SCIENCE FOR EVERYONE

G. I. KOPYLOV

ELEMENTARY KINEMATICS OF ELEMENTARY PARTICLES

MIR

Г.И.Копылов

Всего лишь кинематика

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Elementary Kinematics of Elementary Particles

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Preface to the Second Russian Edition

Elementary Kinematics of Elementary Particles (*It's Only Kinematics* is the literal translation of the Russian title) is the first book written for the layman on the relativistic kinematics of elementary particle interaction. The book begins where books for the general reader on relativity theory usually end. First the author explains, by means of examples, the formulas and concepts of the theory of relativity that will be required in the subsequent discussion. Next he acquaints the reader with a series of problems that are solvable by using the methods of contemporary particle physics. The topics discussed really do concern present-day physics and the problems are ones that are being solved today or were solved only yesterday. These include the discovery of new particles, determination of the quantum numbers of resonance particles, and many others.

The author, Dr. G.I. Kopylov (now deceased), was a prominent scientist in the field of elementary particle kinematics. In this work he skilfully selected from the abundant available material topics that are of general interest, and that can be clearly expounded without oversimplifying their essence. The book was written for persons having a secondary sixth-form (or USA high-school) education and requires a knowledge of only elementary algebra and geometry.

It is very difficult to bring some scientific field or a portion of one within the reach of non-specialized readers without losing some essential details. The task is frequently considered to be entirely impracticable, but such an extreme viewpoint is evidently erroneous for the author of the present book has, in our opinion, successfully overcome this problem.

In the preface to the first Russian edition the hope was expressed that the book "would attract the attention of a wide range of readers interested in nuclear physics and, in particular, school students, students of universities and institutes, and teachers". This indeed happened and the book was quickly sold out.

Dr. G.I. Kopylov died some years ago and therefore, in preparing a second edition, we decided to limit ourselves to only a minimal and most necessary revision, which, to a certain extent, takes account of the spectacular development of particle physics in recent years. A considerable part of the revision was done by his son, Dr. G.G. Kopylov, a physicist.

M.I. Podgoretsky

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Part One
Kinematics for the Lyrical

> ... and, since he had such a strong inclination for reasoning, he wanted to divine how such a tiny atom moves about, and whether it is endowed with ideas, will-power and freedom.
>
> — *VOLTAIRE*, "Micromégas"

Chapter 1
What It's All About

Our aim in this book is to show that much can be made clear in one of the leading fields of modern physics—*elementary particle physics*—even to a reader who has mastered only school mathematics. Particle, or high-energy, physics is the science of the properties, special features and laws of interaction of the tiniest particles of matter. Much here is strange, astonishing and unusual. In fact, everything seems incredible in our first encounter with the world of elementary particles.

We meet particles a hundred thousand times smaller than an atom, which is itself as many times smaller than an apple as the apple is smaller than the earth ("How can that be possible? What can we conceivably discern?"). And their velocity is only just less than that of light ("How can we possibly follow such swift motion?") And their lifespan: some particles exist no longer than 10^{-23} second. Others, disappearing after 10^{-8} s (one hundred millionth of a second), are said in this science to be "long-lived" ("How can we detect such particles? What kind of clock can measure such infinitesimally short lengths of time?").

What about their imperceptibility? There are particles so elusive that they leave barely a trace of their existence on the earth. They appear unnoticed, speed past atoms like incorporeal spirits and then disappear, leaving no trail ("How can we then investigate their properties?").

We have, nevertheless, not only detected all this, but have classified the particles into families, weighed the members of each family, indicated which are related to which, which are transformed into which (and how frequently), which appear the same from all sides and which are not, and much, much more. "How can that be feasible?" the legitimate question is posed. "And if it is (and there is no reason not to believe the people who produced nuclear energy and the laser out of nothing) then what brain waves, what ingenuity of the mind is required to convince oneself that it is so; what intelligence is required to accommodate all this knowledge!"

Well, all in all, such amazement is quite natural. Also understandable is the desire to look into all these marvels. Unfortunately, there is much in this branch of science that does not lend itself to simple explanation. The theoretical ideas are extremely unusual; it proves impossible to hastily explain the principles of the instruments used in nuclear physics to observe fast particles and to measure their direction, velocity, mass, momentum and energy.

But there is one field in this branch of science that the layman can attempt to understand. In this field the complications of the experiment have come to an end and the theoretical complications have not yet begun. This field, in which the scientist is engaged in primary processing of experimental data, is called the *kinematics of elementary particle interaction*, i.e. their collisions and decays. This is a highly interesting field. There is no instrument capable of observing the tracks of uncharged particles, but kinematics enables such particles to be readily noted. So far no instruments can directly measure lengths of time shorter than 10^{-18} s. But kinematics enables lengths of time of the order of 10^{-23} s to be estimated. And all this is achieved without any profound hypotheses, using only a pencil, paper and some simple calculations. Kinematics has to its credit certain quite important feats, including the discovery of such a marvel of nature as the neutrino, a particle that readily pierces the sun; such short-lived tenants of the earth as the neutral pi meson; such strange particles as hyperons and K mesons; such ephemeral structures as the neutral omega meson, and other resonance particles. In short, kinematics was applied in the discovery of all the elementary particles with the exception of the proton and certain charged mesons.*

Of course, these discoveries were not made by kinematics alone; the main role was played by the ingenious instruments that are used in high-energy physics. Still it is indisputable that without kinematics we would be able to make out

* In recent years particles have been discovered with entirely novel properties, which are called charmed particles ("charm" being a strictly scientific term). They also were found with the aid of kinematics.

much, much less using these instruments. Kinematics helps us to see what is beyond the power of apparatus and thereby appropriately concludes the succession of accelerators, targets, bending magnets, electrostatic separators, bubble and spark chambers, and camera lenses aimed at the particle.

We stress the word "concludes" Kinematics sets itself no far-reaching aims of unravelling the mysterious interrelations of particles or the symmetry of nature. It only tries to fill in what has not been observed by the instruments, thereby making them more sharp-sighted without altering them or taking any interest in their design. Just as Sherlock Holmes by simply looking at a person saw ten times more than other people would, a physicist, equipped with a knowledge of kinematics, sees many times more than one without such knowledge.

True, one cannot manage here by only one's imagination. Kinematics is based on precise and careful calculations, rather than considerations of the following kind: "The particle turned this way; that indicates that something pushed it from there. Now what could that be?" Even though the calculations may sometimes be quite complex, they are always based on one and the same simple, well-known principle: in all interactions (decays and collisions) of elementary particles, their total energy and total momentum remain unchanged. If the instruments show that there is less energy or momentum after an interaction, this means that one or perhaps several unobserved particles carried away the deficit. Then other, just as simple, laws may be resorted to that sometimes enable us to identify what has carried away the shortage.

A great many significant conclusions in kinematics are reached, essentially, by means of school algebra. We therefore believe that even an inquisitive sixth former (or high-school student) or a person versed in algebra to the extent taught in the sixth form, having some idea of what vectors are, and having heard of antiparticles and that at high velocities the mass of a body increases with its velocity, can understand the essence of many predictions of kinematics.

The aim of this book is to take the reader into the "kitchen" where many vital discoveries in the physics of elementary particles are made. If you now pluck up courage and read to the end, and if you take the trouble to try to grasp the essence of the calculations and conclusions, you will find that you have understood the mechanism of many discoveries. True, the laws of kinematics are only the first and easiest step into the world of elementary particles. But every journey begins with the first step.

Chapter 2

Mores in the Subatomic World

Sometimes we speak of levels and sometimes of worlds. We begin with the social level and then bound up down the levels like the steps of a staircase: biological, cellular, molecular, atomic, nuclear and the level of elementary particles. We can likewise speak of worlds: the world of the stars, the world of man, the world of bacteria, the world of the atom, etc. Each world has its own laws, its own problems, and has, in general, no concern for the problems of some adjacent world.

Nature evidently acted wisely in establishing such order, separating the different worlds by an invisible and almost impenetrable boundary, endowing each with its own laws. Perhaps this achieves succession, the continuity of development: a catastrophe in one world is unnoticed in other worlds, everything takes its normal course. But it may be that Nature simply wanted to demonstrate its lack of banality.

People, however, unwillingly admit the non-triviality of Nature. All other worlds, those of the atom, the stars and others, they order in accordance with familiar patterns or models. They named the magnet *aimant* in French, which means loving, or devoted; they saw gods in the planets. Then there is a more subtle likening of certain worlds to others: the atom, they contended, resembles the solar system; a man resembles a heat engine and light waves resemble ocean waves. They believed that the laws for the falling of an electron into the nucleus could be derived from the laws for the falling of an apple to the earth, and that the whole world in its entirety might be deduced from the head of a philosopher.

Even now, when we seem to understand that each world has its own problems and laws, the inertia of human reasoning greatly hinders specific penetration of specific worlds. Knowledge is acquired by overcoming habitual conceptions, by rejection of the self-evident in favour of something incomprehensible and unusual (as new laws appear now and then upon first acquaintance).

The farther these two worlds are from each other, meaning the world of the investigator and the world of the item being investigated in a certain science, the more difficult the development of this science and the more the scientist must coerce his imagination and his habitual logic to accept the new concepts. From this point of view, the most difficult sciences are elementary particle physics and astronomy. In this case, it is hard to say why they have leaped ahead, overtaking other sciences studied by mankind.

Either the humanities could not take advantage of their privileged position, or the world of one person differs from that of another person to a greater extent than from that of the atom.

A book for the layman on the kinematics of elementary particle interactions should evidently begin by acquainting the reader with the world of these particles. It will very likely be best to select the most typical inhabitant of this world and to tell about its habits and faculties. This should illustrate the characteristic metamorphoses that elementary particles are subject to, and how the range of these interactions can be restricted (in principle, without knowing the laws governing these interactions) only on the basis of the laws of the conservation of energy and momentum. A more detailed understanding of the metamorphoses and their general laws will be gained later as the reader forges ahead in this book.

One of Many

The most estimable status in the subatomic world is enjoyed by the *proton*. In the first place, it is very massive. Formerly, the proton was thought to be the only massive particle, but later it became necessary to admit that it has a host of relatives. They all have the same family name—*baryon*—but different first names: *neutron*, *lambda*, *sigma*, *xi*, *isobar*, etc. Though they are all more massive than the proton, the proton (and the neutron) retains its previous superiority in number. If you take a walk in the subatomic world, the baryons you will most frequently meet are the proton and the neutron. You will run across all the other members of the family, but only from time to time. Together with the neutron the proton enters into the composition of the atomic nucleus, i.e. it forms the basis for all other more complex worlds.

Secondly, the proton is stable. This means that it has a guarantee of personal immortality. If care is taken to prevent all the rest of the world from having any effect or, at least, only a weak effect on the proton, it is capable of existing forever, of outliving both the stars and the galaxies. All of its brother particles perish sooner or later: a free neutron in 16 minutes on an average, a lambda or xi particle in 10^{-10} s and an isobar even much sooner, in 10^{-23} s. This takes place in the following way. A member of the baryon family disappears all of a sudden without any apparent cause, and in its place another baryon, but not as massive, appears. This is accompanied by the appearance of several (one or two) members of other particle families, such as *mesons*, *leptons* or *photons*. The newborn baryon should also decay, following the same rule. Since the lightest of the baryons is the proton, any heavier baryon is converted, sooner or later, into a proton and, in this guise, finally calms down and acquires the right to immortality.

But what is of especial interest is that all baryons, taken as a whole, are an immortal kinsfolk. Nature decreed that the total number of baryons in the world, in the whole world and in each separate interaction, should remain constant forever. Hence, for instance, when a baryon decays it must produce from itself another baryon (and something else that is not a baryon). From two colliding baryons, two baryons are produced again, maybe of the same kind and maybe not (and, for example, some nonbaryon). To be more exact, in the collision of two protons, for instance, sometimes even three protons may be produced, but then only together with one antiproton (or other antibaryon). Four protons may also be produced, but only with two antiprotons. In short, it is not simply the number of baryons that is conserved, but the number of baryons minus the number of antibaryons. In the simultaneous creation of particles and antiparticles, they compensate one another and this is not considered to be a violation of the conservation law.

It follows that in establishing a constant number of baryons in the world, Nature did not bar the way to their enrichment by new reserves of matter; she simply stipulated that an equal number of new antibaryons are also created. All of our world cannot perish primarily because the baryons, which constitute its main building material, will not disappear. But this does not imply that our universe will exist forever. If some day it is approached by an antiuniverse and the whole conglomeration is converted into a cloud of mesons or photons, this will not contradict any laws of nature.

In exactly the same way, the law of conservation of baryons does not imply that our universe could not have emerged, at some instant in the dim past, out of nothing, or that it has existed from time eternal. We can picture, instead of our universe, a huge reservoir of fast mesons or energetic photons that existed once upon a time. From this reservoir emerged a world-antiworld pair, which then flew asunder in opposite directions. This question is still an open one: some prefer to believe that our part of the universe has existed forever, others like the idea of catastrophes. The law of conservation of the number of baryons is no obstacle to either hypothesis; the question will be answered by other laws and facts.

But let us return to our baryons. The proton stands out among them by the fact that it readily lends itself to outside influence; it is, in particular, especially obedient to man. A proton is electrically charged, and an electric voltage (or field) is capable of accelerating it, repulsing it, deflecting it or supplying it with energy. In short, a proton can be manipulated as one pleases. In exactly the same manner, a constant magnetic field can affect a stream of protons as if it was an ordinary electric current. Such a field cannot accelerate the proton but it can deflect it. These factors make the proton a valuable tool in investigating the subatomic world. By accelerating protons with an electric field they can be supplied with extremely high energy. At this point the proton appears in a new capacity: as a transformer of nature. Incident on another proton, it can produce a considerable number of new particles. That is how new specimens of mesons and baryons are produced. These particles do not exist forever and would have become extinct long ago if they were not created again and again by fast protons.

We could very likely stop here and sum up all that will be needed subsequently. We found that there are two kinds of processes in which elementary particles participate. The first are ones in which the particle itself decays, i.e. disintegrates, into several new particles, and the second are ones where, in the collision of two particles, they are either simply deflected from their previous paths or they create several more new particles.

Experimental physicists have various techniques for registering colliding particles and the results of the collision, as well as decaying particles and into what they decay. They are capable of determining the direction and velocity of particles. Theory sets itself the task of describing the chain of observed events, revealing their mechanism, understanding the causes that impelled the particles to behave as they did and not in some other way, i.e. to determine the principal habits of the particles and their place in the general picture of the world.

The baryon number conservation law that we mentioned above is only one of the simplest laws of the subatomic world. Other laws are more complex and, above all, they are unusual. Sometimes they resemble nothing that happens in other worlds, not to us, not to molecules, not to the stars.

The problem facing the author is to tell about the world of elementary particles, saying almost nothing about the incomprehensible laws. This proves to be possible. It is possible because a law well known to us, the conservation of energy and momentum, concerns the world of elementary particles in full measure. It is just as inconceivable to build a perpetual motion machine out of elementary particles as it is out of spheres, chains and wheels.

If two particles had a store of energy before their interaction, at the instant of interaction this store can neither be depleted nor increased. The same is true of the momentum (recall that the momentum is the product of the mass of a body by its velocity; it is also known as the *linear momentum*). Its store in some isolated group of particles is also constant. If, for instance, the centre of gravity of two particles is first at rest, then, whatever transformations these particles undergo, the centre of gravity of what they are changed into should also be at rest.

"Bookkeepers" of the Subatomic World

The conservation laws were established for large, readily visible bodies. But they did their job so carefully and accurately, they so unerringly squared the accounts of gains and losses, that there was no reason to fire them when physicists began to deal with elementary particles. They were accepted conditionally, up to the first mistake they make. As of today, over three decades of their term of probation has passed, and there has never been any occasion to accuse them of careless bookkeeping in their "ledger"; there has never been even a single case in which the debits and credits did not tally. True, at the beginning of their new career, back in the thirties, they were involved in a controversy. But then they brilliantly proved they were in no way implicated in violating the balance of the accounts, and helped to unmask the true culprit behind the scandal. This turned out to be the young neutrino, who had only then embarked on its career (a shifty rascal and sly boots).

The law of conservation of energy in the decays and collisions of elementary particles looks even simpler than in our world of large-scale phenomena. In our world energy is an entry in many items of the budget: in electricity, in heat, in mechanical motions, in elasticity, in chemistry, and others. But in the subatomic world, when a freely travelling proton collides with a particle, creating several new particles, we can manage by taking into account only two forms of energy: the energy of motion, or kinetic energy, and the so-called rest energy, i.e. the energy consumed in creating the particle itself. It is sufficient to include only these two components in the total energy of the particles.

Why can we neglect other forms of energy?

In some cases, because these energies are extremely low. We can, for instance, forget about the energy mgh of fast-moving particles. It is exceptionally low; the particles are only weakly attracted by the earth.* The energy of a proton in an electric field must be taken into account, but only as long as it is in the field, i.e. as long as it circuits the accelerator. As soon as it emerges into free travel, this energy is entered into the balance account under the item "kinetic energy" Of course, we ought not to ignore the energy transformations that accompany the collision processes of particles. But these processes occur only upon close approach of the particles, take place extremely rapidly, and we are incapable and haven't sufficient time to observe the details of the process and to measure the energy of strong interaction at the very height of the event. Therefore (though against our will) we do not write out the energy balance at that cardinal instant when large sums are transferred from one item of the account to another. But when all is over and the particles leave in different directions, again only two items remain: the rest energy and the energy of motion, and these easily tally. Thus, though we cannot observe and at times

* When the particles themselves have low energy, we must consider the gravitational force. Physicists have a technique for obtaining so-called ultracold neutrons, i.e. ones with extremely low energy. The energy of ultracold neutrons is so low they can be stored in vessels or made to flow along pipes. They are reflected by copper walls instead of simply piercing them as do ordinary particles that leave an accelerator. Hence, if the pipe is directed upward, the ultracold neutrons cannot, in some cases, reach the top; they haven't sufficient energy to overcome the gravitational force. Here, however, we are encroaching on the realm of exotic phenomena.

do not even understand the mechanism by means of which nuclear forces act, we can still predict the results of their action sometimes. The conservation laws restrict the action of any other laws; all others are obliged to accommodate themselves within the limits of the conservation laws. These limits are sometimes so narrow that there is simply nowhere for other laws to display themselves.

When You Know the Mass, You Know the Particle

Another property of elementary particles, their indivisibility, is of prime importance for the successful application of kinematics.

The mass of a large body, as is known, can have any value because the body consists of atoms and can be divided into parts. Now imagine for a moment that there are no atoms and that the energy produced by some special device is accumulated in some kind of reservoirs, remaining imperceptible and shapeless. Then, all of a sudden, after reaching a certain amount, it is instantly transformed into something: a frying pan, a flatiron or a ball. Besides, all the frying pans are absolutely identical and have the same mass, all the flatirons also differ in no way from one another, etc. Imagine that these things would be indivisible: if we banged the frying pan against the flatiron it would not break into two or more pieces, but would remain intact and unimpaired, or would disappear without a trace into the energy reservoir; or, for instance, it would turn into two balls, with the excess energy returning to the reservoir. We would soon get used to such things and would never ask ourselves: "What does a half frying pan look like?". This would sound the same to us as "two and two-thirds wood-cutters".

This, roughly, is the state of affairs as concerns elementary particles. They do not grow from a nucleus as a crystal, they are not built of blocks like a house, they cannot be divided into pieces like a bar of chocolate. They are born all ready, in finished form and of full size. The particles of one kind are all alike, so much so that they cannot be distinguished from one another. The mass is exactly the same for all the particles of one kind and can serve, therefore, as a tag or calling card of the particle. This is what indivisibility of particles leads to.

There remains only a single difficulty: how are we to measure the mass? We could, for example, weigh the particle. But suitable scales have not yet been devised and, what is of most importance, to weigh the particle we first have to stop it. How would we go about doing that? Therefore, the mass of a particle has to be determined in motion as it speeds past atoms and molecules. At this instant, physicists contrive to measure its momentum from how sharply its path is bent in a magnetic field; its energy is measured from the extent of the destruction it brings on, and from other phenomena. These factors are quite sufficient to identify the particle. On the face of it, this seems impossible or, at any rate, highly unusual. It is understandable when the velocity or the temperature or the altitude to which a particle rises is determined from its energy. But to definitely recognize the nature of a body from its energy is something hard to believe; any body can have a given energy, or a given temperature, or a given velocity! Well, what of it? As a matter of fact, this is not so impossible as much as unusual. Why should we judge an article by its energy and momentum when a dozen better methods are available, when we can simply stop the article and look at it? But a particle cannot always be stopped and physicists are obliged to devise more subtle means.

How to Weigh a Bullet in Flight

If necessary, even the mass of a bullet in flight can be determined. You fire the bullet point-blank from a rifle into a box of sand placed in a calorimeter. At the instant the shot is fired you measure the momentum p acquired by the bullet. This can be found, for example, from the recoil of the rifle, i.e. by multiplying the mass of the rifle by its velocity at the instant the shot is fired (owing to the equality of action and reaction, the bullet will have the same momentum). All the energy T, acquired by the bullet is converted into heat when the bullet is brought to a stop by the sand, and can be measured by the calorimeter. What then is the mass m of the bullet? From the formula for the momentum $p=mv$ it follows that the velocity of the bullet is $v = p/m$. Substituting into the formula $T = mv^2/2$ for the kinetic energy, we obtain $T = p^2/2m$, from which it follows that the mass of the bullet is

$$m = \frac{p^2}{2T}.$$

Hence, the mass of a body engaged in mechanical motion can be determined from its store of momentum and energy. This cannot be done, of course, if only its energy or only its momentum is measured.

In the physics of the subatomic world, where visible motion is only of the mechanical kind, there also exists a relation between the mass, total energy and momentum of a particle. Affairs are simplified here because the tiniest particles cannot have any mass; each kind of particle can have only its own mass and no other. Hence, as soon as we have determined the mass of a particle (by the energy and momentum it carries away), we can immediately identify the particle. Very convenient. This couldn't have been done with the bullet. Knowing only its weight, could you determine whether you are dealing with a bullet or with buckshot from a shotgun?

Chapter 3

Energy and Momentum of Fast Particles

In expounding something new it is necessary to base our explanation on something that is already known. We shall assume that the reader is familiar with the formula: $E = Mc^2$.

This formula was discovered by Albert Einstein. We are indebted to him for enabling us to calculate the energy of fast particles and for the knowledge that there are incalculable, unclaimed stores of energy concealed even in a rock half-buried in a vast plane. He derived this formula long before it was required in practice (long before 1919 when the first nuclear transformation was observed). As far back as 1905 Einstein proved that the energy and momentum of a very fast body cannot be calculated by the familiar formulas $E = mv^2/2$ and $p = mv$. He also proved much, much more; he literally upset our usual ideas on all the basic things in our world: motion, space, time, light and mass. Of importance to us for the time being is only what he said about energy and momentum.

The essence of Einstein's discovery can be expounded roughly as follows.

Mass and Velocity

Nothing in the world is faster than light. No kind of light can be faster than any other kind. Any kind of light always travels (in vacuum or, as they now say, in free space) at the same velocity. Therefore, it proves convenient to take the velocity of light equal to unity. All other motion, for instance, that of some body, cannot be faster than the propagation of light, i.e. the velocity of any body is always less than unity. But what about a body that is accelerated by some force for a very long time? We know that any force causes acceleration, and that acceleration increases the velocity. Will not the time come when the velocity of the accelerated body exceeds our unity? But this is impossible; hence, the acceleration must gradually decrease with an increase in velocity. The acceleration must decrease at a rate that prevents the velocity of the body from reaching unity. But what does it signify if the acceleration of a body decreases even though it is subject to the action of a constant force? How can that be? We know another property of motion: the acceleration is inversely proportional to the mass of the body; the heavier the body, the harder it is to accelerate it with the same force. We come to the conclusion then that the acceleration is reduced because the mass of the body increases. In this way both ends meet; as the velocity increases the body becomes heavier, and the previous force can no longer provide the previous acceleration. The acceleration drops and the velocity remains almost constant. Einstein derived the formula indicating the increase in mass as the velocity v of the body approaches unity:

$$M = \frac{m}{\sqrt{1-v^2}}. \tag{3.1}$$

Here m denotes the mass of the body when it is stationary, i.e. when $v = 0$. At a velocity v approaching unity, the denominator of the fraction becomes smaller and smaller, and the fraction becomes larger and larger.

