to know the size of the area element on which it is exerted and the height of the column of liquid above it. By virtue of Pascal's law, nothing else plays any role.

This may seem surprising. Is it possible for the forces acting on the identical bottoms of the two vessels depicted

Figure 7.3

in Figure 7.3 to be the same? Indeed, there is much more water in the vessel on the left. In spite of this, the forces acting on the bottoms are equal to ρghS in both cases. This is greater than the weight of the water in the vessel on the right and less than the weight of the water in the vessel on the left. The sloping walls of the vessel on the left support the weight of the "extra" water, but on the right, on the contrary, they add reaction forces to the weight of the water. This interesting phenomenon is sometimes called the hydrostatic paradox.

If two vessels of different form, but with water at the same level, are connected by means of a tube, water will not flow from one vessel to another. Such a flow could take place in case the pressures in the vessels were different. But this is not the case, and so the liquid in communicating vessels will always stand at one and the same level regardless of their form.

On the contrary, if the water levels in communicating vessels are different, water will begin moving and the levels will equalize.

Water pressure is much greater than air pressure. At a depth of 10 m, water pressure is twice atmospheric pressure, at a depth of 1 km, it is equal to 100 atm.

Oceans have depths greater than 10 km at certain places. The forces due to water pressure at such depths are exceptionally great. Pieces of wood which are lowered to a depth of 5 km are so compressed by this enormous pressure that, after such a "baptism", they sink like bricks in a barrel of water.

This enormous pressure makes great difficulties for investigators of marine life. Deep-sea descents are carried out in steel globes—the so-called bathyspheres or bathyscaphes—which have to withstand pressures greater than 1000 atm.

But submarines can dive to a depth of only 100-200 m.

Atmospheric Pressure

We live on the bottom of an ocean of air—the atmosphere. Each body, every grain of sand, any object situated on the Earth is subject to air pressure.

Atmospheric pressure isn't so small. A force of about 1 kgf acts on each square centimetre of a body's surface.

The cause of atmospheric pressure is obvious. Just as water, air possesses weight and, therefore, exerts a pressure equal (just as for water) to the weight of the column of air above the body. The higher we climb up a mountain, the less air there will be above us and, therefore, the lower will atmospheric pressure become.

One must know how to measure pressure for scientific and everyday purposes. There exist special instruments—*barometers*—for this.

It isn't difficult to make a barometer. Mercury is poured into a tube with one end sealed off. Closing the open end with a finger, one turns the tube upside-down and submerges its open end in a cup of mercury. When this is done, the mercury in the tube will fall, but will not all pour out. The space above the mercury in the tube is undoubtedly airless. The mercury is supported in the

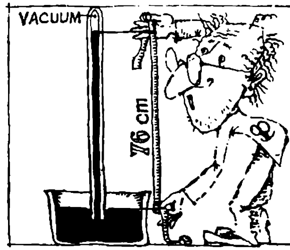


Figure 7.4

tube by the pressure of the external air (Figure 7.4).

Whatever the dimensions of the cup with mercury, and whatever the diameter of the tube, the mercury will stand at about one and the same height—76 cm.

If we take a tube shorter than 76 cm, it will be completely filled by mercury and we will not see any vacuum. A 76-cm column of mercury presses down on the support with the same force as the atmosphere. This mercury column with a cross-sectional area of 1 cm^2 presses down with a force of 1.033 kgf. This number is the volume of the mercury 1×76 cm^3 multiplied by its density and the acceleration of free fall.

As you see, the average atmospheric pressure (usually called standard atmospheric pressure) that is exerted on everything on the Earth is close to the pressure that 1-kg weight exerts on an area of 1 cm^2.

Various units are used in measuring pressures. One often simply indicates the height of a mercury column in millimetres. For example, we say that the pressure is above normal today, it is equal to 768 mm Hg (i.e. of mercury).

A pressure of 760 mm Hg is sometimes called a *standard atmosphere*. A pressure of 1 kgf/cm^2 is called a *technical atmosphere*. Since the difference between a physical atmosphere and a technical atmosphere is very small, from now on we will not distinguish between them.