Mass and Energy

Let us now approach the problem from the other side. Evidently, the force that acted for so long must have been applied by some person or by some kind of engine. Assume that it was an engine. This engine operated for a certain length of time, consuming fuel and expending energy. But energy, as we know, cannot be lost or cannot vanish without leaving a trace. It is evidently transferred to the body being accelerated, and the longer the engine is in operation, the more the energy absorbed by the body. But how can it be absorbed when the velocity of the body cannot in any case exceed unity? This question is simply answered. The energy is spent in increasing the mass of the body. The increase in mass represents the increase in energy. Again everything tallies: the force does work on the body, increasing its energy; the energy is accumulated in the body, increasing its mass. The origin of the famous formula $E = Mc^2$ becomes clear now. We shall write it in the form

$$E = M, \tag{3.2}$$

because we took the velocity of light c to be unity. But do not think that we have derived the formula $E = Mc^2$. It was obtained on the basis of entirely different considerations, and all that we did was to explain its meaning in the simplest possible way.

Let us summarize what has been said, but express these ideas in another manner. Why are new formulas required for the mass and the energy when a body travels at a very high velocity? If the mass of the body did not increase when it is accelerated, then its velocity would continue to increase until, finally, the body would overtake light, but this would contradict experiments. If the energy of an accelerated body would not increase, where would the work done in accelerating it go to?

Answers to Your Questions

"That is all very fine," you say, "but why has nobody ever noticed that accelerated bodies become heavier?"

This really is hard to observe: all that surrounds us travels too slow. Slow in comparison to the velocity of light, i.e. to unity. The accumulation of mass in a body becomes observable only as the velocity of the body approaches the limiting value, whereas the velocity of the fastest rocket is less than 1/10 000. So high is the velocity of light. If the velocity of light was, let us say, 10 km/s, the rocket engineers would have to apply Einstein's formula in their calculations, thereby taking into account the increase in the inertia of the rocket as it approaches this velocity. But if the velocity of light was still lower, for instance 1 km/s, a great many phenomena in the world would take place in a different way and Einstein's mechanics would seem to us to be just as natural as Newton's mechanics do now.

"Just a minute," you ask another question, "don't they contradict each other at our low, customary velocities?"

No, they do not. Einstein took care in his reasoning not to upset the repeatedly tested laws of Newton at low velocities. If the velocity v is very low, the fraction $1/\sqrt{1-v^2}$ becomes equal to $1 + v^2/2$ to a high degree of accuracy (you can check this by substituting, for instance, $v = 0.0001$). The formula for the increase in mass is then transformed to

$$M = m + \frac{1}{2} mv^2. \qquad (3.3)$$

When a rocket flies with the velocity of 30 km/s, then $v = 0.0001$, i.e. its mass is increased by one two-hundred-millionth. It is practically impossible to measure such a change in mass.

We can, instead of equation (3.3), write an equivalent equation, recalling that the mass of a body and its store of energy are the same thing:

$$E = m + \frac{1}{2} mv^2. \quad (3.4)$$

Hence, at low velocities the energy of any freely travelling body consists of two parts: of the part m which does not depend upon the velocity of the body, and the part $mv^2/2$ which increases with the square of the velocity. Just a minute: but $mv^2/2$ is the kinetic energy of the body! This means that Einstein discovered that the kinetic energy (which we usually assume to be the energy of a body in free flight, not subject to the action of any forces) is only a part of the store of energy possessed by the body, and it is only a very small part. The main energy is in the term m, which is the mass that is not affected by the velocity and that the body has even when it is stationary. This could be called the energy of existence.

If a new grain of matter comes into being somewhere, some kind of work must have been done to create it. A stock of energy must have been acquired from some other body or decanted from some source in order to construct the new grain of matter. This energy is in the grain even when it is at rest. This may sound frivolous when we think of ordinary large bodies made up of atoms. We always produce them of ready-made building material (atoms) and expend no energy in creating the atoms. Hence, the energy of existence is of no particular importance in this case; all that is required already exists. The problem of creating matter simply does not arise.

But the transformations due to decay and collision processes of the tiniest particles are quite another matter. Here new kinds of particles are actually created out of previous kinds and the energy they have accumulated, or even out of light alone. Like Ibsen's Button-Moulder, we remelt all the old stuff with no scraps left, and nobody allows us to neglect the energy m.

And the Elephant Was Overlooked

The presence of the term m in the energy equation is so vital that it is worth a more detailed discussion. Why don't we notice this component? Why is it that before Einstein nobody had noticed such vast reserves, "at hand" one might say, that exceed all the energy available at that time by millions and thousands of millions of times? Does that not imply that Einstein was wrong? No, it does not. The fact is that we perceive the change in energy, rather than the energy itself. When kinetic energy changes into potential energy we immediately notice the change because the velocity of the body decreases. If it changes into thermal energy, again we notice it because the body gets hotter. But if the energy is not converted, how can we perceive it? Take the earth, for example. Its kinetic energy is enormous. It whirls around the sun at the mad pace of 30 km/s and its mass is 6 10^{27} g. This is a stupendous store of energy, exceeding our powers of imagination. But who takes any notice? In what way does it reveal itself? Should it be taken into account and entered into the energy balance of conversions that take place with terrestrial bodies? Of course not; it does not change in such conversions of energy; it is dead stock and leads to no change in the balance of energy.

The same is true of the energy m. It remains unchanged in all mechanical, electrical and chemical processes; it is a silent partner on both sides of the energy balance equation and is not worth a rap to anybody. But if we could find some forces capable of pinching off even a piece of m, then m would immediately make itself felt. At first nothing was known about such forces. It's a good thing that the formula

$$E = m + mv^2/2$$

suggested, at least, that such forces are worth looking for. They were found many years later; they turned out to be nuclear forces. In nuclear power plants and in ships propelled by nuclear power, such forces are engaged in nipping off tiny parts from m and converting them into electrical or mechanical energy.

In decay and collision processes involving elementary particles, forces, similar in nature but incomparably greater in magnitude, no longer pinch off piece by piece of the rest mass m. Their activity drastically reconstructs certain building bricks of matter into others, which, at times, in no way resemble the first ones, neither in properties nor in purpose.

Momentum and Velocity

But we have digressed from our direct goal. Now we know how the mass of a body depends upon its velocity:

$$M \quad \frac{m}{\sqrt{1-v^2}}.$$

The energy of the body depends upon its velocity in exactly the same way:

$$E = M = \frac{m}{\sqrt{1-v^2}}.$$

What, then, is to be done with the quantity $p = Mv$, called the momentum of a body? Maybe it also has to be replaced by some other equation?

This turned out to be unnecessary. The momentum is expressed, as previously, by the equation

$$p = Mv,$$

but now M is a quantity depending upon the velocity. This implies that the momentum, like the mass of a body and its energy, can increase without limit as the body accelerates. Hence Newton's statement still holds that the increase in the linear momentum of a body due to the action of a force is proportional to the magnitude of the force and the length of time it acts.* If

* The most general form of Newton's second law is applicable in this case as well:

$$F = \frac{dp}{dt} = \frac{d(Mv)}{dt},$$

i.e. the force is equal to the rate of change of the momentum. Schoolboys use a particular case of this formula the force acts for a sufficiently long time (and in the required direction), the momentum may reach any unlimited high value.

It follows that the formula for the momentum can be written in any of three forms:

$$p \quad Mv$$

$$p = Ev \qquad (3.5)$$

$$p = \frac{mv}{\sqrt{1 - v^2}}$$

and applied as required.

These are the transformed expressions of the energy and momentum that must be used for any body if its velocity is to some extent comparable to the velocity of light, i.e. unity.

Now is just the time to foresee the questions that an inquisitive reader might ask. You may

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in which the mass of the body is constant and only its velocity varies. Hence, the force is equal to the mass multiplied by the rate of change of the velocity, i.e. by the acceleration:

$$F = m\,\frac{dv}{dt} = ma.$$

The general form of the equation is applied, of course, not only in describing the motion of fast particles, but in general in solving problems on the motion of bodies of variable mass, for instance, a rocket. This is exactly what Tsiolkovsky did in deriving his famous formula

$$v = V \ln \frac{M_0}{m}$$

where M_0 is the initial (launching) mass of the rocket, V is the velocity of the exhaust gases ejected from the engine nozzle, and v is the velocity of the rocket when its mass is equal to m.

ask: "How can we calculate the kinetic energy now if the formula

$$T = \frac{mv^2}{2}$$

has turned out to be incorrect at high velocities?"

The answer is that the kinetic energy of a particle is the difference between the total energy of a particle, calculated by the formula

$$E = \frac{m}{\sqrt{1 - v^2}},$$

and its rest energy m, i.e.

$$T = \frac{m}{\sqrt{1 - v^2}} - m.$$

At low values of v the quantity calculated by this last equation differs only very slightly from that obtained by the ordinary equation $mv^2/2$.

Another question is: how can the mass equal the energy with the mass being measured in grams and the energy, for instance, in kilowatt-hours?

But after it has been explained that mass is equivalent to energy, we, knowing the mass of a body, also know its stock of energy. It becomes natural, then, to select units of mass and energy that reveal this equivalence at once. Different units for mass and energy are tolerable only where this equivalence is of no importance, i.e. almost in all phenomena on a terrestrial scale. But where the difference between energy and mass is simply the difference between two aspects of motion (the word "energy" sets off the "store of creative power" of a particle, whereas the word "mass" sets off its inertia properties, its unyielding nature, and one cannot exist without the other), it would be sinful to measure them differently. Hence, in the subatomic world, the unit of measurement is selected so that the energy of a particle is numerically equal to its mass.

But can this be done? Of course. If $E = M$ at $c = 1$, then E and M are naturally to be measured in the same units. In exactly the same manner, if $p = Ev$ and the velocity of a particle is taken as its ratio to the velocity of light, then the momentum p can also be measured in the same units. So long as the processes in the subatomic world do not affect our world, this agreement—to measure energy, momentum and mass by the same unit of measurement—will not lead to any inconveniences; quite the opposite.

What is this unit of measurement? It is called the electron volt (eV). At first this was only a unit of energy and represented the energy acquired by an electron when it passes through a potential difference of one volt in vacuum. One thousand million electron volts (10^9 eV) is equal to one giga electron volts (1 GeV). Measured in these units are both the mass and the momentum, not of large bodies, of course; only of the tiniest ones. This unit is convenient because the mass and energy of a particle are expressed by a small number. The mass of the proton, for instance, is 0.94 GeV and the momentum acquired by protons in the large Dubna accelerator is 10 GeV, etc.

And you might ask this final question: "Is it true that the new mechanics with its new definitions of mass, energy and momentum is required only in the subatomic world, and is of no account in our ordinary world?" No, because among our large machines we have some that cannot be designed according to the laws of Newton's mechanics. These are the particle accelerators (Fig. 1). Their purpose is to accelerate particles, for instance, protons, to velocities near to that of light. At this the proton, according to Einstein's teaching, becomes much more massive.

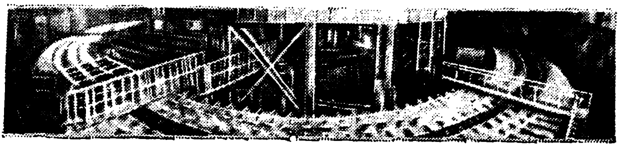

Fig. 1. A run-of-the-mill accelerator.

Its mass grows larger and larger in each revolution along its circular path. With each revolution it becomes more and more difficult to hold the particle in this annular chamber. The forces of the magnetic field are already insufficient to conduct such a massive particle around the circle. It becomes necessary to supply higher and higher current to the electromagnet. In the large Dubna accelerator, for example, where the velocity of the proton does not differ, practically, from that of light, the mass M of the proton at the end of the acceleration period becomes 10 GeV. At the beginning it was equal to 0.94 GeV This means that in 3 s (the length of time required for acceleration), the proton becomes over ten times more massive. By the end of acceleration, the power consumed by the electromagnet increases by a great many times. If you wish to be convinced of the validity of Einstein's formula, watch the wattmeters on the central switchboard to see how reactive load increases.

"Just a minute, just a minute!" exclaims the alert reader. "What is happening here? Electrical power is vanishing at the power station, and ten-fold heavier protons are appearing in the accelerator. Do you mean to say that energy has been converted into mass?"

"What is there so alarming about that?"

"Because that is an erroneous philosophical thesis. Besides, you yourself contended that energy and mass are simply two different shades of the same physical concept."

"In physics, yes; but in common practice an increase in energy does not mean an increase in mass. A teapot does not become heavier because it is heated. Hence, in the everyday sense, there is a huge difference between energy and mass. And when you become a witness to the fact that the power supplied to the accelerator input turns into exceptionally heavy protons at the outlet, you have the right to exclaim in astonishment: 'Electrical energy has turned into the mass of a proton!'"

"Or into its energy ..."

"Or into its energy if we wish to underline the 'store of power' or 'creative potential' of the proton, rather than its 'unyielding nature'. We must get used to the fact that the 'unyielding nature' and the 'store of creative power' of a particle are synonyms. When we get accustomed to this fact, an ineradicable desire arises to banish one of the words, 'energy' or 'mass', and manage with only one. Such attempts are being made by the writers of textbooks and monographs in physics. But in our book, in which the lack of mathematical equations has to be compensated for by verbal expressiveness, we shall employ both synonyms: energy and mass."

"But what about the philosophers?"

"Philosophers differ. Why look for problems where they do not exist? Why pay so much attention to the use of words, where the true meaning is not in words, but in exact relations?"

People with a practical turn of mind are interested in an entirely different matter: "Is it true that the energy evolved in the decay of elementary particles greatly exceeds nuclear energy?"

"Yes, it does. For example, one of the cycles of nuclear reactions that provide the energy of the stars consists in the transformation of four protons into a helium nucleus. Their mass is $0.94 \times 4 = 3.76$ GeV, whereas the mass of helium is 3.73 GeV. Consequently, 0.03 GeV is released, which is less than one percent of the total energy. But in the decay of the neutral pi meson (π^0 meson) into photons the whole mass (100%) of the meson is converted into energy."

"You mean to say we have here a source of energy more powerful than a thermonuclear reaction?"

"By no means. An obstacle is the rareness and instability of such mesons; they cannot be accumulated. And, what is of primary importance, they must be created by expending an amount of energy that is exactly equal to that evolved in their decay. Complete protons, on the other hand, are always available; they are the nuclei of hydrogen. In a thermonuclear reaction we squander stores of energy accumulated by nature; neutral pi meson decay at best only returns the energy spent in creating the mesons."

"Then of what use are they?"

"Mesons, hyperons and their ilk are of use to us for an entirely different reason. They provide clues to the structure of the world."

Chapter 4

More on Energy and Momentum

By now, I hope, the opposition of the reader has been broken down, and he is ready to get accustomed to the new and revised concepts of energy, momentum and mass. We found out that a particle becomes heavier as it is accelerated, that a particle (and any body) cannot acquire additional energy without increasing its mass (inertia) and that at high velocities the increase in momentum is inevitably accompanied by an appreciable increase in mass and energy, because the velocity does not practically increase.

New aspects of the introduced concepts shall appear before the reader in this chapter. We shall find that in acceleration the mass, momentum and energy of a particle increase, that the particle has one (at least one) kinematic characteristic that nothing can change. We shall also find out one astonishing fact: that the mass also depends upon the motion of the instrument that measures this mass. We shall find that there are particles that cannot be at rest, and much, much more.

To begin with let us recall the experiment that we performed in Chap. 2. There we fired a bullet point-blank into a box of sand in order to measure the mass of the bullet, and to write the formula $m = p^2/2T$, which relates the mass of the bullet to its energy and momentum.

By Far the Most Important Equation

Let us see whether such a relation is valid for rapid motion as well. The essence of our calculations in Chap. 2 consisted in the following: we determined the velocity v from one equation and then substituted the obtained quantity into another equation. As a result, the quantity v was eliminated from the equations. Now let us try to get rid of v in the new equations for E and p. This can most easily be done by squaring both sides of the equation for the momentum to obtain

$$p^2 = \frac{m^2v^2}{1-v^2},$$

and then doing the same to the equation for the energy:

$$E^2 = \frac{m^2}{1-v^2}.$$

Next we subtract the first equation from the second:

$$E^2 - p^2 = \frac{m^2}{1-v^2} - \frac{m^2v^2}{1-v^2} = \frac{m^2-m^2v^2}{1-v^2} = \frac{m^2(1-v^2)}{1-v^2}.$$

In final form

$$E^2 - p^2 = m^2. \tag{4.1}$$

This, then, is the required relation between the energy, momentum and rest mass of a par-

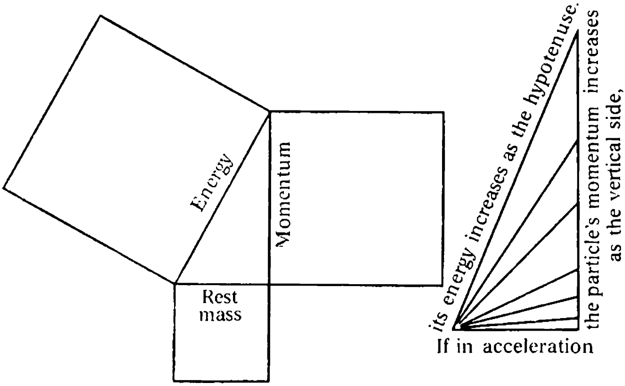

Fig. 2. The Pythagorean theorem in the relativistic* sense.

ticle. It can also be written as

$$E^2 = p^2 + m^2, \tag{4.2}$$

in which case it reminds us of the well-known Pythagorean theorem. If we draw a right triangle (Fig. 2) in which the length of the horizontal side is equal to the rest mass m of a particle, and the length of the vertical side to its momentum p, then the length of the hypotenuse is equal to the total energy E. As long as the par-

* The word "relativistic" means "of or having to do with the theory of relativity".

ticle is stationary, the whole triangle is merged into the horizontal length m, the momentum is equal to zero, and the energy to the rest mass m of the particle. As the particle is accelerated, its momentum begins to increase and, together with the momentum, its energy. At first the change in energy is very slight because in a low triangle the hypotenuse is almost equal to the horizontal side. This is the case of motion at our ordinary everyday velocities and it obeys ordinary Newtonian mechanics. As the velocities become higher and higher, the energy increases faster and faster. At extremely high velocities almost equal to that of light, the triangle is drastically extended upward. In such a high narrow triangle, the hypotenuse is almost equal to the vertical side, i.e. at extremely high velocities the difference between the momentum and the energy of a particle (and, consequently, the mass) is almost obliterated:

$$E \approx p \quad (\text{at} \quad v \approx 1). \tag{4.3}$$

Almost, but never completely. No matter how high the triangle is, the momentum is nevertheless always less than the energy, and less by exactly the amount required so that $E^2 - p^2$ is equal to m^2.

In Einsteinian mechanics, which shows especial interest in the way that various quantities vary with the velocity, the rest mass m is frequently called the *invariant of motion*. Then the difference $E^2 - p^2$ is said to be constant (equal to m^2) and that it is the invariant of motion. No matter how a body is accelerated its momentum and energy grow simultaneously and in unison so that the difference $E^2 - p^2$ does not change.

The formula $m^2 = E^2 - p^2$ can be used to find the rest mass of a particle if its energy and momentum are known.

The rest masses of all known particles, together with other properties, are listed in special tables. To determine whether a particle observed in an experiment is a new item or belongs to the number of particles that have been investigated long ago, it is necessary to measure its energy E and (independently) its momentum p. Then we calculate the difference $E^2 - p^2$, find its square root, and look it up in the table. If there is such a value, fine and good, but it is even better if there is not. This means that you have discovered a new particle. Such a discovery is a tremendous event. It is believed that the whole assortment of existing particles is a manifestation of some kinds of fundamental properties of nature. It is therefore of vital importance to know whether we have already seen all of the elementary particles or whether we have missed any. Attempts are made to classify the known particles to find some definite order in them.

The masses of the particles do not differ greatly in magnitude. The heaviest of terrestrial objects, the earth itself, is about 10^{25} times heavier than an apple. One of the heaviest elementary particles, the omega-minus hyperon, is only 3300 times more massive than one of the lightest, the common electron.*

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* Today much heavier particles are known. The upsilon, for instance, is approximately 20 000 times heavier than the electron.

It can be seen from formula $E^2 - p^2 = m^2$ that the rest mass of any particle determines by how much the increase in its momentum lags behind the increase in its energy. At not very high energies, E and p differ very greatly for heavy particles, but almost not at all for light particles. As the energy increases the three characteristics of the particle merge into one. A proton with a momentum of 10 GeV has an energy of 10.044 GeV and, consequently, its effective mass is also 10.044 GeV But its rest mass is only 0.94 GeV. The remaining 9.104 GeV of mass is the mass of motion (we can also say that the remaining 9.104 GeV of energy is the kinetic energy of the proton).

Unstoppable Particles

Among the elementary particles there are some with a rest mass equal to zero. These are the photon and the two kinds of neutrino, electron-like and muon-like*. When the rest mass of a particle is equal to zero, it is said to simply have none: the particle has no rest mass.

Let us consider the consequences of zero rest mass. We shall begin at the end. The relation $E^2 - p^2 = m^2$ is converted into $E^2 - p^2 = 0$, or $p = E$. The energy of such a particle numerically coincides with the momentum and also with the kinetic energy (no rest mass!), and with the mass. Yes, and with the mass! The formula

* These neutrinos are denoted by the symbols ν_e and ν_μ. The existence of a third neutrino, ν_τ, also with zero rest mass, has been predicted. It has not yet been detected experimentally.

$E = M$ continues to shine in all its splendor. A particle having no rest mass nevertheless has mass, but it is the mass of motion. Do not imagine that this is mere wordplay, that, from force of habit, we are parroting concepts that have no significance. No, the photon really has mass, which is manifested like ordinary mass. The photon has inertia; it is attracted to other bodies according to the law of gravitation. For example, in passing near the sun or the stars, its path is deflected; in approaching the earth vertically the energy of the photon increases as does that of any falling stone. The energy of a photon is proportional to its frequency: $E = h\nu$, where h is the famous Planck constant, equal to 4.14×10^{-24} GeV-s. This means that its frequency is also increased. It is precisely this change in frequency that can be recorded in an experiment. How this is done is described on page 154. As long as the photon is in motion, everything is in perfect order, just the same as for other particles. But, as soon as we make up our mind to stop, or simply to slow down, or, on the contrary, to accelerate the photon, we find that something is wrong with it. A photon cannot be slowed down, and it cannot be speeded up. Its velocity is always equal to unity (as we saw at the end of Chap. 3, in equation (3.5), the momentum is equal to the energy multiplied by the velocity, from which the velocity $v = p/E$, and for the photon $p = E$). It always travels with the velocity of light; it is simply a portion of electromagnetic radiation, a particle of light having the energy E and momentum p. And the velocity of light, with which we began our account in Chap. 3, is constant in free space; we took it to be equal to unity. There, we *took* it equal to unity and now we *obtain* unity for the velocity of light because $m = 0$. Again everything tallies, and that is very pleasant.

Thus, apart from ordinary ones, particles are conceivable (and actually observed) with zero rest mass, particles that cannot be stopped. For these particles the formula

$$E = m/\sqrt{1-v^2} \qquad (4.4)$$

is inapplicable (it becomes $E = 0/0$, and you cannot calculate anything with such an equation). This is what indicates the impossibility of raising the question of the dependence of any characteristics of the photon on its velocity. For the photon its velocity is the same kind of internal and innate characteristic as the mass or charge for other particles.

Do not infer, however, that since photons and neutrinos are incapable of standing stock-still, being slowed down or speeded up, they are not subject to any influence or action. Photons can be diverted; they can disappear, reappear again, and be converted into photons of lower energies, but only in such a way that their velocity remains unchanged. The direction of their velocity can change, however, and this turns out to be sufficient to prevent a photon in motion being distinguished from other particles. Moreover, at superhigh energies, other particles begin to resemble the photon. Their energy, as we could see in using a proton with the energy 10 GeV as an example, becomes closer and closer to their momentum, their velocity differs only slightly from unity, and instruments no longer distinguish them from photons and from one another with respect to these characteristics (though they may greatly differ with respect to other properties).

In Chap. 3 we became acquainted with an extraordinary concept: the energy m of existence of a body. It was stated that any body possesses energy simply because the body exists and is made up of something. It becomes clear now that this term, the energy of existence, is not always convenient and can lead into error if we understand it too literally. Photons really exist (as a matter of fact, we exist owing to solar photons).

But their energy of existence m equals zero. It is more convenient, therefore, to speak of rest energy.

Here the reader may take offence: was it worthwhile to introduce a concept that does not always make sense? It was. The term was not long-lived, but it served its purpose. It helped to store in our consciousness the concept that, in the first place, energy is required (or was required) at some time in the past to create particles, and, in the second place, any existing body is a potential source of energy, even when it rests calmly in one spot. Whereas the term "rest mass" excites no harmful illusions, neither does it arouse our fantasy. It is better not to be angry, but to thank "energy of existence" for its faithful service and to retire it with an honourable discharge. The term has done its duty, let it go.

—1920

A New Concept

To fill the empty place, we shall introduce a new concept; we shall run across it frequently in the future. It is the *relativistic** *factor* γ (it is also called the *Lorentz factor*). This factor indicates by how many times the mass of a particle has increased at a given velocity compared to its mass at rest:

$$\gamma = \frac{M}{m}, \text{ or } \gamma = \frac{E}{m}, \text{ or } \gamma = \frac{1}{\sqrt{1-v^2}}. \qquad (4.5)$$

For a particle at rest $\gamma = 1$; as the velocity increases to unity, γ increases without limit, just as E and p, but, in contrast to them, is a dimensionless quantity, independent of the chosen units of measurement, and therefore very convenient. In the mechanics of ultrafast motion, the velocity is no longer the essential feature of motion it is in mechanics on a terrestrial scale. This is crystal clear: when the velocity of bodies is close to that of light anyway, what is the point of asking what the velocity of this or that particle is? The answer is predetermined: $v \approx 1$. Factor γ is quite another matter: two bodies having almost the same velocities ($v_1 \approx v_2$) may have quite different relativistic factors γ_1 and γ_2 even when $v_1 \approx 1$ and $v_2 \approx 1$.

Though we defined the relativistic factor γ in terms of the dynamic characteristics of the particle (its energy and rest mass), γ actually (as shown by the third equation in (4.5)) depends only on the velocity of a body. Hence the quantity

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* See the footnote to Fig. 2.

γ can be used to characterize the motion not only of particles, but also of objects whose material nature has not been stipulated. A problem, for instance, that frequently arises in physics is the measurement of the properties of a body in various frames of reference, both stationary and moving ones. In such cases, there is no sense in speaking of the energy and momentum of the frame of reference (coordinate system); only its velocity is of importance. But, together with the velocity of the reference frame, we determine γ, which, like the velocity, also characterizes the motion of the frame.

After writing out the conservation laws, the aforesaid is quite sufficient to begin our acquaintance with kinematics proper of particle decay and interaction. But there is one more important concept in Einstein's theory that may come in handy. It reflects the meditations of Einstein and his forerunners on space and time; it concerns the changes in momentum and energy of a body upon changes in the motion of the instruments measuring these quantities; it pertains to the so-called *Lorentz transformations.*

An Exceptionally Important Problem

We are looking at a body travelling at high velocity and, in some way, measure its momentum and energy. Then we ask ourselves: if we ourselves started off in pursuit of the body, would there be any changes in the energy and momentum that we had previously measured? Or if somebody shouts to us that we are not at all stationary, but are travelling in the same direction as the body whose characteristics we had measured, but do not realize that we are, whereas he, an unbiassed observer, took notice, stopped, measured the energy of the same body and obtained an entirely different result, would we believe him?

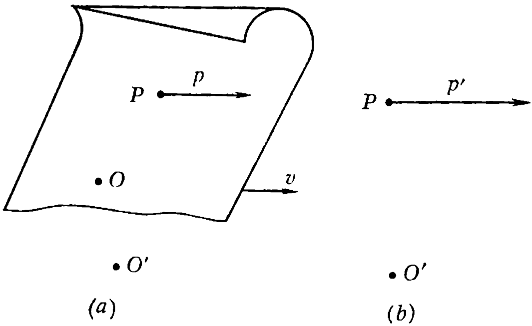


Fig. 3. The simplest Lorentz transformation:
(a) Observer O thinks that particle P has the momentum p, whereas observer O' is of the opinion that observer O is travelling at velocity v in the same direction as particle P (b) From the point of view of observer O', particle P has the momentum p'

Yes, evidently, we would have to believe him. In what way is the energy any better than the velocity? But, there is no doubt that the velocity would be different if the instrument measuring it travels in different ways. The energy of a particle is related to its velocity. But even without this fact it is clear that the particle will affect the instrument measuring its energy in different ways, depending on whether this instrument is at rest with respect to the particle or is in motion, and whether, in the latter case, this motion is slow or fast. And if the particle affects the instrument in different ways, its readings will also differ. When we specify the energy of a particle we must first make clear how, at what velocity and in which direction the instrument that measured the energy was travelling at the time.

This poses the following question. If the energy (or momentum) of a particle was measured by two instruments (two observers) travelling at different velocities, in what way should the readings of the two instruments be related to each other? What is the relation between the energies of one and the same particle measured, as they say, in different frames of reference? Or, in the language we have been employing, if the reference frame, in which measurements yielded the energy and momentum values E and p for a particle, is itself in motion with respect to some new frame (in the same direction as the particle) at the velocity v, what are the energy and momentum of the particle in the new reference frame (Fig. 3)?

The Lorentz Transformations

Let us denote the new energy and new momentum of the particle by E' and p', and characterize the motion of the previous reference frame with respect to the new one by the velocity v and the factor $\gamma = 1/\sqrt{1 - v^2}$. Then it turns out that E' and p' can be expressed linearly in terms of E and p, i.e. they equal the sum of the old energy and momentum multiplied by certain coefficients:

$$\left.\begin{aligned} E' &= \gamma E + \gamma v p, \\ p' &= \gamma p + \gamma v E. \end{aligned}\right\} \tag{4.6}$$

As we can see, the coefficients depend only upon the velocity of the old reference frame with respect to the new one. It is evident from these equations (they are called Lorentz transformations) that the energy and, of course, the momen-

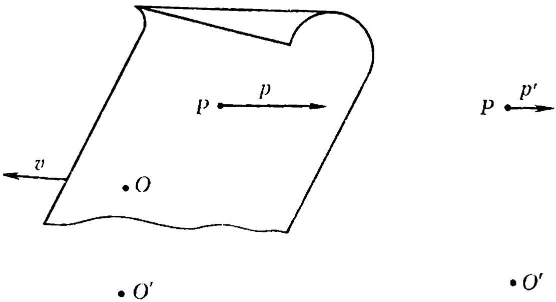


Fig. 4. A Lorentz transformation.
In contrast to Fig. 3, observer O, according to observer O', speeding along at velocity v in the opposite direction.

tum in the new reference frame are greater than in the old frame.

It is quite another matter if the previous reference frame, in which the values E and p were measured, travels (as discovered by the observers in the new reference frame) in the direction opposite to the motion of the body (Fig. 4). Then a minus sign will have to be put before the velocity v and the Lorentz transformation equations take the form

$$\left.\begin{aligned} E' &= \gamma E - \gamma v p \\ p' &= \gamma p - \gamma v E. \end{aligned}\right\} \qquad (4.7)$$

Well, and what if we find that the direction of the particle and the direction of motion of the previous reference frame have nothing whatsoever in common? Assume that we thought we were at rest and measured the energy E and the momentum p of some particle. But while we were busy with our measurements somebody else noticed

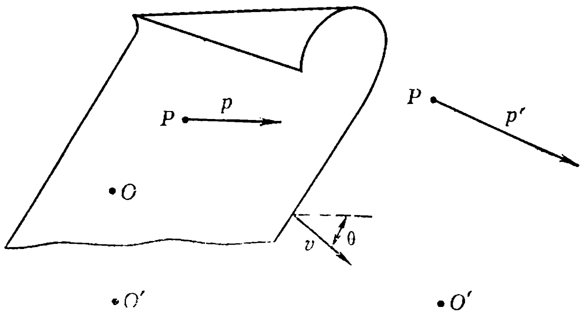


Fig. 5. The general case of Lorentz transformations. From the point of view of observer O', the directions of travel of observer O and of particle P form the angle θ; the momentum p' of particle P, from the point of view of observer O', is shown at the right.

that we were moving, not in the direction of the particle, but toward one side, off at the angle θ with the velocity v (Fig. 5). What do the energy E' and the momentum p' of the particle seem to be to this somebody else?

Travelling Obliquely

The rule here is also simple. We represent the momentum p by a vector, i.e. an arrow that points in the direction in which the particle travelled and is of a length conditionally equal to the magnitude of the momentum. If the momentum is 5 GeV, for instance, we can take one centimetre equal to 1 GeV, and draw an arrow 5 cm long. The greater the momentum, the longer (in a given scale) the vector. Then, on the same drawing, we represent our own direction

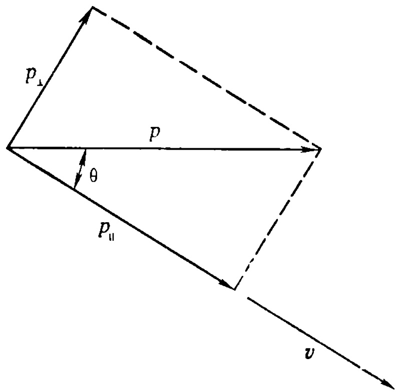


Fig. 6. A Lorentz transformation (stage one).

and our own velocity. Since velocity and momentum are different quantities, we can take any velocity scale we like. Next, we project the momentum vector on the velocity vector by dropping a perpendicular from the head of the momentum vector onto the velocity arrow (or on its extension) and onto the direction perpendicular to the velocity (Fig. 6). We thereby obtain two new vectors. They are called the longitudinal component of the momentum (denoted by $p_\parallel$, and equal to $p \cos \theta$) and the transverse component (denoted by $p_\perp$ and equal to $p \sin \theta$). It turned out that the previous rule, equation

(4.6), concerns only the longitudinal component of the momentum (there the momentum p was in the direction along v and simply coincided with its longitudinal component), i.e. the rule for transforming the longitudinal component $p_{\|}$

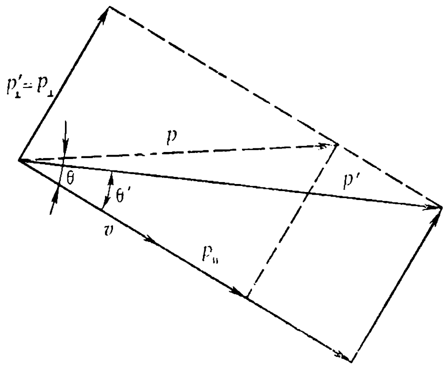


Fig. 7. A Lorentz transformation (stage two).
Observer O passes on the baton to observer O'; this does not alter the transverse component of any momentum, whereas the longitudinal component is changed by a two-member equation; observer O' has to combine the momentum from its two components.

and the energy E is as follows:

$$E' = \gamma E + \gamma v p_{\|}, \quad p'_{\|} = \gamma p_{\|} - \gamma v E. \tag{4.8}$$

This means that we should first calculate $\gamma = 1/\sqrt{1 - v^2}$, and then we can determine the energy of the particle from the first equation of (4.8) and the longitudinal (again the longitudinal) component of the momentum from the second equation. After recalculating, it becomes, for example, equal to 8 GeV. We draw this vector in Fig. 7. What is to be done with the transverse component? It does not change. This means that arrow $p_\perp$ is to be carefully transferred, without being turned in any direction, from Fig. 6 to Fig. 7 and put with its tail at the head of vector $p'_{\parallel}$. The head of the particle's new momentum vector will be at the head of vector $p_\perp$. Next we measure the length of the arrow representing vector p' in the previous scale (1 cm = 1 GeV) and obtain the magnitude of the momentum. How about its direction? This arrow gives the direction as well, i.e. it shows in which direction, in the opinion of the stationary observer (as the observer that ascertained in which direction and how we are moving is conditionally called), the particle is travelling. Recalling the properties of right triangles, we can write the equations for the new angle θ' and the new momentum p'

Hence, in the general form, the Lorentz transformation equations are:*

$$\left.\begin{aligned} E' &= \gamma E + \gamma v p_{\parallel}, \\ p'_{\parallel} &= \gamma p_{\parallel} + \gamma v E, \\ p'_{\perp} &= p_{\perp}. \end{aligned}\right\} \quad (4.9)$$

* If in these equations (4.9) the energy E is replaced throughout by the time t, and the momentum vector $\mathbf{p}$ is replaced by the position vector $\mathbf{r}$, we obtain the transformation equations for calculating the location and time of any possible event in going over from one frame of reference to another. This is exactly the form (i.e. with $\mathbf{r}$ and t) in which Lorentz first wrote the transformation equations. But we shall have no need for the equations with $\mathbf{r}$ and t, and by the Lorentz transformations we shall imply equations (4.9).

Naturally, before employing these equations it is necessary to resolve the momentum vector into its longitudinal and transverse components and, after carrying out the calculations, to piece together again the complete momentum from the new longitudinal and the new (actually the former) transverse component. When p and v coincide in direction, we obtain equations (4.6); when p and v are in opposite directions, we obtain equations (4.7) (the longitudinal momentum is equal to momentum p preceded by a minus sign). And again everything tallies.

Again the Most Important Equation

Just a minute; not everything tallies! There was a rigid relation, $E^2 - p^2 = m^2$, between the previous energy and momentum of the particle; the difference of their squares yielded the square of the rest mass. And now, in the new reference frame, is this equation still valid? It would be too bad if it was not. That would imply that there is one best frame of some kind in which we obtain m^2 after subtracting, and other, worse systems in which something else is obtained instead of m^2. But Einstein pointed out that there were freedom and equality among the frames of reference, and no preferences of some over others. More exactly he said (on the basis of experiments conducted by physicists) that in any frames of reference, moving with respect to one another in straight lines and at constant velocities, all the laws of nature apply equally well. Among others, the law stating that the difference $E^2 - p^2$ is constant in the motion of a particle and is equal to m^2 at all times should hold strictly everywhere. Let us check this statement.

We are to calculate the difference $E'^2 - p'^2$. According to the Pythagorean theorem (see Fig. 7), $p'^2 = p_{||}'^2 + p_{\perp}'^2$. For E', $p_{||}'$ and $p_{\perp}'$ we substitute the expressions from equations (4.9) and begin our calculations:

$$E'^2 - p'^2 = E'^2 - p_{||}'^2 - p_{\perp}'^2$$

$$= (\gamma E + \gamma v p_{||})^2 - (\gamma p_{||} + \gamma v E)^2 - p_{\perp}^2$$

$$= (\gamma^2 E^2 + 2\gamma^2 v E p_{||} + \gamma^2 v^2 p_{||}^2)$$

$$- (\gamma^2 p_{||}^2 + 2\gamma^2 v E p_{||} + \gamma^2 v^2 E^2) - p_{\perp}^2.$$

So far we have only applied the rule for squaring the sum of two quantities. Next, grouping the terms containing the factors E^2, $p_{||}^2$ and $Ep_{||}$ separately, we obtain for the right-hand side of the equation

$$E^2 (\gamma^2 - v^2\gamma^2) + p_{||}^2 (\gamma^2 v^2 - \gamma^2)$$

$$+ 2Ep_{||} (\gamma^2 v - \gamma^2 v) - p_{\perp}^2.$$

The expression within the third set of parentheses is identically equal to zero. That within the first set of parentheses equals $\gamma^2 (1 - v^2)$ and, since $\gamma^2 = 1/(1 - v^2)$ according to equation (4.5), the expression is simply equal to unity. For the same reason, the expression within the second set of parentheses is equal to -1. It follows that there is almost nothing left:

$$E^2 - p_{||}^2 - p_{\perp}^2 = E^2 - (p_{||}^2 + p_{\perp}^2).$$

Next we look again at Fig. 6. Again the same immortal theorem of Pythagoras convinces us that the expression in parentheses is simply the square of the momentum.

Thus, we have proved (see the beginning of the calculations) that

$$E'^2 - p'^2 = E^2 - p^2. \tag{4.10}$$

The last difference is familiar: we became acquainted with it at the beginning of the chapter and made certain that it equals m^2, i.e.

$$E'^2 - p'^2 = m^2.$$

This is an especially significant result. The difference of the squares of the energy and momentum of a particle is, consequently, invariant, i.e. it remains constant not only when the particle is accelerated, but in varying the motion of the instruments measuring the energy and momentum, and in varying the motion of the observer. Various observers, rushing past the particle, will not agree in speaking of its energy or momentum; each observer will insist on his own values, which will be related pairwise by equations (4.9). But all arguments will cease as soon as we ask the observers only a single question: what is the difference of the squares of your particle energy and momentum values? Here they obtain one and the same quantity: m^2. Beautiful, isn't it?

When you master the Lorentz transformations, you can solve a great many interesting problems. Some of them will be solved in the following sections. For the time being, try to solve the following two simple problems.

1. It is obvious that if we increase our velocity from zero to that of a particle, we see that the particle stands stock-still, its momentum becomes zero and its energy is equal to the rest mass. Try, then, to obtain this result by applying the Lorentz transformations. Consider your motion at the velocity of the particle to be the motion of a new frame of reference. Then your prior stationary state will look like motion in the *reverse* direction at a velocity equal to that of the particle. But the velocity of the particle is equal to the ratio of its momentum to its energy (this equation is given above). Thus, substitute the fraction $-p/E$ into the Lorentz transformation equations for v, calculate γ and then calculate the new energy and momentum. In your calculations do not forget what the invariant difference $E^2 - p^2$ is equal to.

2. The Lorentz transformations relate the energy and momentum of a particle in a new reference frame (E' and p') to their values in the previous reference frame (E and p). Assume that the velocity of the previous frame is v with respect to the new frame. The equations express E' and p' in terms of E and p. Quantities E and p are considered to be known and E' and p' to be unknown. Now just imagine that E' and p' are known and E and p are unknown. We obtain two equations with two unknowns. Try to solve them and express E and p in terms of E' and p'. You will see that you again obtain the Lorentz transformation equations, but everywhere with a $-v$ instead of a v. Why did this happen and could you have guessed the result beforehand?

Chapter 5

Conservation of Energy and Momentum

It has already been mentioned that on the microstage where the actors are elementary particles, plays with either of two plots are most frequently performed. The first narrates the story of the decay of what is old, obsolete and corpulent, and the birth of what is new, young and agile. The second plot has to do with the clashes between heroes that are impetuous and striving to be in action and other characters that are stiff and fixed, and the collisions that occur in these encounters. In other, perhaps more appropriate words, these are the spontaneous decay of heavy particles into lighter ones and the scattering of fast particles by stationary ones. Scattering is frequently accompanied by the appearance of new particles*

Forbiddance, the Fundamental Principle

We have previously mentioned that the laws governing these processes are intricate and difficult to explain. In any case, particles cannot be conceived in the form of sacks of peas or in the form of matryoshkas** which fall apart

* In recent years our persistent playwrights have been intensely working on a new plot: the collision of two energetic heroes and its consequences. We are referring here to the technique of colliding beams of elementary particles.

** Hollow wooden dolls painted in Russian peasant dress with successively smaller dolls fitted into them.— *Tr.*

from a collision (in scattering) or from old age (in decay) and their contents spill out (peas or smaller matryoshkas). A better concept is to imagine that the elementary particles live in a society reigned by a police state, and that all relations between the citizens and all changes in the society are governed by a number of prohibitive regulations. The citizens are exceptionally conscientious and strictly obey all the prohibitions. Before committing themselves to any transformation (decay or interaction), they take pains to find out whether there is a law forbidding it. If there is, there is nothing more to be said; no transformation takes place. But if there is no such law, the transformation may occur.

For example, the decay of a heavy neutral K^0 meson with a mass of 0.498 GeV into a positively charged π^+ meson (with the mass 0.140 GeV) and the neutral π^0 meson (with the mass 0.135 GeV) cannot occur because the total electric charge of π^+ and π^0 is positive, whereas the initial meson was neutral, and there is a law that *strictly forbids any change in the total charge in decay processes*. Hence, the decay $K^0 \to \pi^+ + \pi^0$ is impossible. But the decay $K^0 \to \pi^+ + \pi^-$ is not forbidden because one of the pi mesons is charged positively and the other negatively, so that as a whole $\pi^+\pi^-$ is neutral. Also unforbidden by this law is the decay into a K^+ meson and a K^- meson (their masses are slightly less than that of the K^0 meson), i.e. the decay $K^0 \to K^+ + K^-$. Neither are the decay processes $K^0 \to \pi^+ + \pi^- + \pi^0$, $K^0 \to K^+ + K^0 + \pi^-$ and many, many others. Which of these decay processes actually occur depends on whether or not they violate other forbidden transformations. We find, for instance, that the decay processes $K^0 \to K^+ + K^-$ and $K^0 \to K^+ + K^0 + \pi^-$ are forbidden by the following law: *the total rest mass of the particles produced in a decay must not exceed the rest mass of the initial particle.* This directly follows from the law of conservation of energy.

As to the decays $K^0 \to \pi^+ + \pi^-$ and $K^0 \to \to \pi^+ + \pi^- + \pi^0$, they do not violate any laws in force in this world. As a matter of fact, the first decay is sometimes encountered, and sometimes the second. Which will occur in each specific case is impossible to predict beforehand.

There is No God, but Forbiddance

Ponder again, Dear Reader, over the essence of what has been said. It is contended that the fundamental principle ruling the morals and manners of the subatomic world is the following: *everything that is not forbidden is allowed.* This is neither a truism nor empty wordplay. In fact, matters could stand so that a process *A* is not forbidden by any general laws, but still is not observed simply because it is process *A*.

In the subatomic world, you can be sure that if there is a law, then it concerns everybody. It can be readily understood that this is a feature of genuine science, i.e. the absence of exceptions, incompatible facts, allowances for the circumstances, uncontrollable likes and dislikes, etc. Physicists wishing to apprehend which processes occur in one or another case must look over and sort out all the possibilities that are not forbidden by known laws. It is quite another matter that the majority of them are unlikely. Unlikely, but not forbidden and some day, if we are patient, we shall see that they do occur. The skill of a physicist consists, among other matters, in the ability to pick out, first of all, the most probable process among the possible ones.

Of the great many forbidding laws existing in the subatomic world, there is one that is of especial interest to us. You probably guess which one; it is the law of conservation of energy and momentum. It can be stated as follows: *in all collision processes (and in decays), the initial energy of the colliding particles (or decaying particles) is equal to the total final energy of the newly formed particles. The same is true of the momentum.* In this form, this law does not seem to forbid anything, but simply states a fact. Do not, however, permit this to lead you into error. Do you know of no laws that at first sight seem to be inoffensive, but are automatically converted into prohibitions owing to the impossibility of breaking them? It is the same here. As an experimental fact, this is a law like all other laws. But if you attempt to base any theoretical predictions on this law, it immediately shows its teeth.

It is impossible for the total final energy of the remaining particles, after a collision (or decay) process, not to be equal to the initial energy of the colliding particles (or decaying particle). The same is true for the momentum.

In this form the law is immediately converted into an implement of research, forbidding certain processes, flinging the doors wide open to others, and predicting the formation of invisible particles in a third group of processes. Nevertheless, we shall not set off one statement of the law against the other. The affirmative statement, so to speak, is quite sufficient at times.

We will be applying the law of conservation of energy and momentum in what follows. Let us write it in the form of equations separately for the decay of a particle and for the collision of two particles. True, we could manage without equations because these laws are so simple. But mathematics organizes our train of thought in some manner, and this should never be ignored.