Physicists also make frequent use of another unit of pressure, the *bar*; 1 bar = 10^6 dyn/cm^2. Since 1 gf = = 981 dyn, one bar is approximately equal to one atmosphere. More precisely, standard (normal) atmospheric pressure roughly equals 1013 millibars.

The unit of pressure in the SI system is the *pascal* (Pa), which is the pressure produced by a force of 1 N acting on an area of 1 m^2. This is very little pressure, as can be seen from the fact that 1 Pa = 1 N/m^2 = 10 dyn/cm^2 = = 10^{-5} bar.

Computing the area of the Earth's surface with the aid of the formula $4\pi R^2$, we find that the weight of the entire atmosphere is expressed by the enormous figure of $5 \times \times 10^{18}$ kgf.

Barometer tubes can have the most varied forms; only one thing is important: one of the ends of the tube must be sealed off in such a way that there be no air above the surface of the mercury. Atmospheric pressure acts on the other level of the mercury.

Atmospheric pressure can be measured by a mercury barometer with a very great accuracy. Of course, it isn't necessary to use only mercury; any other liquid is suitable. But mercury is the heaviest liquid, and so the height of a mercury column under standard pressure will be minimum. The mercury barometer is not a particularly convenient instrument. It is not good to leave a surface of mercury open (mercury vapour is poisonous); furthermore, this instrument is not portable.

These drawbacks are not shared by aneroid barometers—aneroids (i.e. airless). Everyone has seen such a barometer. It is a small round metal box with a scale and a pointer. Values of pressure are marked on the scale, usually in centimetres of a mercury column.

The air has been pumped out of the metal box. The cover of the box is kept in place by a strong spring, since it would otherwise be crushed by atmospheric pressure.

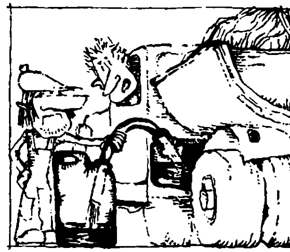


Figure 7.5

With a change in atmospher c pressure, the cover either bends or straightens. The pointer is connected to the cover in such a manner that the pointer moves to the right when the cover is bent.

Such a barometer is graduated by comparing its readings with those of a mercury barometer. If you want to know the pressure, don't forget to knock on the barometer with your finger. The pointer of the dial experiences considerable friction and usually gets stuck at "yesterday's weather"

A simple mechanism—the siphon—is based on atmospheric pressure.

A driver wants to help his friend, who is out of gas. But how can gasoline be poured from the tank of his car? It can't be inclined like a tea-kettle.

A rubber tube comes to his aid. He lowers one of its ends into his gas tank and orally sucks the air out of the other end. Then a rapid motion—the open end is stopped up with a finger and placed at a height below the gas tank. Now the finger can be removed—the gasoline will pour out of the hose (Figure 7.5).

A bent rubber tube is just what a siphon is. The liquid moves in this case for the same reason as through a straight inclined tube. In the final analysis, the liquid flows downwards in both cases.

Atmospheric pressure is necessary for the action of a siphon: it "props up" the liquid and doesn't let the column of liquid in the tube break. If there were no atmospheric pressure, the column would break at the transfer point and the liquid would slip into both vessels.

The siphon starts functioning when the liquid in the right-hand (i.e. the "pouring") part of the tube drops below the level of the liquid being siphoned off, into which the left end of the tube has been lowered. The liquid would otherwise flow back.

How Atmospheric Pressure Was Discovered

Suction pumps were already known to ancient civilizations. Water could be raised to a considerable height with their aid. Water very obediently followed the piston of such a pump.

Ancient philosophers thought about the causes for this and arrived at the following profound conclusion: water follows the piston because nature fears a vacuum and so does not leave any free space between the piston and the water.

It is told that an artisan constructed for the Duke of Tuscany in Florence a suction pump whose piston was supposed to draw water to a height of more than 10 m. But no matter how they tried to begin sucking up water with this pump, nothing came of it. The water rose 10 m with the piston, but after that the piston left the water behind, and so the very same vacuum which nature fears was formed.