Decay

Imagine that some kind of particle O, having the energy E_O and momentum p_O, decayed to several particles, for example, three. We shall denote them by the numbers 1, 2 and 3, their energies by E_1, E_2 and E_3, and their momenta by p_1, p_2 and p_3. Then the conservation of energy can be expressed by the equation

$$E_1 + E_2 + E_3 = E_O, \tag{5.1}$$

and the conservation of momentum by the equation

$$p_1 + p_2 + p_3 = p_O. \tag{5.2}$$

"Just a minute," you say, "something is wrong here. The momentum has lost an essential feature, its direction. The momentum is proportional to the velocity of the particle. To speak of the velocity without mentioning its direction means to say nothing of any significance. It is clear that $p_O = p_1 + p_2 + p_3$ only when the particles 1, 2 and 3 have the same direction as the particle O had. What if that is not the

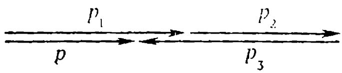


Fig. 8. **Vector addition.**
Vector **p** is the sum of the three ve[illegible] $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$.

case? If, for instance, particle 3 is emitted counter to the first two particles? Then $p_1 + p_2 - p_3 = p_O$ or $p_3 - p_1 - p_2 = p_O$. And what if the first particle travels to the right, the second upwards and the third to the left; what do we do in this case? Do we write a new equation again?" We have simply forgotten that the momentum is a vector, i.e. a quantity characterized by its direction, and that it can and must be represented by an arrow. In order not to write a new equation each time, let us recall the rule for vector addition. We place the tail of one vector at the head of another (the vector must be carefully transferred to its new position, without changing its direction) and draw a new arrow with its tail at the tail of the first vector and its head at the head of the second vector. It will then be unnecessary to replace the plus sign by a minus sign when the vectors point in different directions. Subtraction is obtained automatically (Fig. 8).

Vector Arithmetic

Hence, if we conceive of the sum $p_1 + p_2 + p_3$, in the equation $p_1 + p_2 + p_3 = p_O$, as having been determined by the rule of vector addition, there is no need to write an equation for each new set p_1, p_2 and p_3. To avoid confusion (in the equation $E_1 + E_2 + E_3 = E_O$, for example, addition is carried out according to the well-tested

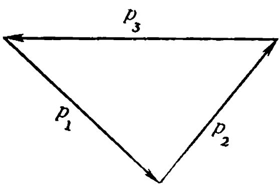


Fig. 9. Vector addition (the sum of the three momenta equals zero).

rule $2 + 2 = 4$), it proves simplest to have boldface letters representing the vectors. Thus

$$\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = \mathbf{p}_O. \qquad (5.3)$$

As soon as we see boldface letters, we recall that they are vectors and must be added in a special way: by carefully transferring them, without changing their direction, and placing them tail to head. You understand, of course, that their sum (according to the distance from the first tail to the last head) may be even less than the added components. In the case illustrated in Fig. 9 the sum of three vectors was found to equal zero because the head of the last vector exactly coincides with the tail of the first.

Collision

It is easy as pie now to write the conservation laws for the second important type of processes—collisions. Assume that after being accelerated in a particle accelerator, particle 1 collides in the target with a stationary proton, particle 2, and, as a result, that both particles disappear. In their place, particles 3, 4, 5, . . ., are produced and they go off in as many directions. The total energy of particles 1 and 2 before they met was equal to $E_1 + E_2$ (incidentally, the second addend here is equal to the rest mass of particle 2), and the total momentum is equal to the momentum p_1 of the incoming particle (the momentum of particle 2 was simply zero). The laws of conservation of energy and momentum can now be written as

$$E_3 + E_4 + E_5 + \ldots = E_1 + E_2, \quad (5.4)$$

$$\mathbf{p}_3 + \mathbf{p}_4 + \mathbf{p}_5 + \ldots = \mathbf{p}_1. \quad (5.5)$$

But, in addition to energy and momentum, other quantities are certain to be conserved in decay and collision processes. These are the same invariant differences of the squares of the energy and momentum $(E^2 - p^2)$ that characterize each particle. We already know that however the particle or observer is travelling, $E^2 - p^2$ is always equal to m^2, the square of the rest mass of this particle. This means that the energy and momentum in our equations are interdependent:

$$E_1^2-p_1^2=m_1^2,\quad E_2^2-0^2=m_2^2,$$

$$E_3^2-p_3^2=m_3^2,\quad E_4^2-p_4^2=m_4^2, \tag{5.6}$$

If we know the energy of a particle, we can, by means of equations (5.6), determine its momentum in magnitude (from the length of the arrow); its direction, in this case, not being limited in any way. This circumstance will prove useful.

Now we have a tool by means of which we can clear up a great deal, in fact all that we shall encounter in this book. It is necessary to solve these equations in each separate case; nothing more. But such a way out would be much too simple. Physics would not be the science that it is if it only solved equations without trying to find out what they stand for. One of the merits of theoretical physics is that it provides us with the possibility of solving equations without solving them, i.e. it enables us to perceive the solution of a problem at once, without complex calculations. One of the founders of quantum mechanics, Paul Adrien Maurice Dirac, said in this connection that he considered he had understood the meaning of an equation if he was capable of conceiving of the features of its solution without directly solving it. We shall see that though we keep the system of equations (5.4), (5.5) and (5.6) in mind, we shall have occasion to solve it only once, and then in the simplest of cases. The rest of the time we shall try our best to write the answer at once, without procrastination, transforming the equations so that each transformation is of physical significance.

Decay Again

To begin with, we can write the conservation laws in more simple form. They are written in our, stationary frame of reference. We stand still and watch the particle decay, record its momentum and then the momenta of the newly produced particles, calculate the energies, combine all of these data and obtain the equations. It is only reasonable, however, to ask: but why us? How would this decay look from the point of view of the particle itself? Or, if you like, from the position of an observer travelling along with the particle?

This question can be readily answered. As far as the observer is concerned, the particle would be at rest, its energy would be equal to its rest mass and its momentum would equal zero. After measuring the energies and momenta of they decay particles, we can write

$$E_1' + E_2' + \qquad = m_0, \tag{5.7}$$

$$\mathbf{p}_1' + \mathbf{p}_2' + . \qquad = 0. \tag{5.8}$$

As can be seen, the terms to the right of the equal sign have been simplified to some extent.

Which equations are correct? The last two equations, (5.7) and (5.8), or the previous equations, (5.1) and (5.3)? The answer is: both systems of equations are correct. One system is simpler, the other a little more complicated, but both are correct. One system is written in the reference frame travelling together with the initial particle, the other in a frame at rest. The equations can be written for any other reference frame as well, for instance, in one travelling together with one of the decay particles. They differ only in how conveniently they are expressed. But convenience is a subjective concept; one person can work more conveniently while sitting at a desk, whereas another may find it more convenient to rush along, racing with a meson. Objectively, all frames of reference are of the same worth, the more so because we have equations (the Lorentz transformations) enabling us to recalculate E and p from one frame to any other, provided that we know how these frames move with respect to one another.

There are two frames of reference that are predominantly preferred by physicists:

1. One is the laboratory (or simply, lab) frame of reference, in which all the energies, momenta and directions of the particles are taken such as they are recorded in the chamber or on the photographic film, i.e. in the instrument at rest in the laboratory.

2. The second type of frame travels along with the decaying particle. It is convenient in that the decay laws can be readily conceived of, because the particle itself is at rest. It is frequently called the co-moving frame of reference, or simply coframe.

We Begin to Reason

We have written the conservation laws. Let us see how the above-mentioned forbiddenness principle with respect to mass can be derived from these laws.

It is forbidden for the total rest mass of particles 1, 2, *to exceed the rest mass of particle O*:

$$m_1 + m_2 + \qquad \leqslant m_O. \tag{5.9}$$

Assume that this is not so, and imagine that we are observing a decay in the co-moving frame of reference. We observe that particle O decays into particles of such masses that

$$m_1 + m_2 + \qquad > m_O.$$

Can this be so? No. As we know, the energy of a particle is always greater than its rest mass (the hypothenuse is longer than a side, see Fig. 2), or, at worst, equal to it. Hence, all the more

$$E_1 + E_2 + \qquad > m_O.$$

But we have violated the law of conservation of energy. Thus, for a successful decay it is necessary that

$$m_1 + m_2 + \qquad \leqslant m_O.$$

But is this condition sufficient? Maybe, even when we comply with this condition (and in the absence of other rules forbidding the process), the decay is not always feasible? No is the answer: this condition is quite sufficient, but we shall postpone the proof until the last chap ters.

Here we may hear from an unsatisfied reader: "Why, then, did we go over to the co-moving frame in our derivations? We should have remained in our own cosy laboratory frame of reference; there your trick would not have worked. Repeating our reasoning in the lab frame, we would show that the inequality $m_1 + m_2 + \quad . \; . > E_O$ is forbidden, but we could not have forbidden the inequality $m_1 + m_2 + \quad . > m_O$. The required condition for decay would then take the form

$$m_1 + m_2 + \qquad \leqslant E_O, \tag{5.10}$$

after which nothing could prevent particles 1, 2, etc. from acquiring energies such that the law $E_1 + E_2 + \qquad = E_O$ is exactly complied with."

We chose a co-moving frame for the observation of particle O because the condition (5.9) derived in this frame is stronger than that obtained in the lab frame. Why decline a more exact limitation? To obtain condition (5.9) in the lab frame, additional calculations are required. We must take into account the requirement for conserving the momentum in a decay process, whereas in the co-moving frame we could manage without it. In other words, condition (5.9) is necessary and sufficient (though we have not yet proved the latter), whereas condition (5.10) is only necessary, but not sufficient (the latter is clear from the fact that it does not coincide with condition (5.9)).

If you have become confused by all these "necessaries" and "sufficients", here is an argument that appeals to your imagination rather than your reason. Assume that in the laboratory frame only condition (5.10) is valid and, according to it, several particles are produced with a total mass greater than m_0 (though less than E_0). Now then, do you really imagine that in the co-moving frame some of the particles disappear just to comply with condition (5.9)?

You Cannot Bind the Boundless

We shall assume that you are now convinced, and shall consider the consequences following from the forbiddance condition

$$m_1 + m_2 + \qquad \leqslant m_0.$$

Thus, *the decay of a light particle into heavier ones is forbidden, no matter how it travels.* A K^0 meson (with the mass 0.498 GeV), for instance, can decay into π^+ and π^- because the total mass of two pi mesons is 0.280 GeV (0.140 GeV each). It can decay into three pi mesons ($\pi^+\pi^-\pi^0$) because their total mass is 0.415 GeV (the mass of π^0 being 0.135 GeV). But nobody ever saw or will see a decay into $\pi^+\pi^-\pi^0\pi^0$; the total mass of four pi mesons is 0.55 GeV, whereas the initial K^0 meson had a rest energy of only 0.498 GeV To whatever energy we accelerate the K^0 meson, under no circumstances can it decay to four pi mesons. Not the energy of motion is of importance, but the rest mass.

This is clear even without any equations. Consider how a decay process occurs at rest. A particle (the K meson) disappears, the energy concealed in its mass is released for other purposes and can be made use of. For each new pi meson to be born, at least 0.140 GeV of the freed energy must be spent on its production, i.e. on the energy of existence of this meson. When three particles are created, then 0.415 GeV of the total energy reserve of 0.498 GeV is expended on their production. The remaining 0.083 GeV is used up in setting them in motion. How they divide this energy among themselves is their own business. In various cases they scatter differently, only taking care that their momenta, combined into a triangle, add up to zero (see Fig. 9), so that conditions (5.7) and (5.8) are complied with. But a fourth meson cannot be created because the remaining 0.083 GeV is insufficient to produce it.

This is a pity, is it not? It would seem sufficient to accelerate a meson in an accelerator to an energy equal to 7, or even 10 GeV, for it to begin to shower protons, antiprotons and even whole, ready-made nuclei. But the law of conservation of energy and momentum stands on guard and prevents mankind from taking advantage of nature. With a single stroke of the pen, as we know, a physicist transfers from the frame in which the proton became heavier to one in which it is at rest and there he finds that no new properties have been imparted to the proton by acceleration.

A Way Out

The whole point is that at the end of the accelerating process, the target—a thin sheet of foil or a polyethylene bar—is inserted into the accelerator, or a stream of hydrogen is released across the proton beam, and the protons collide at full speed with protons or with the nuclei of the target. Then, instead of the conservation laws for decay, we must write equations (5.4) and (5.5), the laws of the conservation of energy and momentum in the collision of two particles (protons collide pairwise rather than in groups). The question is: is it true that even now we cannot obtain a particle heavier than a proton? We shall see.

Assume that several particles are produced. We write that

$$E_1 + E_2 + \qquad = E + m, \tag{5.11}$$

$$\mathbf{p}_1 + \mathbf{p}_2 + \qquad = \mathbf{p}. \tag{5.12}$$

Here E is the energy of a proton that acquired its velocity in the accelerator, $\mathbf{p}$ is its momentum, and m is the mass of the stay-at-home proton, which, suspecting nothing, was at rest in the target until proton No. 1 crashed into it.

Imagine (and this is a very popular course of action) that the protons, after colliding, first formed a certain new particle O with the energy $E_O = m + E$ and the momentum $\mathbf{p}_O = \mathbf{p}$, so that nothing is lost, neither energy nor momentum. Then this new particle decays to the particles 1, 2, Never mind the fact that this may actually not occur at all. The fact is that the conservation laws that we employ are general, and are independent of the specific mechanism of the interaction that occurs. Consequently, whatever the mechanism we assume and imagine, we shall not obtain an incorrect result (provided, of course, that we are interested only in the restrictions following from the conservation laws alone).

The Problem is Reduced to the Preceding One

We have thus reduced the problem of a collision to one on the decay of particle O.

By travelling alongside the particle that is ready to disintegrate, we previously considerably facilitated calculations. Let us do the same here. We measure the energies and momenta of particles 1, 2, . in a frame of reference travelling together with particle O, and write the conservation law. Condition (5.12) now assumes the form

$$\mathbf{p}_1' + \mathbf{p}_2' + \quad = 0, \qquad (5.13)$$

and condition (5.11), the form

$$E_1' + E_2' + \quad =$$

We wrote the equal sign and then hesitated. What do we write next? How much energy will the two protons have together in the frame of reference in which the fictitious particle O is at rest? In the decay of a real particle O we wrote its rest mass at the right. To what is the rest mass of a particle equal if it has the energy $m + E$ and the momentum $\mathbf{p}$? We know that the square of the rest mass is an invariant; it equals the difference of the squares of the energy and momentum. In our case

$$m_O^2 = (m + E)^2 - p^2.$$

After removing the parentheses

$$m_O^2 = m^2 + 2mE + E^2 - p^2.$$

But $E^2 - p^2$ is again an invariant which is the square of the mass of the incoming proton, i.e. m^2. Consequently,

$$m_O^2 = 2m^2 + 2mE. \qquad (5.14)$$

Hence, the law of conservation of energy can be written in the form

$$E'_1 + E'_2 + \quad = \sqrt{2m^2 + 2mE}. \qquad (5.15)$$

But Still . . .

A great physical difference exists between equation (5.15) and the law (5.7) of energy conservation in decays. Previously, a real particle O decayed, which had a constant mass m_O, inherent in only particles of its kind. Now we have the decay of a conditional particle O, and its conditionality is manifested, among other matters, by the fact that the mass m_O of the particle is no longer constant. The greater the energy E of the accelerated proton, the greater the mass m_O (though the invariance of m_O with respect to the changes in the motion of the reference frame remains valid). It is not impossible then for E to reach such high values that even heavy particles can be brought into this world in collisions. Let us see. At the end of the acceleration cycle in the large accelerator in Dubna protons acquire a momentum of 10 GeV. This means that their total energy E at the instant they hit the target is equal to 10.044 GeV (see p. 46).

Then the total energy of all the created particles is

$$m_0 \approx \sqrt{2\times 0.94^2 + 2 \times 10.04 \times 0.94}$$
$$= \sqrt{2 \times 0.94 \times 10.98} \approx \sqrt{20.64} \approx 4.54 \text{ GeV}.$$

See what a store of energy is at the disposal of two protons that met to begin the process of creation! Precise directives on what these 4.5 GeV can be spent do not exist in nature.

Sometimes, for instance, two protons produce another pair like themselves: one proton and one antiproton. Altogether there turns out to be four particles of the same mass, 0.94 GeV each (do not forget that the two initial particles loaned all their energy, including the rest energy, to particle *O* and now demand 0.94 GeV each from the common hoard for their resurrection). Their creation requires $0.94 \times 4 = 3.76$ GeV of energy. The remainder is 0.78 GeV and the whole quartet, three protons and one antiproton, divide this surplus in some manner between themselves and immediately go off, carrying their share in the form of energy of motion.

In other cases, in addition to the two former protons, several mesons may be produced. Let us calculate the greatest number of mesons that can appear in such a collision. To resurrect themselves the protons expend $0.94 \times 2 = 1.88$ GeV of energy. They can give all the rest ($4.54 - 1.88 = 2.66$ GeV) to progeny. The creation of one charged meson requires at least 0.14 GeV, so that as many as 19 mesons can be produced. True, all the available energy will be spent on their creation, and the whole family

(2 protons + 19 mesons) will remain stationary; it will not have enough energy to crawl apart. We have forgotten, however, that all this occurs in a reference frame travelling together with the conditional particle O. Its velocity with respect to the accelerator is tremendous:

$$v = \frac{10}{10.98} = 0.91.$$

At this same velocity our family, as a single whole, breaks away from the target.

It turns out that the higher the energy that the protons acquire in the accelerator, the more extensive their capacity to create new particles, heavier ones and in a greater quantity. (Their capacity, but not their duty. They may just as well not create anything new and simply fly apart.) But if one of the newly created particles subsequently decays spontaneously, its large store of energy will be of no avail. With whatever energy it rushes along, a reference frame can always be found in which it is at rest. Therefore, the total mass of the particles that are formed after its destruction can in no case exceed its own mass.

Chapter 6

Kinematics in the World of Accelerators

Accelerators are, no doubt, the largest physical instruments that ever existed. Journalists and poets like to write about them. Journalists report on their graphic rhythms. Poets write about the girls attending the cyclotron. Film directors ask the girls to dance on the electromagnet.

I also find difficulty in tearing myself away from this subject. Let us spend a little more time estimating what an accelerator is capable of. You have some idea of its design. It consists of an annular channel (see Fig. 1) located between the poles of a large circular magnet. Electrically charged particles are injected into the channel. They begin to travel in a circle owing to the action of the magnetic field; the electric field accelerates them and they accumulate mass. Then a target is inserted into their path and the rest you already know. This will be sufficient for the time being.

What is an accelerator capable of?

We have seen that if the energy of an accelerated proton increases from m to E (in a 10 GeV particle accelerator, for instance, to 10 GeV), then, after hitting the target, the total rest mass of all the remaining and newly produced particles cannot exceed $m_0 = \sqrt{2m(m+E)}$. This enables particles heavier than the proton (hyperons) or antiparticles (antiprotons) to be created. All by themselves, without the participation of man, these particles may appear in cosmic rays, but since their appearance is unexpected and they live only a fraction of a second, they are hard to detect. An accelerator is an accurately controlled source of such particles; usually located near it are instruments that fish out the particles.

The larger E is, the more we should expect from the accelerator. Near the city of Serpukhov, where the continental shield approaches closest to the earth's surface, an accelerator that imparts protons a momentum of 70 GeV has been in operation for many years.* Let us see what this installation is capable of. At the end of the acceleration cycle each proton turns out to be 74 times as massive as it was in the beginning ($70 \quad 0.94 = 74$). The value of m_0 will equal 11.6 GeV. This energy is sufficient for the creation of about seven protons and five antiprotons:

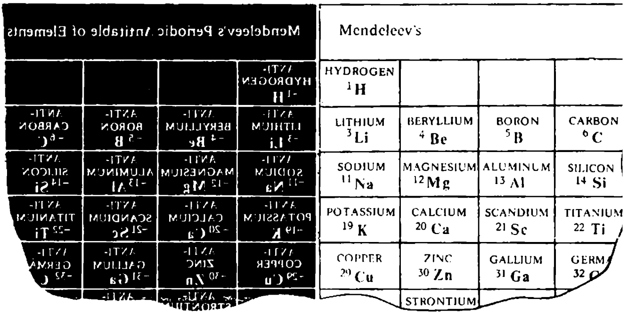
$$0.94 \times (7 \quad 5) \quad 11.28.$$

In the same way as all the matter of the earth, the sun and the galaxy consists of protons and neutrons, there may well be other galaxies consisting of antimatter, i.e. atoms of antihydrogen, antideuterium, antihelium, anti-iron, anti-uranium, etc. All the atoms of these antielements consist of antiprotons and antineutrons that form antinuclei about which positrons (as antielectrons are called) rotate.

It is unlikely that we will ever reach these limits of the universe. But, as a matter of fact, what is there to prevent us from creating antimatter on our earth? If an accelerator can simultaneously produce several antinucleons (protons and neutrons are called *nucleons*, i.e. particles of the nucleus; this clears up the definition of an antinucleon), they can combine to form antinuclei. The nucleus of antideuterium can be obtained from an antiproton and an antineutron;

* Two more powerful accelerators have been built more recently: a 300 GeV accelerator in Switzerland and a 400 GeV one in the USA.

the nucleus of antihelium from two antiprotons and two antineutrons, etc. There is nothing that forbids them to combine in this way. But we know that in physics *all that is not forbidden is allowed.* As a matter of fact, antimatter has



10. Chemical elements and antielements.

really been produced in the Serpukhov accelerator: nuclei of antihelium-3 were detected in 1970, and nuclei of antitritium in 1974 (even earlier, in 1965, antideuterons were discovered in one of the American accelerators). They do not last long, however; they soon collide with atoms of matter, miniature atomic explosions occur that are detectable only by special instruments, and all is transformed into mesons and radiation. But physicists manage to examine and investigate these phenomena. Thus Mendeleev's periodic table is being added on from the other end, from the prehydrogen end rather than the end with the transuranium elements. (If we move from the end of the table to its beginning, the number of baryons in the nuclei of the elements becomes lesser and lesser. The lithium nucleus has 6, helium 4 and hydrogen only 1. It would seem that we have reached the limit. But, in the same way as the thermometer is graduated for temperatures below zero, we can agree upon the disposal of nuclei with —1, —4, —6, etc. baryons on the other side, or "back" if you like, of Mendeleev's table. These will be nuclei with 1, 4, 6, etc. antibaryons, i.e. the nuclei of antihydrogen, antihelium, antilithium, ...) This is how Mendeleev's "antitable" began to be filled in (Fig. 10).

We should not, however, suppose that the Serpukhov accelerator was built just for this purpose. This example has been presented to give some idea of the extensive capability of accelerators.

What Are Two Accelerators Capable of?

Another large accelerator is in operation near Geneva in Switzerland, at its very border with France. When it was built, it was the largest in the world, designed for 25 GeV It is no longer the largest. Physicists of the European Council for Nuclear Research, abbreviated CERN (Conseil Européen pour la Recherche Nucléaire), could not reconcile themselves to this situation. That is how the project of the colliding-beam accelerator was evolved. Proposed by this project was the construction of an even larger annual channel, the so-called intersecting storage ring (Fig. 11) near the existing accelerator, but on the other side of the border. Accelerated protons are injected into the ring from two sides. Two proton round dances are obtained in the ring, rotating in opposite directions, without coming into contact. In the course of accelerator operation, newer and newer portions of accelerated

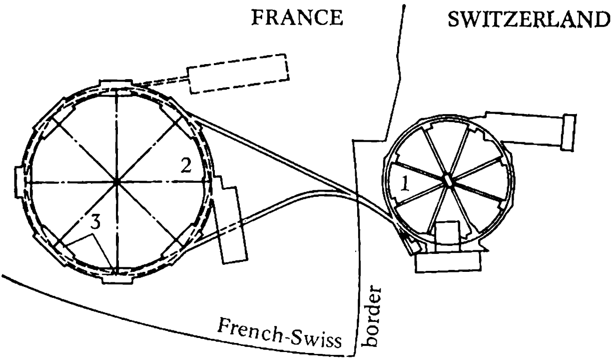


Fig. 11. Reconstruction of the CERN accelerator.
1—the old accelerator; *2*—where the round dances are held (storage ring); *3*—points (eight in all) where collisions take place.

protons are injected. When there are enough of these "immigrants", the round dances are aimed at each other and at eight points of their intersection (the rings are not exactly circular) pairs of protons begin to collide intensively. Eight points of intersection are better than one because they enable us to install eight times as many recording instruments.

Let us figure out to what consequences this "collision on the French border with Switzerland" can lead. Again, as before, we imagine that at the instant of collision of two protons each with an energy of 25 GeV, at first a fictitious particle O is formed, which immediately decays. As we know, we can estimate the number of particles produced in a decay from the rest mass m_0 of the initial particle (their total mass cannot exceed this value). Hence, we only have to find m_0. We write the conservation laws in our, i.e. laboratory, frame of reference.

The momenta of the protons from opposing round dances at the instant of collision are the same in value, but opposite in direction. Then their sum, i.e. the momentum of the fictitious particle O, is equal to zero. This implies that particle O is at rest. Thus its energy is, in fact, its rest mass. What is it equal to? The sum of the energies of two colliding protons. If their momenta are equal, then so are their energies. Hence, the mass of the fictitious particle is simply twice the rated energy of the accelerator:

$$m_O = 50 \text{ GeV}$$

This is a very high energy; it would be sufficient to produce even antioxygen. In a single 70 GeV accelerator we can produce a store of energy equal to 11.6 GeV, whereas in two colliding beams of 25 GeV each the store of energy equals 50 GeV What conventional accelerator does such an "accelerator with an attachment" correspond to? If the energy of a conventional accelerator equals x, then the usable energy it produces is equal, as we know, to $m_0 = \sqrt{2m^2 + 2mx}$. We wish to find out at what value of x this m_0 will be equal to $2E$, i.e. the doubled energy of the double accelerator.

Equating $2E$ and m_0 we find that

$$2m^2 + 2mx = 4E^2,$$

$$2mx = 4E^2 - 2m^2,$$

$$x = 2E^2/m - m,$$

or, discarding the small quantity m, we obtain

$$x = 2\frac{E^2}{m}.$$

If $E = 25$ GeV, then x 2 625/0.94 1330 GeV Thus a 25 GeV accelerator with an attachment is equivalent in i.s capacity to a conventional 1330 GeV accelerator. These marvels are due to the fact that 1330 GeV is the energy that rushes past us, whereas the arrangement of two colliding beams enables us to obtain 50 GeV right off the bat, and then utilize it completely to create new particles.

The CERN project was realized many years ago. Colliding beams of protons, each with an energy of 30 GeV, have revealed much that is interesting about collisions in the region of 1500 GeV

With Us in Siberia

Let us examine the equation $x = 2E^2/m$ again. It is clear that the less the mass of the colliding particles, the higher the gain in energy. Hence, colliding-beam accelerators have been built in many countries for electron + electron and electron + positron beams. One such accelerator was built in Novosibirsk (Fig. 12). It is not at all large, occupying only a part of a large room

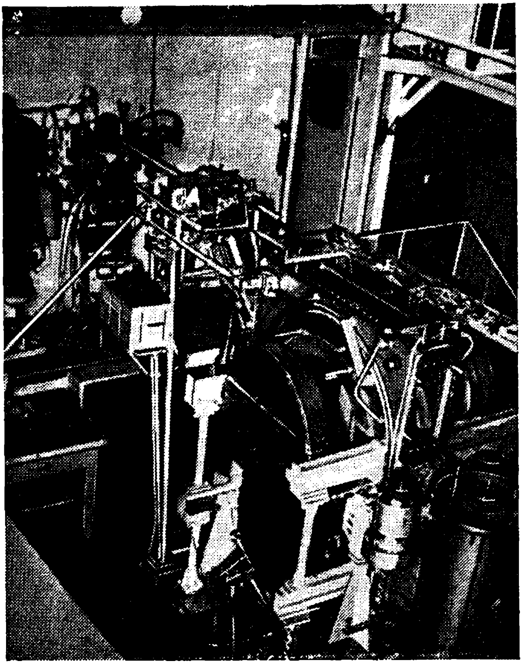


Fig. 12. General view of the Novosibirsk colliding-beam accelerator.

(with a high ceiling, it is true), but it accelerates electrons to an energy of 0.13 GeV The electrons are fed into two circular magnetic racecourses (one metre in diameter) where they are stored for the time being, and then the two opposing beams are aimed at each other. Their collision is equivalent to the production and decay of a particle with the mass 0.26 GeV. This is an immense value for an electron, which is lighter than a proton by a factor of 1840! If we had wanted to create particles with a mass of 0.26 GeV by accelerating electrons in a conventional accelerator and making them collide with electrons

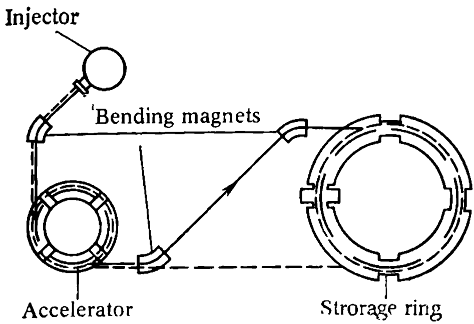


Fig. 13. Principle of the colliding-beam accelerator. Electrons (full line) and positrons (dash line) are injected into the accelerator; they collide in the storage ring.

at rest, we would have had to build an electron accelerator with a rating of

$$x = 2\frac{0.13^2}{0.00051} \approx 68\ \text{GeV}.$$

This would have been a huge structure.

In another accelerator built in Novosibirsk (Fig. 13), electrons with an energy of 0.7 GeV collide in the storage ring with positrons of the same energy. An equivalent accelerator of the conventional type would have to impart an energy of 2000 GeV to the electrons.*

* Oher accelerators with colliding beams of electrons and positrons are in operation in the USA, FRG, France and Italy. In the very largest, the colliding beams consist

The calculations that we have carried out here do not excel in profundity. We acquired no knowledge of the secrets of accelerator operation, nor did we get even a notion of the true nature of the problems faced by the designers of these structures. But our knowledge of kinematics proved sufficient to understand how much more efficient this newer type of accelerator, applying the colliding-beam principle, is than the previous types.

Chapter 7

How Particles Are Discovered

Little by little we have learned to freely manipulate the laws of conservation and invariant quantities. We are now ready to advance another step. It was mentioned at the very beginning of this book that kinematics helps us to see what instruments have overlooked. The time has come for us to take a look and see how this is done. Assume that we wish to detect an uncharged particle. Such a particle leaves no traces. Since it is uncharged it does not strip the electrons it encounters from their orbits, and slips by unnoticed. Sometimes, of course, it may have a head-on collision with some nucleus. This immediately draws attention because the tracks of new particles diverge from this spot. But nuclei are tiny, and it is a rare neutral particle that runs across one. But it would be a good thing to find the

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of particles with an energy of 16 GeV The equivalent energy rating of an accelerator with a fixed target is 1 000 000 GeV.

route of a flying uncharged particle each time that one is created. The following three examples explain how this is done.

First example: unstable particles.

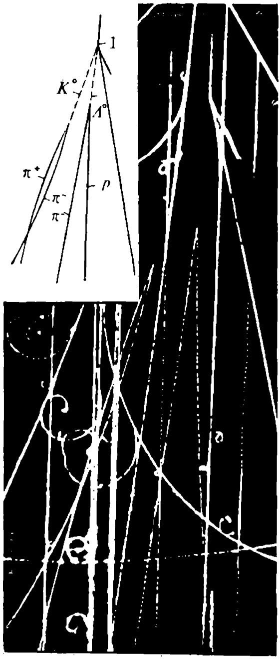

Fig. 14. The creation of a pair of strange particles. Three tracks emerging from point 1 constitute a so-called star; lower down we see two "forks" with their vertices aimed at the star (this is an up-to-date photograph, not one of those mentioned in the text).

At the end of the forties and the beginning of the fifties of our century, physicists recording the tracks of particles coming from outer space to the earth by means of a Wilson cloud chamber began to observe an interesting phenomenon more and more often (Fig. 14). Among the many stars—traces of particles produced in the collision of a fast proton with an atomic nucleus—they often saw a pair of tracks emerging from a single point and seeming to soar in the void. One of the names given to this phenomenon was a "fork". When you look at a fork you get the impression that somewhere in empty mid-air a pair of charged particles suddenly appear and go off in different directions. Such pair production, you guess, is probably due to the decay of an invisible neutral particle. Where could it have come from? It was noticed that the forks are frequently close to stars. The idea suggests itself that along with the many charged particles created in a star, invisible neutral particles are also produced. Fortunately, these particles are unstable and soon decay into two charged ones: one with a positive and the other with a negative charge. It is they that form the fork. Maybe new neutral particles are also produced in the decay, but we cannot see them.

How could we check whether the guess was correct? Additional observations were required. It was necessary, in the first place, to find out what particles form the prongs of the fork. It was found that sometimes the prongs are traces of π^+ and π^- mesons, whereas in other cases one trace belongs to a proton and the other to a π^- meson. It follows that the neutral particle, whose existence was suspected, decayed either according to the scheme

$$V^0 \to \pi^+ + \pi^-,$$

or according to

$$V^0 \to p + \pi^-$$

(the unknown particle was evidently denoted by a V to remind us of the fork). But if this actually is the decay of some particle, we can put the whole apparatus of Einstein's invariants into operation. Do you recall what this means? Whatever the way in which a particle is rushing past and whatever the direction in which we happen to be moving, the difference of the squares of the energy and momentum of the particle remains constantly equal to the square of its mass. But how can we find the energy and momentum of an invisible particle? Here, precisely, is where the conservation laws came in handy.

The energy E, momentum p and mass m of the particles leaving the tracks were determined for each pair-production fork. When the track of a particle is registered on photographic film, the more its path is bent by a magnetic field, the less its momentum; the longer it keeps travelling without stopping, the higher its energy. In this way, the energy, rest mass and momentum of a particle travelling by can be determined from length and curvature of its track.

The energy of the invisible particle was obtained by adding the energies of the two visible ones. The momentum of the invisible particle was found by adding the momenta of both visible ones. But here there was a subtle point that we already know about: they were added like vectors (Fig. 15). As we know, the sum of two vectors depends, not only on their magnitudes, but on the angle between them as well. The angle here was known; it is equal to the apex angle of the fork prongs. This addition can most simply be carried out graphically by superposing the vectors of the particle momenta, drawn to some scale, onto the tracks of the particles. Then we transfer one of the arrows without turning it in any way to the head of the other one. The distance from the tail of the first vector to the head of the second is, as we know, the sum of the matics. Nowadays, however, electronic computers have come to the aid of the physicist (a person can no longer cope with the required volume of calculations). Special automatic measuring machines measure the length, curvature and other characteristics of the tracks and, on the basis of these measurements, calculate the momentum, mass and energy of the Λ^0 particle. The presumable K^0 meson is treated in the same way. Moreover, the machines find the fork closest to the star and measure the coordinates of the fork and star. All of these data are delivered (sometimes they are transmitted directly by cable) to the computer. What does the computer check? First it makes sure, according to equation (7.1), that the mass obtained concerns a Λ^0 particle. But this is not all. The computer next calculates the direction of the vector $\mathbf{p}_1 + \mathbf{p}_2$ and compares it with the direction from the star to the apex of the fork. According to the law of conservation of momentum, these directions must coincide. Only when all the checks tally can we consider that the tracks indicate the decay of a Λ^0 particle that leaves the star.

Well, and what next? What are we driving at?

The most interesting is what comes afterward. The laws of creation of the Λ^0 and K^0 particles are investigated, an attempt is made to guess the forces acting on them at the instant they come to light. The K^0 meson is the subject of especially persistent research. In the family of elementary particles, the K^0 particles keep themselves aloof; they have many strange, unusual properties. In 1956 the explanation of one of these properties led to an upheaval in science.

At the present time, the K^0 mesons have set about another uprising in physics, perhaps even more thorough than the previous one*.

Second example: cascade hyperons.

The first of the hyperons, Λ^0, was discovered in 1950, but scientists kept on finding previously unknown relatives in their family for many years. The Ω^- (omega-minus) hyperon turned up in 1964.

Film Star

As long as six months before its discovery, the existence of the Ω^- particle was predicted by the

* A nut with a right-hand thread can be just as readily screwed on a right-hand screw as a left-hand nut on a left-hand screw. Everyone is accustomed to such equality of rights of right- and left-handedness, and thought that that is the way all things should be, now and for evermore. But in 1956 they found that in K meson decays and like processes this equivalency is violated. Physicists call this parity nonconservation in weak interaction. An entirely outlandish conclusion on the properties of space suggested itself: that right and left are not equivalent in space. To "rescue space" it was proposed that "right-hand things" made up of particles are equivalent to the same kind of "left-hand things" consisting of antiparticles. This hypothesis was immediately tested experimentally and it seemed to agree excellently with the experimental facts. Raptures over the discovery of a new law of nature—the law of conservation of combined parity—did not last very long. It was soon found that in the decays of K^0 mesons, combined parity was also not conserved, though by only a very small amount. The nonconservation is very small, but the problem is a huge one and has not yet been fully understood.

young and promising Soviet physicist Tim Suvernev. This was the subject of a film, "No. 1 Newton Street", made shortly afterward, and almost forgotten at the present time. The film showed how its hero, listening to a concert in the Conservatory, frantically jots down in a notebook

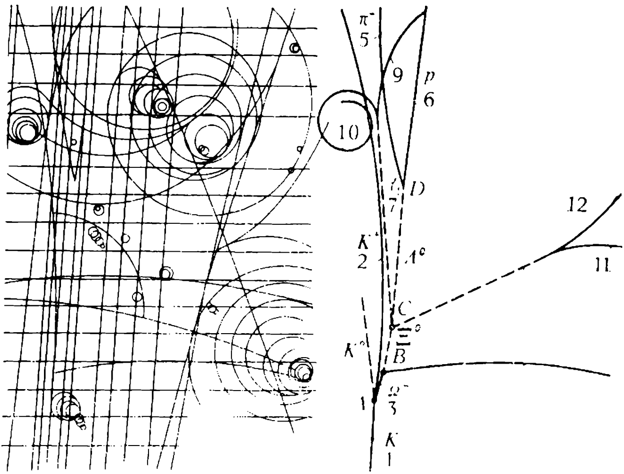

Fig. 16. Creation and decay of the Ω^- hyperon.

the reaction $K^- + p \rightarrow \Omega^- + K^+ + K^0$ that had just occurred to him. The page in the notebook was shown covering the full width of the screen. Soon after the film was produced, experiments were conducted in the USA with the aim of obtaining this reaction. The predictions were confirmed with exceptional accuracy. This concerned the way by which the particle is created, the decay mode, the mass predicted beforehand,

and even the name—omega-minus—of the new particle.

Let us examine the photograph on which the Ω^- hyperon was first seen (Fig. 16). Of interest to us is how the conservation laws behaved in this case.

The photograph was taken on January 31, 1964 in the Brookhaven National Laboratory, which has a powerful accelerator in operation. It was necessary to examine 50 000 photographs of interactions that occurred in the liquid-oxygen bubble chamber before the one illustrated here was found. It is difficult, without sufficient practice, to make head or tail of the spider web of lines, but that is only because you haven't seen the 49 999 earlier pictures. If you had, you would have immediately seen what was noticed by the 33 American physicists* conducting the experiment. You would have seen a new, previously unwitnessed particle.

Down the Cascade

Separated out at the right of Fig. 16 is the essential part of the photograph. The solid lines show what is also visible in the photograph; the dash lines show what was conjecture. Let us examine them more closely. Line *1* is the track of a K^-

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* Gone are the times in which an experiment in high-energy physics could be performed by a small group of researchers. Today, such complex and diverse equipment is required for a good experiment that only a large collective can cope with it. For this reason we frequently find scientific papers with several dozens of authors, sometimes as many as a hundred.

momenta in the same scale. After determining the total energy $E_1 + E_2$ and the total momentum $\mathbf{p}_1 + \mathbf{p}_2$, we can calculate the mass of the required particle:

$$M_V = \sqrt{(E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2}. \quad (7.1)$$

After many pair-production forks had been observed and M_V had been calculated for each

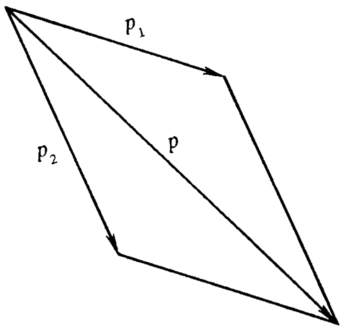

Fig. 15. Vector addition.

one, it was found that all the M_V values obtained could be divided into two classes: in the $V^0 \rightarrow$ $\rightarrow p + \pi^-$ decays the value of M_V was always approximately equal to 1.11 GeV, whereas in the $V^0 \rightarrow \pi^+ + \pi^-$ decays, the value of M_V varied in the neighbourhood of 0.49 GeV*.

* Do not forget that exact measurements are never obtained in physics, and that all such values are always approximate. This constitutes a basic difference between the natural sciences and the humanities in which everything is always known with absolute exactitude.

To what conclusion did this lead? It was proof of the existence of two, rather than one, neutral particles Λ^0 and K^0 (lambda-zero and K-zero). One, the heavier (it was denoted by Λ^0, overturning the fork) decays as follows:

$$\Lambda^0 \rightarrow p + \pi^-;$$

whereas the other, a lighter one, decays by the reaction

$$K^0 \rightarrow \pi^+ + \pi^-.$$

What if differing values had been obtained for M_V in one or both of the classes? This would imply that a part of the energy and momentum of V^0 is carried away by new invisible particles. Fortunately, this did not happen.

Today, Λ^0 and K^0 are the most ordinary and most frequently encountered specimens of the so-called strange particles. They are unstable, i.e. after appearing they soon decay spontaneously. From the distance from the star to the point of decay, the path length of the Λ^0 and K^0 particles is determined. After calculating their velocities ($v = p/E$), their lifetimes can be readily found. The Λ^0 particle has an average lifetime of 10^{-10} s (travelling several centimetres during this time). The lifetime of the K^0 mesons is approximately the same. The K^0 mesons are among the most interesting bodies in nature.

Though K^0 and Λ^0 mesons were discovered many years ago and have been accepted into the family of elementary particles, though tens of thousands of these particles pass by physicists in various experiments, each separate K^0 or Λ^0 particle is identified as before by means of kinemeson ejected from the accelerator with a momentum of 5 GeV. At point A it collides with a proton, a hydrogen nucleus (that cannot be seen), and the pair, K^- and p, are transformed into the particles Ω^-, K^+ and K^0. The particle K^0 departed, without decaying, beyond the limits of the photograph and remained unnoticed. The track of particle K^+ is marked by the number *2*; it turns to the left because the chamber was placed in a strong magnetic field, which turned all particles with a positive charge to the left and those with a negative charge to the right. The short track *3* is that of the sought Ω^- hyperon. At point B it decayed into a π^- particle (track *4*) and an invisible Ξ^0 (xi-zero) hyperon. This latter, after a slightly longer flight than the Ω^- hyperon itself, decays, in its turn (at point C), into two, again invisible, particles: π^0 and Λ^0. The Λ^0 hyperon travelled a comparatively long way before it decayed (at point D) into a proton (track *6*) and a π^- meson (track *5*). As to the π^0 meson, its lifetime, as a rule, does not exceed 10^{-16} s and it does not have time enough, in essence, to leave its birthplace. (Its average path length is $c \times 10^{-16} = 3 \times 10^{10} \times 10^{-16} = 3 \times 10^{-6}$ cm.) The π^0 meson decayed at point C into two photons (tracks *7* and *8*). These photons are invisible, but, fortunately, an event happened to each of them that is rarely observed in hydrogen; passing too close to some nucleus, they were transformed into two charged particles each: an electron and a positron (tracks *9* and *10* are from the particles produced by one photon and tracks *11* and *12* are from the other) that are turned by the terrific force (called the Lorentz force) of the magnetic field. This cascade of interactions can be written in the form of the following chain of reactions:

$$
\begin{array}{l}
K^- + p \to \Omega^- \quad K^+ + K^0 \\
\qquad\qquad \lfloor\!\longrightarrow \Xi^0 + \pi^- \\
\qquad\qquad\qquad \lfloor\!\rightarrow \Lambda^0 + \pi^0 \\
\qquad\qquad\qquad\qquad\qquad \lfloor\!\rightarrow \gamma_1 + \gamma_2 \\
\qquad\qquad\qquad\qquad\qquad\qquad \lfloor\!\rightarrow e^+ + e^- \\
\qquad\qquad\qquad\qquad\qquad\qquad \lfloor\!\longrightarrow e^+ + e^- \\
\qquad\qquad\qquad\qquad \lfloor\!\rightarrow \pi^- + p
\end{array}
\qquad (7.2)
$$

As we can see, many particles flashed by unnoticed in the chamber, and we cannot manage without kinematic laws here. Let us see how to make sure that the whole picture of the cascade has been correctly represented by the diagram (7.2).

Dersu Uzala * at Work

In the first place, from the shape and nature of the visible tracks, taking all precautions, the physicists made sure that tracks *4* and *5* most likely belong to π^- mesons, track *2* to a K^+ meson, track *6* to a proton, and tracks from *9*

* A Nanaian hunter and guide, he was a real-life East Siberian version of Fenimore Cooper's Pathfinder and was made famous by V. K. Arsenyev, Soviet explorer of the Far East, ethnographer and writer, and by the Japanese director Akira Kurosawa in the 1975 film "Dersu Uzala". — *Tr.*

through *12* to electrons and positrons. The momenta of the visible particles were determined from the curvature of their tracks.

Calculating by equation (7.1) the mass of the invisible particle that created the fork *5-6*, a value of 1.116 GeV was obtained, which is exactly the mass of the Λ^0 hyperon. Adding the momentum vectors of particles p and π^-, they found the magnitude and direction of the momentum of the Λ^0 hyperon. Then, adding (vector addition!) the momenta of particles *9* and *10*, they obtained the momentum of the invisible photon γ_1 (it was found to be equal to 0.082 GeV) and, naturally, its direction. In exactly the same way, the momentum (0.177 GeV) of the photon γ_2 was found, and its direction also became known. At this point the first test of their line of reasoning began. The arrows representing the momenta of Λ^0 (*DC*), γ_1 (*7*) and γ_2 (*8*) met in almost a single point (with a discrepancy of only 1 mm in the horizontal direction and 3 mm in a vertical plane. All the events occurring on the photograph extend over a length of 200 cm). This signifies that some invisible particle really did decay at this point into a Λ^0 hyperon and into something else that immediately decayed to two photons, γ_1 and γ_2. It was not hard to guess that this "something else" could not be anything but a π^0 meson. But this had to be checked by calculations.

Again the laws of kinematics and the invariance of the rest mass were put to work. If the momenta of two photons are known in both magnitude and direction, it is almost child's play to find the rest mass, by means of the same equation (7.1), of the particle that decayed into the photons. The value 0.1351 ± 0.0015 GeV was obtained and it is in excellent agreement with the known mass of the π^0 meson, equal to 0.135 GeV.

The next problem was to find what invisible being decayed at point C into a Λ^0 hyperon and a π^0 meson. Again a candidate is on hand: everyone knows that the so-called cascade hyperon Ξ^0 with the mass 1.314 GeV willingly decays to a Λ^0 hyperon and a π^0 meson. Was it possible to check this hypothesis? Indisputably! Adding together the energies of the Λ^0 and π^0 particles, the physicists calculated the energy of the supposed parent; adding vectorially the momenta of Λ^0 and π^0 (the momentum of π^0 was obtained by adding the momenta of γ_1 and γ_2, which, in turn, were found by adding the momenta of particles *9*, *10*, *11* and *12*) they obtained the momentum of the parent. Then equation (7.1) was applied again, and it provided the value 1.316 ± 0.004 GeV.

The agreement was excellent. And to top it off, the momentum of particle Ξ^0 came almost exactly to point B.

Is it worthwhile to relate in detail what happened next? The reader guesses without trouble that the calculated momentum of Ξ^0 and the measured momentum of π^- turned out, after being added together, to be directed along prong BA and enabled (again by equation (7.1)!) the mass of particle Ω^- to be determined. What a triumph of theoretical physics! Two years earlier, the American physicist Murray Gell-Mann and the young Japanese physicist Susumu

Okubo* had predicted that the Ω^- particle would decay in exactly this way and, above all, that its mass would be 1.686 GeV. Kinematic calculations yielded the value

(1.686 ± 0.012) GeV!

As they say, no explanation is required. But still, run over all the reasoning again to see into what a tight knot the checkups and assumptions are tied, to see how all the assumptions are reliably checked and how the final conclusion follows from the premises with unyielding logic.

Before going on, try to find what has remained unproved in reaction (7.2) and how to prove it.

Third example: how can nondecaying, neutral particles be seen?