When Galileo was asked to explain the cause of this failure, he answered that nature really dislikes a vacuum, but only up to a certain point. A disciple of Galileo, Evangelista Torricelli (1608-1647), evidently used this case as an excuse to perform his famous experiment in

1643 with a tube filled with mercury. We have just described this experiment—the constructing of a mercury barometer is precisely Torricelli's experiment.

Taking a tube of height more than 76 cm, Torricelli created a vacuum over the mercury (it is often called a *Torricellian vacuum* in his honour) and thus proved the existence of atmospheric pressure.

By means of this experiment, Torricelli cleared up the misunderstanding of the Duke of Tuscany's artisan. In fact, it is easy to see how many metres water will humbly follow the piston of a suction pump. This motion will continue until the column of water with an area of 1 cm^2 acquires a weight of 1 kgf. Such a column of water will have a height of 10 m. This is why nature fears a vacuum ..., but only up to 10 m.

In 1654, 11 years after Torricelli's discovery, the action of atmospheric pressure was graphically demonstrated by the Burgomaster of Magdeburg, Otto von Guericke (1602-1686). It wasn't so much the physical essence of the experiment as the theatricality of its performance that brought the author renown.

Two copper hemispheres were connected by an annular washer. The air was pumped out of the sphere so obtained through a pipe attached to one of the hemispheres, after which it was impossible to separate the hemispheres. A detailed description of Guericke's experiment has been preserved. The atmospheric pressure on the hemispheres can now be calculated: for a diameter of 37 cm, the force was approximately equal to 1000 kgf. In order to separate the hemispheres, Guericke ordered that two teams of eight horses each be harnessed. Ropes passing through the rings attached to the hemispheres were tied to the harnesses. The horses proved unable to separate the Magdeburg hemispheres.

The forces supplied by eight horses (exactly eight and not sixteen, since the second team harnessed for greater effect could have been replaced by a hook nailed to the wall, with no change in the force acting on the hemispheres) were not enough to break the Magdeburg hemispheres.

If there is a cavity between two bodies in contact, these bodies will not come apart because of atmospheric pressure.

Atmospheric Pressure and Weather

Pressure fluctuations caused by the weather are very irregular. At one time people thought that pressure alone determines the weather. Therefore, the following inscriptions have been placed on barometers up to the present day: clear, dry, rain, storm. You can even find the inscription "earthquake".

Changes in pressure really do play a big role in changing the weather. But this role is not decisive. Average or standard pressure at sea level is equal to 1013 millibars. Pressure fluctuations are comparatively small. The pressure rarely falls below 935-940 millibars or rises to 1055-1060.

The lowest pressure—885 millibars—was registered on August 18, 1927, in the South China Sea. The highest—about 1080 millibars—was registered on January 23, 1900, at the Barnaul station in Siberia (all figures are taken with respect to sea level).

A map used by meteorologists analyzing changes in the weather is depicted in Figure 7.6. The lines drawn on the map are called isobars. The pressure is the same along each such line (its value is indicated). Note the regions of the lowest and highest pressures—the pressure "peaks" and "pockets".

The directions and strengths of winds are related to the distribution of atmospheric pressure.

Pressures are not identical at different places on the Earth's surface, and a higher pressure "squeezes" air into

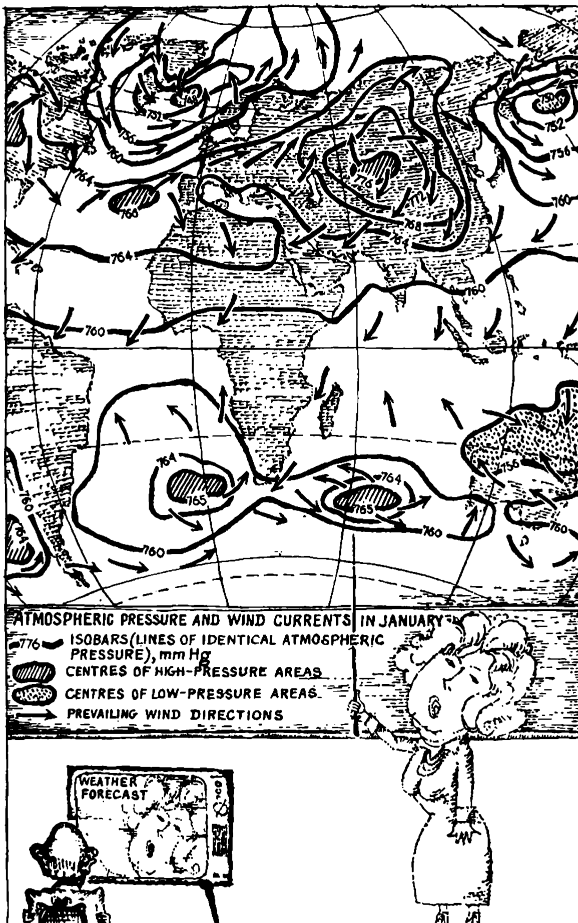

ATMOSPHERIC PRESSURE AND WIND CURRENTS IN JANUARY
776 ISOBARS (LINES OF IDENTICAL ATMOSPHERIC PRESSURE), mm Hg
CENTRES OF HIGH-PRESSURE AREAS
CENTRES OF LOW-PRESSURE AREAS
PREVAILING WIND DIRECTIONS
WEATHER FORECAST

Figure 7.6

places with a lower pressure. It would seem that a wind should blow in a direction perpendicular to the isobars, i.e. where the pressure is falling most rapidly. However, wind maps show otherwise. The Coriolis force interferes with air pressure and contributes corrections which are very significant.

As we know, a Coriolis force directed to the right of the motion acts on any body moving in the Northern Hemisphere. This also pertains to air particles. "Squeezed out" of places of higher pressure and into places where the pressure is lower, the particle should move across the isobar, but the Coriolis force deflects it to the right, and so the direction of the wind forms an angle of about 45° with the direction of the isobar.

A strikingly large effect for such a small force! This is explained by the fact that the obstacles to the action of the Coriolis force—the friction between layers of air—are also very insignificant.

The influence of the Coriolis force on the direction of winds at pressure "peaks" and "pockets" is even more interesting. Owing to the action of the Coriolis force, the air leaving a pressure "peak" does not flow in all directions along radii, but moves along curved lines—spirals. These spiral air streams twist in one and the same direction and create a circular whirlwind displacing air masses clockwise in a high-pressure area. Figure 2.16 (see p. 94) clearly shows how a radial motion is converted into a spiral motion under the action of a constant deflecting force.

The same thing also happens in a low-pressure area. In the absence of the Coriolis force, the air would flow towards this area uniformly along all radii. However, along the way air masses are deflected to the right. In this case, as is clear from the figure, a circular whirlwind is formed moving the air counterclockwise.

Winds in low-pressure areas are called *cyclones*; winds in high-pressure areas are called *anticyclones*.

You shouldn't think that every cyclone implies a hurricane or a storm. The passing of cyclones or anticyclones through the city where we live is an ordinary phenomenon related, it is true, more often than not to a change in weather. In many cases, the approach of a cyclone means the coming of bad weather, while the approach of an anticyclone the coming of good weather.

Incidentally, we shall not embark on the path of a weather forecaster.

Change of Pressure with Altitude

Pressure falls with an increase in altitude. This was first clarified by the Frenchman Florin Périer in 1648 on the instructions of Blaise Pascal. Mt. Puy de Dôme, near where Périer lived, was 975 m high. Measurements showed that the mercury in a Torricellian tube falls by 8 mm when this mountain is climbed.

A fall in air pressure with an increase in altitude is quite natural, for a smaller column of air then presses down on the instrument.

If you have ever flown in an airplane, you should know that there is an instrument on the front wall of the cabin indicating the altitude of the airplane with an accuracy to within tens of metres. This instrument is called an altimeter. This is an ordinary barometer, but it has been calibrated to show heights above sea level.