As a matter of fact, how can they be seen? We know about the creation of a Λ^0 hyperon because it very soon, after 10^{-10} s, decays into charged particles. But what can we know about the production of a neutron? Its lifetime is about a quarter of an hour; most neutrons travel about in the chamber without decaying. Collisions between neutrons and nuclei are also rare events. Or what can we do about the neutrino,

* It is precisely their predictions that are played up in the film "No. 1 Newton Street" mentioned above. Credit should be given to the science advisor of the film (the physicist V V. Shekhter of Leningrad), who selected from the numerous predictions of Gell-Mann's theory just the very one that was confirmed first of all. The fact that the particle was predicted in the film (and by a mythical Tim Suvernev) is, of course, only a joke, but it foretells its near discovery.

which does not, practically, interact with anything at all? Or let us consider the π^0 meson. It does decay, but only into neutral particles—photons—that we rarely succeed in noticing.

Where Have All K-zeroes Gone?

In order to understand how these things are done, we return again to the photograph on which the Ω^- hyperon was first observed. From what phenomena did the experimenters know that a pair of heavy mesons, K^+ and K^0, were produced together with the Ω^- hyperon? For no K^0 can be seen in the photograph. It simply dashed along the chamber without decaying. What clue was there indicating its creation?

Naturally, it was the law of conservation of energy and momentum. The momentum of the initial K^- meson was known (5 GeV). The momentum of the created K^+ was measured from the curvature of its track. The momentum of the created Ω^- hyperon was calculated in passing when its mass was determined (it was simply equal to the vector sum of the momenta of all the particles obtained in the decay of the Ω^- hyperon: two π^- mesons, two electrons, two positrons and a proton). The energies of the K^- and K^+ mesons and of the Ω^- hyperon became known together with their momenta. It was quite natural curiosity to see whether the sum of the energies of the K^+ meson and the Ω^- hyperon was equal to the sum of the energies of the initial particles, the K^- meson and the proton (the latter was at rest so that its energy was equal to its mass). Another question was whether the vector sum of the momenta of the K^+ meson and the Ω^- hyperon coincided with the momentum of the K^- meson (since the proton was at rest, its momentum equalled zero). As it turned out, no coincidence was found. Well then, the 33 physicists reasonably decided, besides the K^+ meson and the Ω^- hyperon, there were other invisible particles of some kind that carried away the unbalance (as physicists usually call this phenomenon).

No trouble was encountered in determining the energy and momentum carried away by the invisibles. The next step was also easy to take; it consisted in finding the difference in the squares of the energy and momentum that were carried away. This difference is invariant; it yields the square of the rest mass of the particle that carried away the energy and momentum (or, if there were several such particles, we would obtain the rest mass of the fictitious particle that decays to the invisible ones).

Missing Mass

Calculations of the rest mass of the invisible particle yielded the value 0.5 GeV. It could only be a K^0 meson, whose mass is 0.498 GeV. If the energy and momentum had been carried away by two particles, say K^0 and π^0, the mass of the fictitious particle could not be less than the sum of the masses of the K^0 and π^0 mesons, i.e. not less than 0.63 GeV, but the calculated value was only 0.5 GeV. We can therefore write

$$K^- + p \rightarrow \Omega^- + K^+ + K^0,$$

the more so because Gell-Mann stated beforehand that the Ω^- hyperon cannot be created in any way except together with two K mesons.

The mass of a particle (fictitious or real) that carries away energy and momentum is called *missing mass.* If all the initial energy is denoted by E (in our case the sum of the energies of the K^- meson and the proton) and the initial momentum by $\mathbf{p}$ (the momentum of the K^- meson in our case), and if the energies and momenta of the particles that are *visible* at the end are denoted by E_1, $\mathbf{p}_1$, E_2, $\mathbf{p}_2$, . the missing mass M_x can be calculated by the equation

$$M_x = \sqrt{(E - E_1 - E_2 - \ldots)^2 - (\mathbf{p} - \mathbf{p}_1 - \mathbf{p}_2 - \ldots)^2}. \tag{7.3}$$

To subtract vectors $\mathbf{p}_1$, $\mathbf{p}_2$, etc. from vector $\mathbf{p}$ is just the same as to add to vector $\mathbf{p}$ vectors $\mathbf{p}_1$, $\mathbf{p}_2$, etc. on which the heads and tails have changed places.

When the missing mass is equal to the mass of some particle, this means that the energy and momentum were carried away by this particle. When there are no particles with such a mass, the energy and momentum were stolen by several particles. These are the principles on which the sleuthing of the Sherlock Holmes of physics are based.

This way it is possible to learn of almost every event of (single!) neutron or π^0 meson creation. All we need to do is to measure the momenta of all the visible particles as accurately as possible.

Results of Reactions

Using the large chambers that were built in recent years, momenta are determined with quite high accuracy. This immediately enabled physicists to find out what reaction they obtained on a photograph. The fact is that the principle "all that is not forbidden is allowed" is especially effective at high energies. If a proton collides with an antiproton, the creation of a rather large number of π mesons is allowed, provided that their total charge equals zero (the proton has a plus and the antiproton a minus charge). Nature makes use of this opportunity. The number of created π mesons varies greatly from collision to collision. A multitude of different reactions occur pellmell. For example:

$$\bar{p} + p \to \pi^+ \quad \pi^-, \tag{7.4}$$

$$\bar{p} + p \to \pi^+ \quad \pi^- \quad \pi^0, \tag{7.5}$$

$$\bar{p} + p \to \pi^+ + \pi^- + \pi^0 + \pi^0, \tag{7.6}$$

$$\bar{p} + p \to \pi^0 + \pi^0 + \pi^0 + \pi^0, \tag{7.7}$$

$$\bar{p} \quad p \to \pi^+ \quad \pi^+ \quad \pi^- \quad \pi^-, \tag{7.8}$$

$$\bar{p} + p \to \pi^+ + \pi^+ + \pi^- \quad \pi^- + \pi^0, \tag{7.9}$$

$$\bar{p} + p \to \pi^+ + \pi^+ \quad \pi^+ + \pi^- + \pi^- \quad \pi^-. \tag{7.10}$$

In appearance, reactions (7.4), (7.5) and (7.6) do not differ; only the pair $\pi^+\pi^-$ is visible in all three. In exactly the same way, there is no difference in the appearance of the reactions (7.8), (7.9) and (7.10). This is where the concept of missing mass comes in very handy. By measuring the tracks of the charged particles we find the momenta and energies of the π^+ and π^- mesons. The momentum of the antiproton is known beforehand (before entering the chamber, all the antiprotons created in the accelerator pass through a special channel which sorts out only particles with a definite momentum). All of these data are fed into a computer which calculates the missing mass by means of equation (7.3). We then obtain values either close to zero, or close to 0.135, or values exceeding 0.27.

When the missing mass is equal to zero, or, more exactly, when the missing energy and the missing momentum are equal to zero, this indicates that there were no particles involved except the visible ones. Hence, we deal with reaction (7.4) if we observed two tracks, reaction (7.8) if we observed four, and reaction (7.10) if we observed six. If the missing mass equals zero and the missing energy and momentum are nonzero, a photon was created.

When the missing mass equals 0.135 GeV, one π^0 meson is produced (reactions (7.5), (7.9), etc.) in addition to the visible particles. The missing momentum $\mathbf{p} - \mathbf{p}_1 - \mathbf{p}_2 - \ldots$ is its momentum. The missing energy $E - E_1 - E_2 - \ldots$ is its energy. Hence, we can see the invisible π^0 meson by means of kinematics just as well as the π^+ and π^- mesons.

When the missing mass exceeds 0.27 GeV, either two π^0 mesons were created or heavier neutral particles. Here kinematics, as a rule, denies its assistance, and there are no more possibilities of direct analysis.

In any case, the conservation laws enable us to separate cases in which only charged particles are produced from those in which a single neutral particle is additionally produced. This is much more important than it may seem and, as soon as the measurement of the missing mass became sufficiently precise, a whole new cycle of discoveries began in elementary particle physics. Entirely unheard-of kinds of particles were disclosed, which, on the one hand, could not even be identified as being particles, but, on the other, possessed many of the features of particles. The list of elementary particles began to lengthen at a threatening rate. At first theoretical physics was taken aback by this torrent of new particles, but subsequently competed with the stream of newly discovered particles by means of an even more powerful tide of predicted ones. This process is in full swing at the present time. Many feel that these two head-on waves will elevate physics to a new level.

Chapter 8

How Resonance Particles Are Discovered

Four physicists (three Americans and one Serbian) were conducting experiments on the annihilation of antiprotons in 1961 at Berkeley (just across the Bay from San Francisco and the home town of the University of California). Annihilation, or demolition (from the Latin word annihilare, meaning "to bring to nothing") is the phenomenon mentioned at the end of the preceding chapter. When, after colliding, two heavy particles disappear and, in their place, several light particles appear, this is annihilation. Instead of sedate nucleons, the acknowledged building bricks of matter, unstable particles are formed, unsuitable for creating nuclei.

The physicists placed a large hydrogen bubble chamber in the path of the antiprotons rushing headlong out of the accelerator. Then they adjusted the automatic apparatus and the photographic cameras, which began to make snapshots of the collisions and decays that occurred. The reactions were of the type of (7.4) through (7.10). The antiprotons of the beam, together with the protons of hydrogen, are transformed into groups of π mesons. The momentum of each antiproton was 1.61 GeV, the mass m_0 of the proton-antiproton system is 2.29 GeV (you can check this by means of equation (5.14)), so that up to fifteen π mesons could be formed. The researchers decided to pick out the photographs in which only four-prong stars could be found. Of the 2500 four-prong stars (Fig. 17), they further picked out 800 photographs about which it was safe to say that, in addition to the four visible particles, another invisible one, the π^0 meson, was concealed. Using equation (7.3), they calculated the missing mass and obtained 0.135 GeV, which is the value characteristic for the π^0 meson. This indicated that they were concerned with the process

$$\bar{p} + p \rightarrow \pi^+ + \pi^- + \pi^0 + \pi^- \quad \pi^+. \qquad (8.1)$$

What was their interest in this particular reaction due to? The point is that several years earlier, in studying the internal structure of the proton and neutron, many theoretical physicists

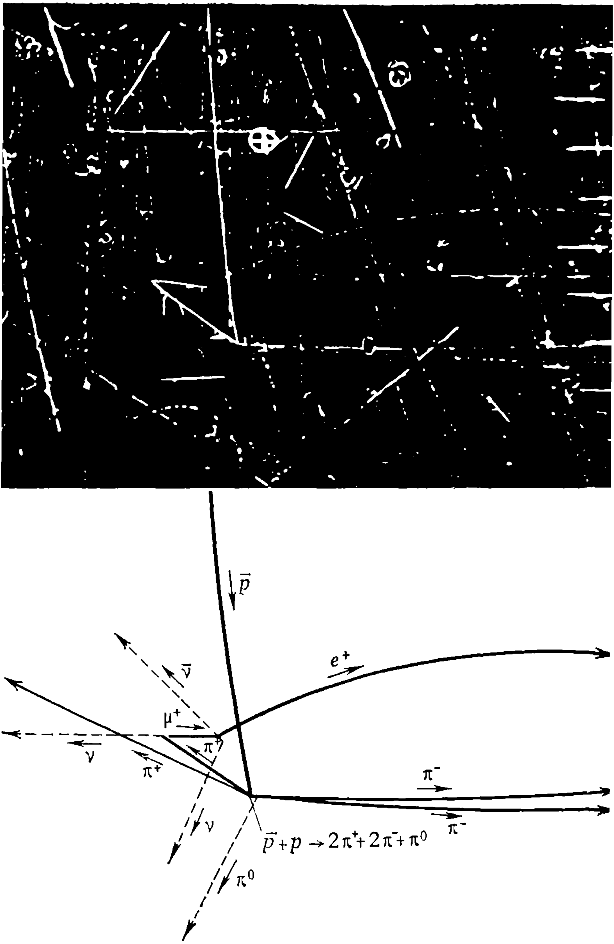

Fig. 7. Annihilation.

came to the conclusion that there must be a heavy uncharged particle capable of decaying to three π mesons. The best way to obtain a large number of mesons straight off is annihilation, and this is why reaction (8.1) was resorted to to find the new particle.

They expected the mass of the particle to be about 0.67 GeV and that it would exist about 10^{-23} s. During this time it could move from its birthplace not more than 10^{-13} cm, and there seemed to be no hope of distinguishing its point of creation (annihilation) from its point of decay. Practically all the four prongs in the photographs emerged from a single point.

Searching for the Pearl

What were the chances of finding this particle? Assume that sometimes it actually was created and immediately decayed into π^+, π^- and π^0 mesons, i.e. that the course of annihilation was

$$\bar{p} + p \rightarrow \omega^0 + \pi^+ + \pi^-$$
$$\quad \hookrightarrow \pi^+ + \pi^- + \pi^0. \qquad (8.2)$$

If we only knew which of the π^+ and π^- mesons were created in the decay of the sought particle (which was arbitrarily called ω^0, i.e. omega-zero), the problem could have been readily solved by means of kinematics. It would be necessary to sum up the energies of the π^+ and π^- mesons, adding to this the energy of the invisible π^0 meson and thus obtaining the energy of the ω^0 particle. After doing likewise with the momenta, we could calculate the momentum of the omega-zero particle. Then we could have calculated the invariant quantity, the mass of the appearing and immediately disappearing particle:

$$m_\omega = \sqrt{(E_1 + E_2 + E_3)^2 - (\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3)^2}. \qquad (8.3)$$

What is this mass equal to? To 0.67 GeV? But this value was proposed as a purely tentative guess. It would have been better, evidently, to proceed in the same manner as when the Λ^0 and K^0 particles were discovered. Instead of specifying the mass of the ω^0 particle beforehand, they should have waited to see what value of m_ω is obtained in all the annihilation four-pronged stars without exception.

"Hold on, all the rest is clear!" you may exclaim. "The mass that was obtained everywhere: on the first photograph, on the second, on the third and so forth, up to the eight-hundredth, is the mass of the ω^0 particle. Everything tallies beautifully! Hurrah for and long live kinematics!"

But I am compelled to remind you of the "if" with which we began our line of reasoning. If we only knew which of the π^+ and π^- mesons were created in the decay of the meson. But this is precisely what we do not know! Assume that sometimes ω^0 mesons were actually produced. And what if they were not? What happens then to the combination (8.3)? It will obviously not be equal to the mass of the ω^0 meson, but what will it be equal to? This last question can be readily answered: even if no ω^0 particle is created, the reaction $\bar{p} + p \rightarrow \pi^+ + \pi^- + \pi^0 +$

$+ \pi^- + \pi^+$ can be written in the form

$$\bar{p} + p \rightarrow X \quad \pi^- + \pi^+$$
$$\quad \downarrow \pi^+ + \pi^- + \pi^0$$

Only X would not then be a recorded real particle (ω^0), but, instead, a certain fictitious particle, whose mass is not constant, varying from time to time, from one photograph to another. Hence, the invariant mass of the three, the quantity

$$m_X = \sqrt{(E_1 + E_2 + E_3)^2 - (\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3)^2},$$

would have a new variable value in each photograph. Within what values would it vary? Within those stipulated by the conservation laws. The lowest value of m_X would be one in which decay of the fictitious particle into three π mesons would still be possible, i.e. 0.415 GeV (the creation of a π^+ meson requires at least 0.140, that of a π^- meson also 0.140, and the creation of a π^0 meson requires 0.135 GeV). The maximum value of m_X should be such that the whole available mass of the proton-antiproton system (2.29 GeV) would just be sufficient to produce the particles π^+, π^- and X:

$$m_X = 2.29 - 0.14 - 0.14 = 2.01 \text{ GeV}.$$

Within these limits (from 0.415 to 2.01), however, not a single value of the invariant mass is neither forbidden, nor favoured, nor to be encountered much more often than values close to it in magnitude (provided that particle X is not real, but fictitious, invented for convenience).

Everything seems to be clearing up. Let us apply some calm reasoning.

If ω^0 mesons had been created in each case (and if we knew which of the mesons were obtained in the decay of the ω^0 meson), i.e. if reaction (8.2) was the one that always occurred, then all the invariant masses m_X of the triplet, π^+, π^- and π^0 mesons, on all of the photographs would be found to equal the same value: the mass of the ω^0 meson.

On the other hand, if no ω^0 mesons were produced at all, i.e. only the direct reaction (8.1) occurred, then practically all values of the invariant mass m_X of some (any) triplet, ranging from 0.415 to 2.01 GeV, could be found on the various photographs.

Find the Irregularity

What if reaction (8.2) occurs in one half of the cases and reaction (8.1) in the other half? What happens and what do we obtain? Reaction (8.1) will yield any invariant masses. In any definite interval of values, say 0.5 to 0.6 GeV, there will be approximately the same number of values of m_X as in the adjacent interval, from 0.6 to 0.7 GeV; and in the interval from 0.6 to 0.7 approximately the same amount as in the interval from 0.7 to 0.8 GeV, etc.

What will reaction (8.2) yield? Here all the masses m_X will be concentrated close to a single value, the mass of the ω^0 meson. We could compile a table showing how many times the invariant mass of the meson triplet is found in the interval from 0.5 to 0.6, in the interval from 0.6 to 0.7, etc. Then the mass interval containing the mass of the ω^0 meson will immediately stand out and draw our attention. From interval to interval this "how many times?" will vary smoothly and at some place there will be a sudden jump. There will be too many triplets at this point and therefore the mass of the ω^0 meson is within this interval.

This, then, is the idea on which the search for the invisible particle is based: in the absence of an ω^0 meson, we cannot expect certain values of m_X to be more frequent than others. If, of course, the creation of an ω^0 meson is rare, the surplus of cases with $m_X \approx m_\omega$ over other values of m_X will be small, so that the jump remains unnoticed. But what is there to prevent us from trying anyway? What if ω^0 mesons are frequently produced and the table immediately reveals them?

"Yes, that seems reasonable, but what are we going to do about the fact that we do not know which π^+ and π^- mesons are produced by the decay of the ω^0 meson, and which are produced directly from the system $\bar{p} + p$?"

"Well then, that only means that we have to try all the combinations of π^+, π^- and π^0 mesons. Of five particles

$$\begin{matrix} \pi^+ & \pi^- & \pi^0 & \pi^- & \pi^+ \\ 1 & 2 & 3 & 4 & 5 \end{matrix}$$

a neutral triplet can be obtained in four different ways: of particles 1, 2, 3; of particles 5, 2, 3; of particles 1, 4, 3 and of particles 5, 4, 3. In order not to miss the triplet that could be created from the ω^0 meson we have to try all of them, i.e. calculate m_X for each triplet.

This, of course, makes it necessary to seek the ω^0 meson on a background of a considerably greater number of unnecessary events, but there is no other way.

An Explanation for the Layman

I do not know whether I have set forth the idea of the search with sufficient clarity. In the case that I have not, let us consider a similar situation from real everyday life. A Martian visiting the earth (and what could be more real?!) can establish the date of World War II by counting the numbers of people of various ages that live in Europe. He finds that the least amount in the age bracket from 37 to 40 years. Or, by investigating the numbers of sick-leave certificates that were issued by the doctors of a certain city in the months January, February, etc., he can establish what month the current wave of grippe attacked the city.

What do these examples, for instance the former, have in common with the matter we are discussing? The invariant mass of the meson threesomes is the age. The various ranges of m_X values represent the age rounded off to a whole number of years. The counting of the number of people of a definite age corresponds to the calculation of the frequency of specific m_X values. The slight change in the birth rate from year to year meets the requirements that, in the absence of a ω^0 meson, the frequency of various m_X values varies smoothly from range to range. The drastic reduction of the birth rate during the war, analogous to the surplus of cases with the ω^0 meson, is the predominance of one value of m_X over all the others.

We do not actually know the line of reasoning followed by the physicists that discovered the ω^0 meson, but we are quite sure that they proceeded in the following way. In each photograph they measured the momenta and directions of the four charged mesons, calculating the missing momentum. This provided the momentum and direction of the fifth (π^0) meson. Its energy was calculated in the same way. To make sure that the particle really was a π^0 meson, they calculated its invariant mass $\sqrt{E_\pi^2 - p_\pi^2}$. Then for each of the four possible neutral combinations of three π mesons, they calculated the invariant mass of the missing particle by equation (8.3). At the same time, the same calculations were carried out for the four meson threesomes with the charge ± 1, i.e. for the threesomes π^+, π^+ and π^-; and π^-, π^- and π^+, and for the two threesomes with the charge ± 2, i.e. for the threesomes π^+, π^+ and π^0; and π^-, π^- and π^0. This last operation was carried out just in case there happens to be a particle that decays into π^+, π^+ and π^- mesons, or into π^+, π^+ and π^0 mesons.

After all this work had been done on all 800 photographs, the physicists began to count the number of times the invariant mass m_X was found to be within the 0.02 GeV range from 0.42 to 0.44 GeV, within that from 0.44 to 0.46 GeV, etc. Instead of compiling a table, the results of the count were represented in a graph (Fig. 18). Plotted along the horizontal axis are the m_X values; along the vertical axis are the

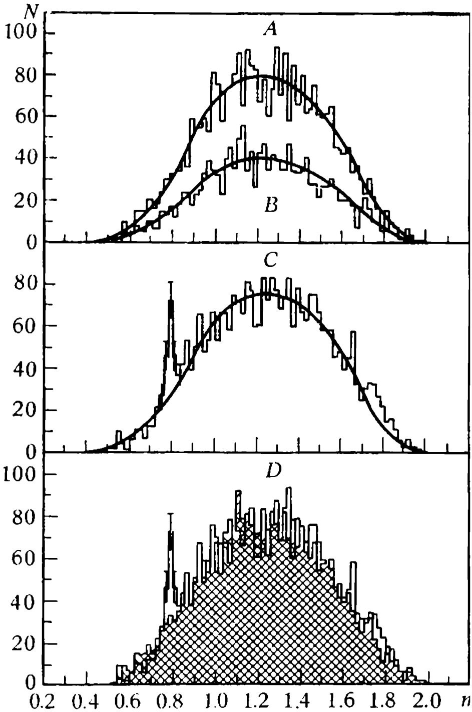

Fig. 18. Discovering the ω^0 resonance pa
The upper graph shows how frequently the va
found for the threesomes $\pi^+\pi^+\pi^-$ and $\pi^-\pi^-\pi^+$ (c
$\pi^+\pi^+\pi^0$ and $\pi^-\pi^-\pi^0$ (curve *B*). The same for π
by curve *C*, whereas curve *D* represents the su

frequency with which they were fou
two upper graphs (*A* and *B*) the
plotted for the threesomes, π^+, π^+ a
π^+, π^+ and π^0 mesons. Though the curv

line, as you can see, but the jumps from range to range are quite small and are of a random nature. No values of m_X are found to be appreciably more frequent than others. Curve C is, as they say, a horse of a different colour. It represents the frequency of m_X values found in investigating the combination of π^+, π^- and π^0 mesons. Though the jumps are also small over most of the curve, within several of the ranges, from 0.76 to 0.82 GeV, the smoothness of the curve is obviously violated. A peak crops up on a smooth rise. In this range there are many more threesomes (93 more) than there should be if the curve rose smoothly (there should have been about 98 threesomes). This surplus is what reveals the existence of the particle being sought. The mass of the particle, the location of the centre of the peak, turned out to be equal to 0.787 GeV. Calculations of the height of the peak lead to the conclusion that reaction (8.2) occurs in 10% of the cases. Only in this way could the peak have reached its height.

Much time has passed since 1961. The discovered ω^0 particle has been investigated many times during these years. It is known now that the tip of the peak is actually located at the invariant mass 0.7828 GeV. Of especial interest is the fact that though the peak is very narrow, it is not infinitely narrow.

If we removed all the photographs on which the triplets of π^+, π^- and π^0 mesons were not created by the decay of the ω^0 meson, and if we calculated the invariant mass for the remaining triplets that we know for sure to be produced by an ω^0 meson, we would not always obtain the value 0.7828. The reason is not because we cannot measure the momenta of the mesons with sufficient accuracy and determine the mass with sufficient precision. By no means! Even with absolutely precise determination, the masses of

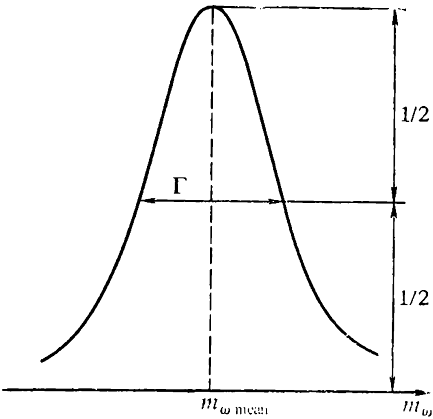

Fig. 19. Mass distribution curve for the resonance particle.