Pressure falls with an increase in altitude; let us find a formula of this dependence. We single out a small layer of air with an area of 1 cm^2 located between altitudes h_1 and h_2. The change of density with altitude is hardly noticeable within a layer which is not too large. Therefore, the weight of the volume of air we have singled out (it is a small cylinder of height $h_2 - h_1$ and base area of 1 cm^2) will be

$$mg = \rho (h_2 - h_1) g$$

This weight is just what yields the fall in pressure caused by rising from altitude h_1 to altitude h_2, that is

$$\frac{p_1 - p_2}{\rho} = g (h_2 - h_1)$$

But according to Boyle's law, which should be known to the reader (and if not, he will find it in the second book, p. 41), the density of a gas is proportional to its pressure. Consequently,

$$\frac{p_1 - p_2}{p} \propto h_2 - h_1$$

On the left is the fraction by which the pressure grew when the altitude was lowered from h_2 to h_1. Hence, a growth in pressure by one and the same per cent will correspond to identical drops of $h_2 - h_1$.

Measurements and calculations in complete agreement with each other show that the pressure will fall by 0.1 of its value for each kilometre rise above sea level. The same also holds for descents into deep shafts under sea level—the pressure will increase by 0.1 of its value when we descend by one kilometre.

We are talking about a change of 0.1 from the value at the previous altitude. This means that during an ascent of 1 km, the pressure decreases to 0.9 of the pressure at sea level; during an ascent through the next kilometre, it will become equal to 0.9 of 0.9 of the pressure at sea level; at an altitude of 3 km, the pressure will be equal to 0.9 of 0.9 of 0.9, i.e. 0.9^3, of the pressure at sea level. It is not difficult to continue this reasoning further.

Denoting the pressure at sea level by p_0, we can write out the pressure at altitude h (expressed in kilometres):

$$p = p_0 (0.87)^h = p_0 \times 10^{-0.06h}$$

A more precise number is written in parentheses: 0.9 is the rounded-off value. The formula presupposes the identical temperature at all altitudes. But as a matter of fact, the temperature of the atmosphere changes with altitude and does so, moreover, in accordance with a rather complicated law. Nevertheless, the formula yields fairly good results and may be used for altitudes up to hundreds of kilometres.

It is not hard to determine with the aid of this formula that on the top of the Elbrus—about 5.6 km—the pressure will fall by a factor of approximately two, while at an altitude of 22 km (the record height of a stratospheric balloon's ascent with people), the pressure will fall to 50 mm Hg.

When we say that a pressure of 760 mm Hg is standard, we must not forget to add, "at sea level" At an altitude of 5.6 km, the standard pressure will not be 760, but 380 mm Hg.

Along with pressure, air density also falls with an increase in altitude according to the same law. At an altitude of 160 km, not much air will remain.

In fact,

$$(0.87)^{160} = 10^{-10}$$

The air density at the Earth's surface is equal to about 1000 g/m^3, which means that according to our formula there should be 10^{-7} g of air in 1 m^3 at an altitude of 160 km. But in reality, as measurements performed with the aid of rockets show, the air density at this height is ten times as great.

Our formula gives us an even greater underestimation for heights of several hundreds of kilometres. The change of temperature with altitude and also a particular phenomenon—the decay of air molecules under the action of solar radiation—are responsible for the fact that the formula becomes useless at great heights. Here we shall not go into this.

Archimedes' Principle

Let us hang a weight on a spring balance. The spring will stretch and show how much the weight weighs. Without taking the weight off the spring balance, let us submerge it in water. Will the reading of the spring balance change? Yes, the weight of the body seems to decrease. If the experiment is done with an iron kilogram weight, the "loss" in weight will constitute approximately 140 grams.

But what is the matter? For it is clear that neither mass of the weight nor its attraction by the Earth could have changed. There can be only one cause of the loss in weight: an upward force of 140 gf acts on the weight submerged in water. But where does this buoyant force discovered by the great scientist of antiquity, Archimedes, come from? Before considering a solid body in water, let us consider "water in water" We conceptually single out an arbitrary volume of water. This volume possesses weight, but does not fall to the bottom. Why? The answer is obvious—the hydrostatic pressure of the surrounding water prevents this. This implies that the resultant of this pressure in the volume under consideration is equal to the weight of the water and directed vertically upwards.

If this volume is now occupied by a solid body, it is clear that the hydrostatic pressure will remain the same.

Thus, as a result of hydrostatic pressure, a force acts on a body immersed in a fluid. The force is directed vertically upwards and is equal in magnitude to the weight of the fluid displaced by the body. This is *Archimedes' principle.*

It is said that Archimedes lay in a bath-tub and thought about how to determine whether or not there is any silver in a gold crown. A person taking a bath distinctly feels

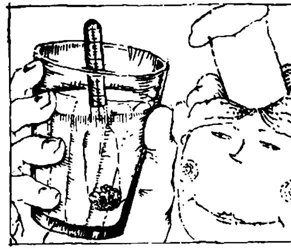


Figure 7.7

a buoyant force. Suddenly the principle came to light, presented itself to Archimedes in its remarkable simplicity. With a cry of "Eureka!" (which means "I found it!"), Archimedes jumped out of the bath-tub and ran into the room containing the precious crown in order to immediately determine its loss of weight in water.

The loss of weight of a body in water will be equal to the weight of the water displaced by the body. Knowing the weight of the water, we shall immediately determine its volume, which is equal to the volume of the crown. Knowing the weight of the crown, we can immediately find the density of the material out of which it was made and, knowing the density of gold and silver, find the fraction of silver in the crown.

Archimedes' principle is valid, of course, for any fluid. If a body of volume V is immersed in a fluid of density ρ, then the weight of the displaced fluid—and this is just the buoyant force—will be equal to $\rho g V$

The working of simple instruments controlling properties of fluid products is based on Archimedes' principle. If alcohol or milk is diluted in water, its density will change; but it is possible to judge its composition on the basis of its density. Such a measurement is simply and easily performed with the aid of an areometer (Figure 7.7). An areometer lowered into a liquid will be immersed to a greater or smaller depth depending on its density.

An areometer will be in a state of equilibrium when the buoyant force becomes equal to the weight of the areometer.

Divisions are marked off on an areometer, and the density of a liquid is read from the marking which appears at its surface. Areometers applied for the control of alcohol are called alcoholometers, and those for the control of milk lactometers.

The average density of a person's body is somewhat greater than unity. Anyone unable to swim will drown in fresh water. Salt water has a density greater than unity. The salinity of the water in most seas is insignificant, and its density, although greater than unity, is less than the average density of the human body. The density of the water in the Bay of Kara-Bogaz-Gol in the Caspian Sea is 1.18. This is greater than the average density of the human body. It is impossible to drown in this bay. One can lie on the water and read a book.

Ice floats on water. The preposition "on", incidentally, is somewhat out of place here. The density of ice is about 10% less than that of water, so it follows from Archimedes' principle that approximately nine-tenth of a piece of ice is submerged in water. It is precisely this circumstance that makes it so dangerous for ocean liners to come across icebergs.

If a balance scale is in equilibrium in air, this does not imply that it will be in equilibrium in a vacuum. Archimedes' principle refers to air to the same degree as to water. A buoyant force equal to the weight of the displaced air acts on a body in air. A body "weighs" less in air than in a vacuum. The greater the volume of a body, the greater will be its loss of weight. A ton of wood loses more weight than a ton of lead. To the humorous question of which is lighter, there is the same kind of answer: a ton of lead is heavier than a ton of wood if they are weighed in air.

The loss of weight in air is slight as long as we are considering small bodies. However, in weighing a piece the size of a room, we would "lose" several tens of kilograms. For exact weighing, the correction due to the loss of weight of large bodies in air should be taken into account.

The buoyant force in air permits us to construct balloons, aerostats and dirigibles of various types. For this one must have a gas lighter than air.

If a balloon of volume 1 m^3 is filled with hydrogen, 1 m^3 of which has a weight equal to 0.09 kgf, then the lift—the difference between the buoyant force and the weight of the gas—will equal

$$1.29 \text{ kgf} - 0.09 \text{ kgf} = 1.20 \text{ kgf}$$

1.29 kg/m^3 is the density of air.