Mass values are plotted along the horizontal axis, and how frequently these values are encountered (in relative units) along the vertical axis. Also shown is the width (variance) of distribution.

the triplets will differ in the various photographs, though all the triplets owed their formation to the same ω^0 mesons. Most often, of course, we would find triplets with masses close to 0.7828 GeV, but some would have a mass of about 0.782, about 0.775, about 0.79 GeV, etc. If we were to plot a curve like the ones in Fig. 18 for the decays $\omega^0 \rightarrow \pi^+\pi^-\pi^0$, i.e. plot the number of times we find one or another value of the mass

($\pi^+\pi^-\pi^0$) along a vertical axis, we would obtain a peak rather than needle in the curve. Its vertex would be located at the value 0.7828 GeV and its width halfway to the vertex would equal 0.0094 GeV (Fig. 19).

For us this is an entirely new phenomenon. Does it follow that the particle has no definite mass? But can that be possible? Is not the mass of a particle a fixed quantity, once and for all, can it yield to the influence of something?

On the Threshold of New Discoveries

Maybe we have discovered the existence of corpuscles even smaller than elementary particles? Maybe the ω^0 meson consists of these corpuscles in exactly the same way as atoms are made up of protons, neutrons and electrons? Maybe, in the same manner as the mass of a frying pan is reduced if we break off a piece, small pieces are broken off the ω^0 meson from time to time so that its mass varies from case to case? Evidently, then, the mass of these corpuscles is much less, say by a factor of one hundred, than 0.0094 GeV, i.e. less than that of an electron. Maybe the ω^0 meson is the first sign of a new subelementary world?

Maybe the point is that energy and momentum are not conserved in decay processes? We do not actually measure the energy of the ω^0 meson itself; we measure the energy and momentum of three π mesons. Maybe we should, in fact, proclaim the "law of violation of energy conservation" in decay processes, and not be at all surprised that the mass of the ω^0 meson is not conserved?

Maybe Einstein's theory of relativity is to blame? Maybe what we call an invariant quantity (the difference $E_\omega^2 - p_\omega^2$) is no such thing? Maybe this difference varies with the energy or something else? Then again there is nothing to be surprised about.

The Discovery Fell Through

But reality turns out to be much more sensible and, at the same time, much more fantastic.

Matters are quite different from what we have surmised above. We need not resort to the aid of any corpuscles. We shall have to leave alone the law of energy conservation. The theory of relativity will, as before, continue its triumphant march.

What happened is that quantum mechanics, the mechanics of the interaction and motion of minute particles, reminded us of its existence. So far we had no need to deal with it. This, in fact, was what our initial intention consisted of: to relate about elementary particles all that requires no knowledge of the theory of their motion and transformation; to tell how the conservation of energy restricts the morals and manners of the subatomic world. I thought this could be done without resorting to quantum mechanics.

Surrender

But it could not. One of the basic laws of quantum mechanics—the indeterminacy principle—gives the following explanation for the effect we have observed. The point is that the ω^0 meson has such a short lifetime that its mass does not have enough time to "become established". The certainty with which the mass of an unstable particle is registered depends upon the mean lifetime of the particle. Only stable (eternal and nondecaying) particles have a fixed mass that does not vary. If, however, the particle is free to decay, then its mass is capable of varying within some limits. The longer the lifetime of the given kind of particle, the closer it is to stability, the more precisely its mass can be recorded. For this reason, the mass of the π^+ mesons, with a lifetime of 10^{-8} s and the mass of the Λ^0 hyperons, with an average lifetime of 10^{-10} s, and even that of the π^0 mesons, with a lifetime of only 10^{-16} s, are practically constant. Their variations cannot be registered by up-to-date instruments.

But if you succeed in finding particles that have a mean lifetime of 10^{-23} s, you will be astonished by the discord in their masses. The difference will reach 0.005 and 0.010 GeV and even more. But they will be one and the same kind of particles and have the same properties. All their properties will be the same except for one: their mass.

You may ask what kinds of properties can such minute objects have (even Voltaire's Micromégas knew that the smaller a body is, the less properties it has). Actually, there are very many. I did not enumerate them before, not wanting to enunciate incomprehensible words. But now I shall name them to demonstrate how many properties particles have. Particles differ from one another by their lifetime (as you know), electric charge (as you also know), baryon charge, lepton charge, spin, isospin, space parity, charge parity, strangeness, helicity, decay mode, form factor, magnetic moment, force of interaction with other particles, etc. Not so few, as you can see. In addition, they are characterized by their mass. So far, nobody has noted that any of these properties depends upon the mean lifetime. But the mass does.

The fact that such a dependence must exist was known beforehand. The founders of quantum mechanics warned a long, long time ago that this would happen. The indeterminacy principle predicted that if it becomes necessary in physics to deal with processes which last an extremely short time, the energy evolved in these processes will change from time to time. This had already been observed in other phenomena as well. The law relating the duration t of a process and the uncertainty Γ in the energy is formulated as follows:

$$\Gamma\ (\text{GeV}) \times t\ (\text{s}) \approx h/2\pi = 6.6 \times 10^{-25}\ \text{GeV-s}. \tag{8.4}$$

Here h is Planck's constant*.

---

* The lifetime of the excited state of an atom $\tau \approx 10^{-8}$ s. After this it emits a quantum of visible light with the energy $E = h\nu$. The uncertainty relation can then be written in the form

$$\Delta\nu \times \tau = 1/2\pi$$

where $\Delta\nu$ is the uncertainty in the frequency. Hence, it is equal to about 10^8 s^{-1}. The frequency itself of visible light $\nu \approx 5 \times 10^{14}$ s^{-1}, so that the relative uncertainty of the frequency is $\Delta\nu/\nu \approx 10^{-6}$. Consequently, spectral lines are not infinitely narrow.

With respect to our acquaintance, the ω^0 meson, the uncertainty Γ of its mass is the width (variance) of the curve in Fig. 19, i.e. 0.0094 GeV. Then, applying equation (8.4), we can calculate how long on an average the particle exists from the instant of its creation to the instant of its decay. We obtain 7×10^{-23} s.

The Clock-Balance

Hence, we have measured the lifetime of the ω^0 meson without measuring any times. We have in hand a most unique clock. It is not enough that it measures intervals of time a thousand million times shorter than any other clocks can, but it measures them without measuring time.

Let us follow again the explanation of how these clocks work. We wish to ascertain the mean lifetime of a particle X. We find out that it decays and measure the energies and momenta of the particles that are created from it with the greatest feasible accuracy. Each time we calculate the invariant mass of this group of particles, i.e. the mass of particle X itself. We try to record as many decays as possible. Rounding off the mass values to an accuracy of 0.01 GeV, for instance, we plot a curve with the rounded-off values along the horizontal coordinate axis and the number of times they were found along the vertical axis. We obtain a curve similar to the one in Fig. 19. The location of its vertex indicates, to an accuracy within 0.01, the average mass of particle X. Besides, the distance between the branches of the curve, halfway to the vertex, is the quantity Γ. Substituting its value into equation (8.4), we obtain the mean lifetime of particle X.

Can we use this clock to measure the lifetime of the Λ^0 hyperon? Judge for yourself. The Λ^0 hyperon has a lifetime of 10^{-10} s. Hence the difference in mass of various Λ^0 hyperons will be of the order of

$$\Gamma = \frac{6.6 \times 10^{-25}}{10^{-10}} = 6.6 \times 10^{-15}\ \text{GeV}.$$

No instrument exists that can detect such a small difference in energy.

Can we make use of this clock to measure the lifetime of the π^0 meson? No, we cannot: $\Gamma = 6.6 \times 10^{-25}/1.8 \times 10^{-16} = 4 \times 10^{-9}$ GeV. This is also beyond the limits of accuracy of up-to-date experimental procedure. The lifetime of the π^0 meson has been measured, but by a different method.

Then what kind of times can our clock measure? Approximately 10^{-22} s and less. Our ω^0 meson lies just on the boundary of experimental feasibility. We are still incapable of measuring a variance in energies of the order of 0.001 GeV, especially if we take into account the fact that the peak is never as evident as shown in Fig. 19, but always appears on a background of other phenomena (as in Fig. 18).

Besides the ω^0 meson, do other particles exist with a similarly short lifetime? Yes, they do. Before the discovery of the ω^0 meson, the rho meson (denoted by ρ), for example, had been found. It decays into two π mesons, its average mass fs 0.763 and the average variance Γ in masses is 0.106 GeV Thus, its lifetime (found by substituting into equation (8.4)) is approximately only one tenth of that of the ω^0 meson, and equals 6×10^{-24} s. Other particles, such as the Y-zero, isobar, etc., have similarly short lifetimes.

But their *too* short lifetime brought about a situation in which they were not considered to be particles at all. It was thought that the ρ meson is a pair of π mesons that do not go off at the instant of their creation, but hang around each other for a certain length of time (6×10^{-24} s) and leave each other only then. They were given a different name, not particles, but resonances: the rho resonance, the Y resonance, stressing, by this word, the ephemeral and unstable nature of such formations. (I am obliged to use words here that do not have an exact meaning, such as ephemeral nature, hang around each other, etc. Actually, a more or less precise theory exists concerning these phenomena, and all inaccuracies are stipulated. But it happens to be a mathematical theory. If we attempt to translate the language of mathematics into that of visualizable conceptions, we obtain nothing except vague words. Gradually crystallized out of these words tre such terms as "resonances".)

A Flood of Discoveries

The discovery of the ω^0 meson marked a definite turning point in the sentiments of scientists.

The ω^0 meson was undoubtedly a "resonance" since it had such a short lifetime. At the same time, it resembles a particle in all of its features:

its mass varied from time to time by only 1 to 1.5%. It became clear that no impassable abyss exists between resonances and particles, that formations with a lifetime of 10^{-23} or 10^{-24} s are just as full-fledged candidates for the status of elementary particles as the Λ^0 hyperon or π^0 meson. The precision and powerful demonstrative capacity of the experiment itself had an immense impact. Physicists began feverishly to discover new resonances. They proceeded, in general, in the same way as previously. They picked out a definite reaction, calculated the invariant masses of all the combinations of particles they found in it, and then looked to see whether any values of this mass were found more frequently than neighbouring values.

Some experimenters were carried away by unwarranted enthusiasm, others announced exaggerated results: if an insufficient number of photographs are made, some mass values may be found more frequently (by pure chance) and others more rarely. It was quite easy, when wishing at all costs to make a discovery, to pass a random accumulation of mass at one point as the detection of a new particle. But gradually rigid rules were approved for the acception of new peaks into the resonance family. Up to the present several dozens have been discovered, and there is still no end in sight. Each new resonance (still, as previously, they are called resonances, though it is understood that there is no difference between them and particles) is met with great interest, because physicists hope that when a great many have been accumulated, they will begin to understand the relations between the resonances and their place in the picture of the world.

In conclusion, I wish to underline the difference between resonance particles and particle particles that makes the strongest impression on us. A resonance differs from a particle in that a photograph in which a particle decays can, as a rule, be shown. But nobody can show you a photograph recording the decay of a resonance.

If you ask a physicist to show you a photograph recording the decay of a π^0 meson into two photons, he can look for and find such a photograph (Fig. 20). True, you will not see the π^0 meson there, but the physicist knows that it has been there because the invariant mass of the two photons yields a value close to the mass of the π^0 meson. If you next ask him to show you a photograph of the process in which a ω^0 meson

Fig. 20. Creation and decay of the π^0 meson.

Shown at the lower left is what you should pay attention to in the photograph: γ_1 and γ_2 are the photons produced in the decay; γ_1 produced the electron-positron pair e_1e_1, whereas γ_2 produced a whole shower. The straight tracks at the bottom belong to other particles created together with the π^0 meson.

is created and then decays, the physicist will make a helpless gesture. He cannot give you one photograph. He can give you *two hundred* and say "Among them at least one hundred 'belong' to the ω^0 meson" But which ones? "That is something nobody knows" This is understandable, because the decays $\omega^0 \rightarrow \pi + \pi + \pi$ are found on a background of π meson triplets, whose invariant mass turns out *by mere chance* to be close to that of the ω^0 meson. We are incapable of separating the ω^0 meson from the background. This constitutes the difference. The Ω^- hyperon was discovered by finding a single suitable photograph, whereas the experimenters succeeded in discovering the ω^0 meson without having on hand even one trustworthy case of a $\omega^0 \rightarrow \rightarrow \pi + \pi + \pi$ decay, but having a great many untrustworthy ones.

Here physicists are in the same boat as geologists who are given a great number of samples of some mineral and told, "A part of the samples are not the mineral, but simply rock. But nobody knows which are which. The properties of the rock are unknown. Those of the mineral are also unknown. Please report on the properties of the mineral. Do not rely on separating the mineral from the rock by some kind of chemical analysis, they are inseparable, ..." These are exactly the conditions under which physicists work in investigating resonances (see also Chap. 16).

Part Two

Kinematics for the Schoolboy

A brilliant young student of Worms
Majored in physics for four terms.
As he approached BS stature,
He got farther and farther from nature,
This paperized physicist of Worms.

Chapter 9

The Momenta Hedgehog

We shall deal in this chapter with the principal properties of the decay of a particle at rest into two particles. The problem is to be formulated as follows. Assume that a π^0 meson at rest decays to two photons. Can we find out beforehand the energy that these photons will have? Or, in another case, a Λ^0 hyperon disintegrates in motion to a proton and a π^- meson. What did the energies of its successors seem to be to this hyperon in the last instant of its existence? Or, if you cannot stand such profanation of elementary particle transformation, what did these energies seem to equal to you, if you travelled alongside the hyperon and were the unintentional witness of his event?

We Solve a System of Equations

Let us solve this problem in its most general form. Assume that a stationary particle *O* of mass m decays to two particles, *1* and *2*. We know the rest masses of particles *1* and *2* beforehand: they are m_1 and m_2. What can be said about the energies E_1 and E_2 and the directions of these particles?

It can be said with respect to the energies that added together they equal the energy of the initial particle, i.e. its mass m:

$$E_1 + E_2 = m. \tag{9.1}$$

As to the directions, we know that they should be such that the momentum vectors $\mathbf{p}_1$ and $\mathbf{p}_2$ of the two particles add up to equal zero (as was the momentum of the particle O at rest):

$$\mathbf{p}_1 + \mathbf{p}_2 = 0. \tag{9.2}$$

Besides, we know another property of moving particles: whatever their energies and momenta, a certain combination of these properties is independent of their motion:

$$E_1^2 - p_1^2 = m_1^2, \tag{9.3}$$

$$E_2^2 - p_2^2 = m_2^2. \tag{9.4}$$

We shall have to solve this system of equations, (9.1) through (9.4). The solution is quite simple. Look at equation (9.2). Recall the rule for adding vectors. The addition of two vectors yields zero only when the tail of the first vector coincides with the head of the second and vice versa. It can be seen in Fig. 21 that

$$p_1 = p_2. \tag{9.5}$$

The momenta of particles *1* and *2* are equal in magnitude, but opposite in direction. Hence, if we subtract equation (9.4) from equation (9.3) we obtain

$$E_1^2 - E_2^2 = m_1^2 - m_2^2. \tag{9.6}$$

Next we divide equation (9.6) by equation (9.1) and obtain

$$E_1 - E_2 = \frac{m_1^2 - m_2^2}{m}. \qquad (9.7)$$

Now we add equations (9.7) and (9.1). After

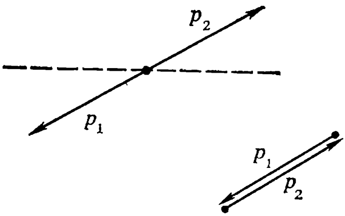

Fig. 21. Decay of a particle at rest (the sum of the two momenta equals zero).

cancelling, we obtain

$$2E_1 = m + \frac{m_1^2 - m_2^2}{m},$$

from which

$$E_1 = \frac{m^2 + m_1^2 - m_2^2}{2m}. \qquad (9.8)$$

If you wish to find E_2 you interchange masses of particles *1* and *2*:

$$E_2 = \frac{m^2 + m_2^2 - m_1^2}{2m}. \qquad (9.9)$$

This is the algebraic solution of our system of equations. I can inform geometry fans that the problem of the decay of a particle of mass m to two particles of masses m_1 and m_2 fully coincides with the following school geometry problem

(Fig. 22): the sum of the sides of the triangle equals m, and their projections on the base are equal to m_1 and m_2; find the sides of the triangle. You will see that the same system of equations, (9.1) through (9.5), are obtained. For your

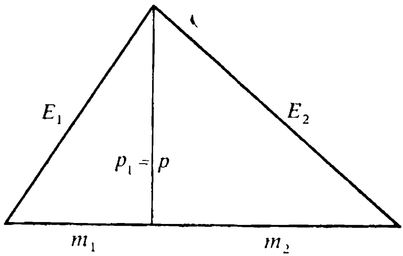

Fig. 22. The geometric solution.
Solving this triangle means you have solved the problem of decay to two particles.

benefit, I have indicated on the triangle the quantities E_1, E_2 and $p = p_1 \quad p_2$. If you prefer the geometric method of solving this problem, try to obtain equation (9.8) by purely geometric means.

It should be easy to understand by just looking at the triangle that the problem has a solution only when $m \geqslant m_1 + m_2$. Here again we meet our old acquaintance, one of the forbidden reactions: *the sum of the masses of the products of decay must not exceed the mass of the initial particle.*

Let us try to find an understanding of the solution obtained. We have come across an interest tng feature. In the decay of a particle at rest ihe energies of its two offspring cannot have just any values. They depend upon the rest masses of the particles involved: on m, m_1 and m_2. The energy values are predetermined by these masses. But in decay into two identical particles, their energies are determined only by the mass of the parent particle:

$$E_1 = E_2 = \frac{m}{2}. \qquad (9.10)$$

For instance, when a stationary π^0 meson with a mass of 0.135 GeV decays to two photons, it simply divides its energy in half: 0.0675 GeV to one photon and the same amount to the other. But the momentum of the progeny always depends upon their mass (it is the same for each one and is equal to $\sqrt{E_1^2 - m_1^2}$ or $\sqrt{E_2^2 - m_2^2}$). The heavier they are, the less their momentum as they go off. At $m_1 = m_2 = m/2$, the particles to which particle O decays do not go anywhere but remain at the point of decay. This happens, as a matter of fact, not only when their masses are equal, but with any masses, provided that $m_1 + m_2 = m$. To understand why this is so, just imagine what the altitude of the triangle in Fig. 22 would be if $m_1 + m_2 = m$. At $m_1 + m_2 = m$, what we have is more like something falling into parts, rather than decay into particles.

Thus, we are already capable of calculating beforehand with what energy particles will be created in one or another kind of decay. This enables us to identify particles which have a typical mode of decay.

It is known, for instance, that K^+ mesons (having a mass of 0.494 GeV) and π^+ mesons (with a mass of 0.140 GeV) have approximately the same lifetime and decay into the same particles: the μ^+ meson (mu-meson, or muon, with the mass 0.1057 GeV) and the neutrino (mass 0).*

Practical Conclusions

This decay "at the end of the trail" may be like the one shown in Fig. 23: the track of the K^+ or π^+ meson, more and more sinuous (owing to the loss of velocity the particle meanders between the atoms; the lower its velocity, the greater the angle through which it can be turned by an atom happening to be nearby, and, as a result, it loses still more velocity), is suddenly interrupted (the particle comes to a stop) and ends with a knee, which is the track of the outgoing muon. The neutrino leaves no track.

Can we determine from our photograph what has decayed: a K^+ or a π^+ meson? One procedure takes into account the fact that the degree of sinuousity, or crookedness, at the end of the trail differs for a K^+ particle and for a π^+ meson. But there exists another procedure that is based on kinematics. It consists in measuring the path of the muon. The higher the momentum, the longer the path, and the momentum in the decay

$$K^+ \to \mu^+ + \nu$$

is not at all the same as in the decay

$$\pi^+ \to \mu^+ + \nu.$$

* This is practically the only decay mode for the π^+ meson; for the K^+ meson it is one of many, but quite frequent.

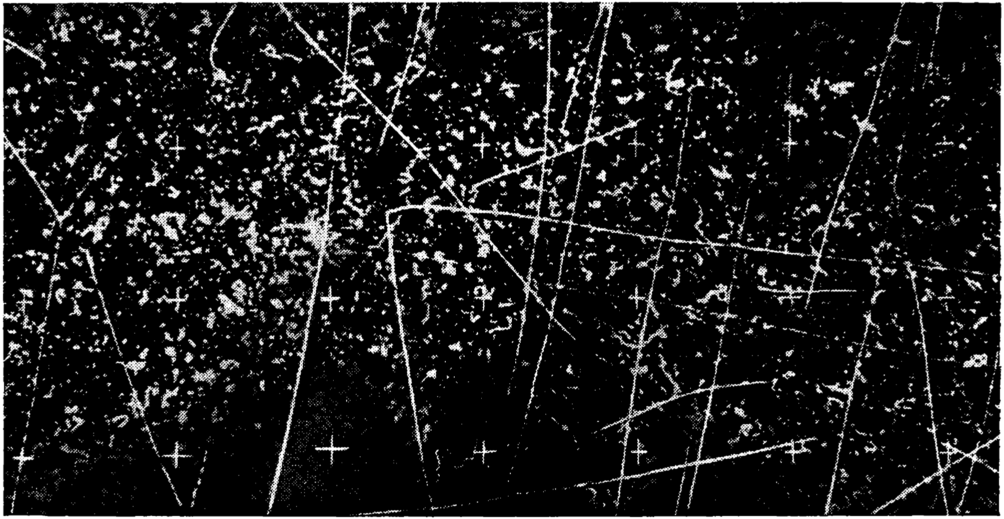

Fig. 23. A case of the decay $K^+ \rightarrow \mu^+ + \nu$.
The track running from the right across the other tracks is the trace of the K^+ meson; the track downward from the knee is the outgoing μ^+ meson. Just before the knee, the direction of the K^+ track was changed. This proves that the particle was at rest during the decay.

Fig. 24. The decay $\pi^+ \rightarrow \mu^+ + \nu$.

The π^+ meson itself was evidently created in the reaction $\pi^- + p \rightarrow \pi^- + \pi^+ + n$; the μ^+ meson decayed to e^+ and a ν pair

In the former case it equals 0.236 GeV (check by equation (9.8), and then by equation (9.3)), whereas in the latter it is only 0.0298 GeV. This implies that all μ^+ mesons obtained in the decay of a stopped K^+ meson will have one path length in a given substance, and the μ^+ mesons obtained from the decay of a π^+ meson will have an entirely different path length (Fig. 24). Usually, both procedures are applied to eliminate any doubt about what decayed, a K^+ particle or a π^+ meson.

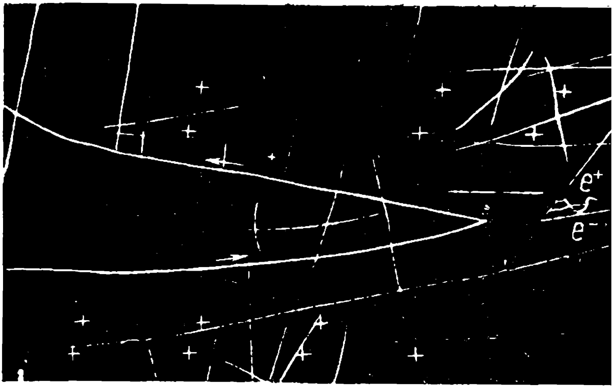

Fig. 25. A case of the decay $K^+ \rightarrow \pi^+ + \pi^0$.
At the end of its trail the π^+ meson comes to a stop and decays to μ^+ and ν. The decay of π^0 produces two photons, one of which, near some nucleus, creates the e^+e^- pair visible in the photograph.

In exactly the same way, on the basis of path length, you can distinguish the decay

$$K^+ \rightarrow \mu^+ + \nu$$

from the decay

$$K^+ \rightarrow \pi^+ + \pi^0,$$

provided that the decay occurred with the K^+ particle at rest (i.e. if the knee was preceded by a highly sinuous track). The momentum of the π^+ meson should equal 0.205 GeV, instead of 0.236 GeV, the momentum of the μ^+ meson (Fig. 25).