Hence, a load of about a kilogram can be attached to such a balloon, and this will not prevent it from flying above the clouds.

It is clear that with relatively small volumes—of several hundred cubic metres—hydrogen balloons are capable of raising considerable loads into the air.

A serious defect of hydrogen aerostats is the inflammability of hydrogen. Together with air, hydrogen forms an explosive mixture. Tragic accidents have marked the history of the creation of aerostats.

Therefore, when helium was discovered, people started filling balloons with it. Helium is twice as heavy as hydrogen and the lift of a balloon filled with it is smaller. But will this difference be significant? The lift of a 1-m^3 balloon filled with helium is found as the difference 1.29 kgf — 0.18 kgf = 1.11 kgf. The lift has decreased by only 8% At the same time, the advantages of helium are obvious.

The aerostat was the first apparatus with whose aid people rose in the air. Aerostats with a hermetically

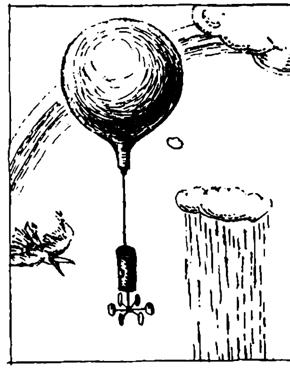


Figure 7.8

sealed car have been used up to the present day for investigating the upper layers of the atmosphere. They are called stratospheric balloons. They rise to a height of more than 20 km.

Balloons equipped with various measuring devices and transmitting the results of their measurements by radio (Figure 7.8) are widely used at the present time. Such radiosondes contain miniature radio transmitters with batteries which report on the humidity, temperature and atmospheric pressure at various heights by means of prearranged signals.

One can send an unguided aerostat on a long journey and determine rather accurately where it will land. For this it is necessary that the aerostat climb to a great height, of the order of 20-30 km. Air currents are extremely stable at such heights, and the path of the aerostat can be calculated quite well beforehand. When necessary, one can automatically change the lift of the aerostat by letting out gas or throwing off ballast.

Aerostats on which a motor with a propeller was installed were previously used for flights. Such airships were streamlined. Airships lost the competition with airplanes;

even in comparison with planes of 30 years ago, they are clumsy, difficult to control, move slowly and have a "low ceiling". It is believed that airships would be advantageous for carrying cargo.

Extremely Low Pressures. Vacuum

A vessel which is technically empty still contains an enormous number of molecules.

Molecules of gas constitute a considerable hindrance in many physical instruments. Radio tubes, X-ray tubes, accelerators of elementary particles—all these instruments require a vacuum (derived from the Latin *vacuus* meaning "empty"), i.e. space free of gas molecules. There should also be a vacuum in an ordinary electric lamp. If air enters a lamp, it will oxidize and immediately burn out.

In the best vacuum instruments, vacuum of the order of 10^{-8} mm Hg is produced. A completely negligible pressure, it would seem: the level of mercury in a manometer would move by a hundred-millionth of a millimetre if the pressure changed by such an amount.

However, there are still several hundred million molecules in 1 cm^3 at this meagre pressure.

It is interesting to compare the void of interstellar space with such a vacuum—there one finds an average of one elementary particle of matter in several cubic centimetres.

Special pumps are employed in order to obtain vacuum. An ordinary pump removing gas by means of the motion of a piston can create a vacuum of at best 0.01 mm Hg. A good or, as one says, high vacuum can be obtained with the aid of a so-called diffusion (mercury or oil) pump in which gas molecules are caught up in a stream of mercury or oil vapour.

Mercury pumps, bearing the name of their inventor, Langmuir, start working only after a preliminary exhaustion to a pressure of about 0.1 mm Hg; such a preliminary rarefaction is called a forevacuum.

This is the way it works. A small glass container is connected to a vessel with mercury, an evacuated space and a forepump. The mercury is heated and the forepump carries away its vapour. The mercury vapour captures molecules of the gas along the way and brings them to the forepump. The mercury vapour condenses (cooling by means of running water is provided for), and the liquid trickles down into the vessel from which the mercury began its journey.

A vacuum obtained under laboratory conditions, as we have just said, is still far from empty in the absolute sense of the word. A vacuum is greatly rarefied gas. The properties of such a gas may differ essentially from those of an ordinary gas.

The motion of the molecules "forming a vacuum" changes its character when the mean free path of a molecule becomes greater than the dimensions of the vessel containing the gas. The molecules then rarely collide with each other and travel in straight zigzags striking against first one and then another wall of the vessel. We shall speak in detail about the motion of molecules in the second book. It is known to the reader that mean free path of a molecule in air at atmospheric pressure is equal to 5×10^{-6} cm. If we increase it by a factor of 10^7, it will be 50 cm, i.e. will be noticeably greater than an average sized vessel. Since the mean free path is inversely proportional to the density, and hence also to the pressure, the pressure must be 10^{-7} of atmospheric pressure, or approximately 10^{-4} mm Hg.

Even interplanetary space is not entirely empty. But the density of the matter in it is about 5×10^{-24} g/cm^3. The main component of interplanetary matter is atomic hydrogen. At the present time, it is considered that cosmic space contains several hydrogen atoms per 1 cm^3. If a hydrogen molecule were enlarged to the size of a pea and placed in Moscow, its nearest "cosmic neighbour" would prove to be in Tula.

Pressures of Millions of Atmospheres

We daily come across high pressures exerted on small surfaces. Let us estimate, for example, what the pressure will be at the point of a needle. Assume that the tip of a needle or nail has a linear dimension of 0.1 mm. This implies that the area of the point will be about 0.0001 cm^2. If a rather modest force of 10 kgf acts on such a nail, then the tip of the nail will exert a pressure of 100 000 atm. It's no wonder that the pointed objects so easily penetrate deeply into dense bodies.

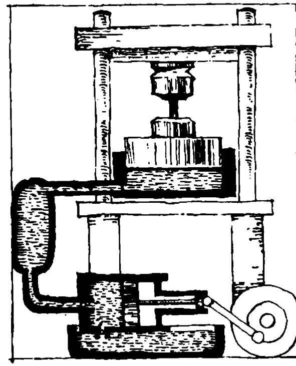
It follows from this example that to create high pressures on small surfaces is quite a common thing. The situation is completely different if the question is to create high pressures on large surfaces.

The creation of high pressures under laboratory conditions is accomplished with the aid of powerful presses, for example, hydraulic ones (Figure 7.9). The force of the press is transmitted to a piston of small area, and the piston forces its way into the vessel within which we wish to create a high pressure.

Pressures of several thousand atmospheres can be created in this manner without any particular difficulty. But in order to obtain ultrahigh pressures, we must complicate the experiment, since the material composing the vessel cannot withstand such pressures.

Here nature has met us half-way. It turns out that metals become considerably stronger under pressures of the order of 20 000 atm. Therefore, an apparatus for

Figure 7.9



obtaining ultrahigh pressures is submerged in a liquid which is under a pressure of the order of 30 000 atm. In this case, one is able to create pressures of several hundred thousands of atmospheres (but again with a piston). The highest pressure—400 000 atm—was obtained by the American physicist Percy Williams Bridgman.

Our interest in obtaining ultrahigh pressures is far from idle. Phenomena which are impossible to induce by other methods can occur at such pressures. Artificial diamonds were obtained in 1955. A pressure of 100 000 atm and, in addition, a temperature of 2000 K were required for this. Ultrahigh pressures of the order of 300 000 atm on large surfaces are formed during explosions of solid or liquid explosive materials—nitroglycerine, trotyl, etc. Incomparably higher pressures attaining 10^{13} atm arise within an atomic bomb during its explosion. Pressures during an explosion exist for a very short time. There are constant high pressures deep inside celestial bodies including the Earth, of course. The pressure at the centre of the Earth is equal to approximately 3 million atmospheres.