In addition, the derived equations (9.8) and (9.9) are essential in investigating another kind of decay: decay "in travel". We shall deal with this type of decay in Chap. 11, and now I shall tell you about what gave this chapter its name: the momenta hedgehog.

The Momenta Hedgehog

By solving the system of equations (9.1) through (9.4) we determined the energies of particles *1* and *2*. But you probably noticed that nothing was said about their directions. The fact is that they may have any directions. Vectors $\mathbf{p}_1$ and $\mathbf{p}_2$ are equal in length (and their length is determined only by the masses m, m_1 and m_2) and opposite in direction, but whatever their direction, it is all the same, their sum is always equal to zero. This implies that there are no forbidden directions of motion of the offspring of particle O. But we know that *all that is not forbidden is allowed*. As a matter of fact, from time to time, from photograph to photograph, the direction of the kink, or knee, of the μ^+ meson (in the decay $K^+ \rightarrow \mu^+ + \nu$) is found to differ. Each new decay of particle O into particles *1* and *2* may take place in some new direction. This is what enables us to construct the decay momenta hedgehog.

Imagine that we have accumulated an innumerable heap of photographs of the decay $K^+ \rightarrow \rightarrow \mu^+ + \nu$. What will happen if we position them so that their points of decay coincide and draw all the momentum vectors of the μ^+ meson? The point of decay will then bristle with thousands of arrows. Their length will be the same, but we shall find practically any directions. This, then, is our momenta hedgehog (Fig. 26).

All of its spines are identical, and there is no place on it that we can touch without getting pricked. You must think of this hedgehog each time you deal with the decay of a particle into two and ask yourself in which directions the daughter particles might travel. The hedgehog will remind you: in any.

To the readers who have no use for counterparts from animate nature in a book on physics,

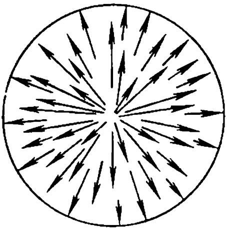

Fig. 26. Decay momenta hedgehog of a particle at rest.

I state simply: the locus for the heads of the momentum vectors of particle *1* is a sphere of radius p with its centre at the point of decay. The same sphere is the locus for the momenta of particle *2*.

Chapter 10

What Colour Are Elementary Particles?

This is a question that I have asked many physicist friends. Not one of them was surprised, and each tried to describe his idea of what particles look like. The question was asked separately and none of them knew what answers the others had given.

The discord was incredible. The more astonishing the fact that many of the opinions coincided with respect to the proton. The general notion was that it must surely be black (if not red).

It is of interest that only theoretical physicists found it possible to express their idea on this question. It turned out that experimental physicists never see particles in colour. Only one said that he thought the electron was green (and, even in this case, it proved to be the trace of an electron on an oscilloscope screen that he had in mind). Hence, this resulted, at the same time, in a good test enabling a theorist to be distinguished from an experimenter. It may help a physics student to choose what will suit him, or her, better. If you do not sense the colour of particles, do not become a theoretical physicist.

A reader inexperienced in science may ask: "What a strange way to settle scientific problems by a majority vote? Don't physicists know the true colours of particles?"

This question is fully within the competence of kinematics, and I cannot refrain from discussing it. The colour of a body is determined by the frequency of the light waves travelling away from it. If the body is self-luminous, these are waves emitted by the body itself. Otherwise, they are waves reflected from the body. The former is simpler, and we shall define the colour of a particle as the colour of the light waves spontaneously radiated by it. The colour of an atom is also the colour of the light it emits. Assume for the sake of simplicity that the particle is at rest.

It is known from quantum mechanics that the radiation of light is a flux of photons, or light quanta. The frequency ν of light is related to the energy E of the photons by the equation

$$E = h\nu, \tag{10.1}$$

where h is Planck's constant:

$$h = 4.14 \times 10^{-24} \ \text{(GeV-s)}. \tag{10.2}$$

Hence, if we wish to determine the colour of particle X (or atom X) of mass m, we should find the energy of the photons it emits and convert it into frequency. This is, in essence, the problem on the kinematics of decay to two particles. Let us imagine for the moment that the particle emits photons, one after the other, but remains, of course, unchanged. We will perceive this flux of photons from the particle as its glow. The process is

$$X \to X + \gamma. \tag{10.3}$$

The mass m of the initial particle X is not less than the sum of the rest masses $(m + 0)$ of particles X and γ, so that our necessary and sufficient condition for the possibility of decay

$$m \geqslant m_1 + m_2 \tag{10.4}$$

is complied with. Let us now find the energy of γ. Into equation (9.9) $(E_2 \quad (m^2 + m_2^2 - m_1^2)/2m)$ we substitute the values $m_1 = m$ and $m_2 = 0$. But what do we obtain: $E_2 = 0$!

The answer is unexpected. We find that the energies of photons and, consequently, their momenta as well are equal to zero. Our assumed beam of light carries neither energy nor momentum. There will simply be no radiation.

Now we begin to understand why physicists are compelled to solve the problem on the colour of particles by resorting to democratic principles, i.e. by an analysis of public opinion. We have proved the theorem stating that particles have no colour. This, evidently, is why the proton seems to many physicists to be black. And all atoms should appear black. When, for instance, atoms of incandescent sodium emit photons, they do not cease to be atoms of sodium. This is process (10.3) again:

$$\mathrm{Na} \rightarrow \mathrm{Na} + \gamma. \qquad (10.5)$$

Hence, atoms of sodium or any other atoms cannot radiate colour.

...We frantically turn over our knowledge of physics: where were we tripped up? It is true, of course, that sodium engaged in radiation does not cease to be sodium. But light is radiated by *excited* atoms of sodium. What do we mean by "excited"? Simply energy stored up beforehand; having surplus energy. Surplus energy means surplus mass. This, then, is the crux of the matter! When common salt shines with a yellow colour in the flame of a Bunsen burner, the reaction is

$$\mathrm{Na}^* \rightarrow \mathrm{Na} + \gamma \qquad (10.6)$$

(where the mass of Na* is greater than that of Na), rather than reaction (10.5). If we now wish to find the colour of particle X, we have to look for a process

$$X \to Y + \gamma, \tag{10.7}$$

where particle Y must be lighter than particle X. Then everything turns out fine: a photon is emitted with the energy

$$E_\gamma = \frac{m_X^2 - m_Y^2}{2m_X}; \tag{10.8}$$

and, after dividing E_γ by 4.14×10^{-24} we find the colour of the particle.

Just for sport let us see how the frequency of the light emitted by sodium is calculated by this equation. An excited atom weighs only slightly more than an unexcited one, i.e. in the decay $Na^* \to Na + \gamma$, $2m_X$ in the denominator of equation (10.8) can be replaced by the sum $m_{Na^*} + m_{Na}$. After cancelling we obtain

$$E_\gamma = m_{Na^*} - m_{Na}.$$

This is the well-known equation for the frequency of light radiated by an atom in its transition from an excited state to its ground state. Usually, however, it is written in the form

$$h\nu = E^* - E.$$

But we know that the energy of an excited state and the mass of an atom are one and the same, but expressed in different words.

So, everything has been cleared up and it remains to look for processes in which elementary particles spontaneously emit photons. Unfortunately, such processes are exceptionally few and far between. The following reactions are decays in which the colour of the particles can be determined:

$\pi^0 \rightarrow \gamma + \gamma$,

$\eta^0 \rightarrow \gamma + \gamma$ (η^0 is a resonance with the mass 0.550 GeV),

$\omega^0 \rightarrow \pi^0 + \gamma$,

$\Sigma^0 \rightarrow \Lambda^0 + \gamma$ (Σ^0 is a hyperon with the mass 1.192 GeV).

But even here an unpleasant surprise lies in wait for us: the energy of photons is so high that the eye does not sense it as light. We will simply obtain radioactive radiation.

We can calculate, for instance, the frequency of the photons from the decay $\Sigma^0 \rightarrow \Lambda^0 + \gamma$. Substituting $m_\Sigma = 1.192$ and $m_\Lambda = 1.115$ into equation (10.8) yields $E_\gamma = 0.0745$ GeV, which corresponds to a frequency of $\nu = 1.8 \times 10^{22}$ s^{-1}. The eye cannot perceive a frequency higher than 10^{15} s^{-1}.

Thus, nothing came of our interest in the colour of particles. Now what is the moral in all this, as they used to ask in the nineteenth century? Or the dry residue, as they ask in the twentieth? What have we found out?

In the first place, we found that in radiating light, atoms become lighter. This is quite clear: light carries away energy, and energy is equivalent to mass.

In the second place, we found out that an atom before radiation and the atom after radiation are *different* particles, just like the Λ^0 and Σ^0 hyperons in the reaction $\Sigma^0 \rightarrow \Lambda^0 + \gamma$. The differences, of course, are not so large, because the energy of the photons are much less, but they are real differences. Unexcited helium, for instance, is an inert gas. Excited helium, on the contrary, participates in reactions.

We shall not be astonished now to hear the Σ^0 hyperon called the excited state of the Λ^0 hyperon. The concept that certain heavy particles are excited states of lighter particles is a useful one. It helps to establish family ties between elementary particles.

Still another piece of information that we gained is that condition (10.4), the necessary and sufficient condition that the decay $O \rightarrow 1 + 2$ will take place, requires an additional stipulation. When particles *1* and *2* have rest masses, then $m_O \geqslant m_1 + m_2$ is quite valid. But when particle *2*, for example, has no rest mass, then the inequality becomes strict:

$$m_O > m_1.$$

This is understandable: at $m_O = m_1 + m_2$, particles *1* and *2* are created fixed, whereas a photon cannot stand still.

The Mössbauer Effect

Finally, we should not forget the fifth conclusion. As a photon is emitted it carries away, not only energy, but momentum as well. The momentum of the photon is simply equal to the energy $h\nu$. Particle Y receives an equal momentum $h\nu$ in the opposite direction: in radiating light the atom is subjected to recoil. This recoil is very small; the energy of visible light is in-significant. In the radiation of yellow light with the frequency $\nu = 5 \times 10^{14}$ s^{-1}, for instance, the recoil momentum of the atom is

$$h\nu \approx 2 \times 10^{-9} \text{ GeV}.$$

It is quite another matter when an elementary particle decays or we observe a radioactive decay. There we cannot ignore this recoil (in the preceding example $p_r = E_\gamma = 0.0745$ GeV).

The reaction in radioactive gamma decay (emission) is

$$N^* \rightarrow N + \gamma,^1 \tag{10.9}$$

with the transition of an excited nucleus from a state with greater mass to one with less mass, emitting a photon and undergoing recoil. The recoil energy is equal as usual to

$$R = \frac{M_N v_N^2}{2} = \frac{p_r^2}{2M_N} = \frac{(h\nu)^2}{2M_N}.$$

In our example

$$R = (0.0745)^2/(2 \times 1.15) \approx 2 \times 10^{-3} \text{ GeV}.$$

It becomes clear then why in the bulk of the substance this photon is not immediately captured by another unexcited nucleus. To accomplish the reverse process

$$N + \gamma \rightarrow N^*$$

it is necessary for the nucleus N to have exactly the same momentum (in magnitude and direction), and the same kinetic energy R, as the nucleus N in the decay reaction.* But nuclei N

* Only in this case can we conform to the laws of conservation of energy and momentum.

are almost stationary or, to be more precise, their velocity and energy in ordinary thermal motion are much less than the velocity and energy received in recoil. (The kinetic energy of thermal motion can be determined by the equation $E = 3kT/2$, where $k = 8.62 \times 10^{-14}$ GeV/deg, i.e. Boltzmann's constant. At room temperature, the energy is $E \approx 2.5 \times 10^{-11}$ GeV, which is much, much less than the required amount R.)

When, however, photons are emitted by atoms, the recoil energy of the atom is much less than the energy of thermal motion. At the frequency $\nu = 5 \times 10^{14}$ s^{-1} of the radiated light, the recoil energy of a sodium atom (whose mass is about 22 GeV) is equal to

$$R = \frac{(h\nu)^2}{2M} = \frac{(4.14 \times 10^{-24} \times 5 \times 10^{14})^2}{2 \times 22} \approx 10^{-19} \text{ GeV},$$

whereas the energy of thermal motion is 10^{-11} GeV Therefore, among the countless amount of atoms there could quite possibly be some that have the proper momentum, equal to the recoil momentum of the atom that emitted the light, and the light will be absorbed.

The German physicist Rudolf Ludwig Mössbauer was the first to comprehend that when nuclei N* and N are bound in the points of a crystal lattice, the recoil momentum of the photon in decay (10.9) is sometimes shared by the whole crystal. The mass of the crystal is very large and everything looks as if N* and N are "particles" of incredibly large mass (equal to the mass of the crystal). It is obvious that such a "particle" remains stationary after gamma radiation. To absorb such a photon it is also necessary for its momentum to be taken up by the whole crystal. This is feasible when the energy of the γ quantum is not too high. Mössbauer reasoned that in radioactive γ decay of nuclei in a crystal, the photons will be absorbed again in the same crystal (or in another one, but of the same material) in the same manner as occurs in the radiation of light by atoms. He confirmed his hypothesis experimentally. The sensitivity of the Mössbauer effect is so great that when the photon-emitting crystal is raised several metres above the absorbing, or receiving, crystal, absorption no longer takes place. As the photons "fall" with gravity they lose potential energy, gain frequency and reach the nuclei with an energy differing from that required for the process $N + \gamma \rightarrow N^*$. This experiment confirmed (once again) Einstein's concept that photons possess a mass of motion that differs in no way from the mass of motion of other particles.

Chapter 11

Relativistic Transformations of the Momenta Hedgehog

There is not much good in the hedgehog if a particle decays while it is at rest. It is quite another matter when it decays in travel. We can, of course, follow the particle in a thought experiment, in which case it will seem to be at rest, and we can repeat all the reasoning of Chapter 9. But it is not in our power to make physical instruments follow the particle. It is therefore of prime importance to comprehend how decays look to a stationary observer. And not only those of a single particle, but of a great many particles of the same type.

To discover a new particle, it is sometimes sufficient to record it at least once. This is what happened, for example, in the case of the Ω^- hyperon. But to determine the properties of a particle, a single observation is hardly enough. Most of the properties can be cleared up only after investigating a great many interactions of a single kind of particle. These properties are manifested as a certain average property of the whole set of particles being investigated. It may be of interest, for instance, to find out in which direction such and such particles are most frequently emitted in such and such a process, or what recoil a proton is most frequently subject to in such and such a reaction, and how often the reaction occurs.

There is, therefore, one feature of the kinematics of elementary particles that distinguishes it from the kinematics, for example, of an icicle falling from the eaves of a roof, or that of a rocket in its flight to the moon. Elementary particle kinematics is concerned, not only with the motion and decay of separate single particles, but also with the average characteristics in the decay of a great number of particles of the same type. It poses such questions as: how many, on an average, do we find particles with the same energy or ones travelling in the same direction, etc.? In the kinematics and the statistics of particle decay and particle collisions, the questions "how much?" and "how often?" usually go hand in hand.

Up to this time we have, in essence, dealt with the kinematics of a single, separate decay or collision. When we drew the decay hedgehog, we first raised the question about how a thousand decay processes, such as $K^+ \to \mu^+ + \nu$, would look if they all occurred at a single point. But we saw nothing of any particular interest: an ordinary round hedgehog, indicating that the μ meson may have any possible direction. A decay in travel, however, substantially alters the picture. The momenta hedgehog is transformed: certain spines become shorter, others longer; at certain spots they become denser, at others, more sparse. This leads to interesting problems.

Now let us find out how a great number of processes in which a particle *O* decays into particles *1* and *2* will look if we observe these decays while we are stationary. We shall assume that all the particles *O* have the same momentum **p** in both magnitude and direction (and, naturally, the same energy E).

We Solve Without Solving

Our problem can be solved in different ways. We can proceed directly by writing the conservation laws

$$\left.\begin{aligned} E_1 + E_2 &= E \\ \mathbf{p}_1 + \mathbf{p}_2 &= \mathbf{p} \end{aligned}\right\} \qquad (11.1)$$

and solving these equations, taking into account the invariance of the combination $E^2 - p^2$. But we can also resort to a roundabout way. We shall, of course, favour the roundabout way over the direct one, according to the proverb: the furthest way about is the nearest way home.

We shall be led by the Lorentz transformations. Recall their meaning and form. Assume that in the previous frame of reference (for example, the one travelling together with particle *O*), the momentum of a particle (for instance, *1*) was equal to p^*, the energy to E_1^*, the longitudinal projection of the momentum was denoted by $p_{\parallel}^*$ and the transverse projection by $p_{\perp}^*$.* In the new frame of reference (for instance, the laboratory frame), the same quantities will be denoted by letters without asterisks. Then the transverse projection $p_{\perp}$ remains unchanged:

$$p_{\perp} = p_{\perp}^*, \tag{11.2}$$

whereas the longitudinal projection turns out to be a combination of the quantities $p_{\parallel}^*$ and E_1^*, i.e.

$$p_{\parallel} = \gamma p_{\parallel}^* + \gamma v E_1^*. \tag{11.3}$$

The energy is also expressed by a linear combination. Thus

$$E_1 = \gamma E_1^* + \gamma v p_{\parallel}^*. \tag{11.4}$$

The factors γ and γv in equations (11.3) and (11.4) depend only upon the velocity of the pre-

* We have omitted the subindex 1 for the momentum p of particle *1*, firstly, for the sake of simplicity, and secondly, because in the reference frame in which particle *O* is at rest, the momenta of particles *1* and *2* have the same magnitude.

vious reference frame with respect to the new one, i.e. the velocity, in our case, of the particle O. The factors equal

$$\gamma = \frac{E}{m} \text{ and } \gamma v = \frac{p}{m}. \qquad (11.5)$$

The first of these fractions is simply the definition of the Lorentz factor, and the second follows from the first if we recall that $v = p/E$.

Thus, instead of solving equations (11.1) in a stationary frame of reference, we can solve them in the co-moving frame, i.e. the one in which particle O is at rest (which we learned to do in Chapter 9), by finding the momenta of particles *1* and *2*, and transforming these momenta into ones in the stationary frame according to the Lorentz formulas. This simple algebraic substitution, i.e. the solution of the system of equations (11.1), is done with equations (11.3) and (11.4), in which E_1^* and $p_{||}^*$ are to be replaced by their values from Chapter 9.

Being true, however, to our love of geometry, we shall obtain this solution graphically. In the (co-moving) system in which particle O is at rest, all solutions are depicted as points on a sphere of radius p^* (the sharp points of the spines of the momenta hedgehog). For the time being, it is sufficient to draw a single cross section of this sphere: a great circle (also of radius p^*). The coordinates of any point Q of this circle are equal to the longitudinal $p_{||}^*$ and transverse $p_{\perp}^*$ components of the momentum vector having its head at point Q. Just look at what the Lorentz transformations do to this circle. If they were of the form

$$p_{\perp} = p_{\perp}^{*},$$
$$p_{\parallel} = \gamma p_{\parallel}^{*},$$

they would simply stretch it γ-fold horizontally (the vertical coordinate of each point remaining unchanged, and the horizontal coordinate stretched γ times). The figure obtained in stretching the circle is an ellipse (Fig. 27). But the formulas also have the additional member $\gamma v E_1^*$. Thus

$$p_{\parallel} = \gamma p_{\parallel}^{*} + \gamma v E_1^*.$$

It further increases each stretched horizontal coordinate by the amount $\gamma v E_1^*$. *But this amount is the same for all points of the circle*: γ, v and E_1 depend only on the mass of the particles, i.e. m, m_1 and m_2 and on the energy of particle O, and we specified the values of these quantities beforehand by stating that we are dealing with a definite decay reaction and with particles having definite momenta. Hence, the addend $\gamma v E_1^*$ simply transfers all points of the ellipse a distance $\gamma v E_1^*$ to the right, i.e. the ellipse as a whole is shifted to the right with respect to the initial circle (Fig. 28).

Thus, the Lorentz transformations convert a circle into an ellipse by stretching it to the right and left, and then shift it as a whole to the right. Next we revolve the circle and ellipse about axis **p**.

Our circle becomes a sphere (our previous momenta hedgehog) and the ellipse, an ellipsoid of revolution (Fig. 29). This then is our transformed momenta hedgehog of particle *1*. We have proved that the locus of heads of the momentum

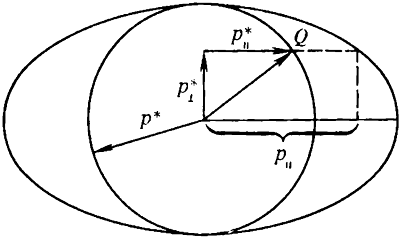

Fig. 27. The geometric meaning of the first member of equation (11.3).
The circle of radius p^* was stretched into an ellipse.

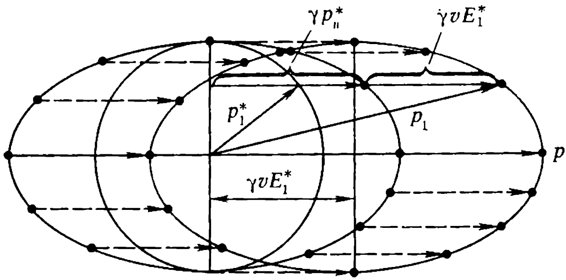

Fig. 28. The geometric meaning of equation (11.3).
The ellipse was shifted by the distance $\gamma v E^*_1$ to the right.

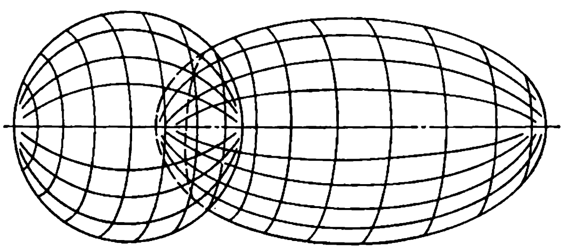

Fig. 29. Decay hedgehogs of a particle at rest (at left) and of a particle in motion (at right).

vectors of particle *1*, formed upon the decay of particle *O* having the specified momentum **p**, is an ellipsoid of revolution. In other words, if we observe, while standing at one place, a great number of decay processes of particles *O*

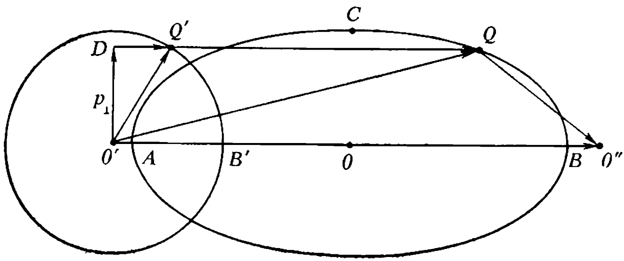

Fig. 30. An anatomical analysis of the hedgehogs Fig. 29.

with the same momentum **p** (in both magnitude and direction), and imagine that all their points of decay are made to coincide, the momenta of particles *1*, taken *in toto*, are confined within the ellipsoid of revolution, the locus of their heads. The shape and position of the ellipsoid are determined only by the values of m, m_1, m_2 and the vector **p**. Let us see just how this happens.

The centre *O* of the ellipsoid is displaced, as we mentioned above, with respect to the centre *O'* of the sphere, by the distance $\gamma v E_1^*$ in the direction of vector **p**. Since $\gamma v = p/m$ and $E_1^* = (m^2 + m_1^2 - m_2^2)/2m$, then (Fig. 30)

$$\overrightarrow{O'O} = \frac{E_1^*}{m}\mathbf{p} = \frac{m^2 + m_1^2 - m_2^2}{2m^2}\mathbf{p}. \qquad (11.6)$$

The minor semiaxis of the ellipsoid was directed across the circle-stretching procedure and therefore remained unchanged. It is equal to p^*. Recall that p^* is the altitude of the triangle in Fig. 22. You can see for yourself that p^* is determined by an equation resembling the one that expresses the altitude of a triangle in terms of its area and base (a corollary of Hero's formula expressing the area of any triangle in terms of its sides). Thus

$$\overline{OC} = p^* = \frac{\sqrt{(m-m_1-m_2)(m-m_1+m_2)(m+m_1-m_2)(m+m_1+m_2)}}{2m} \quad (11.7)$$

Naturally, the major semiaxis is equal to γp^* because that, precisely, is the amount that the horizontal radius $O'B'$ was stretched. Hence

$$\overline{OB} = \frac{E}{m} p^*. \quad (11.8)$$

An ellipse, the cross section of our ellipsoid, can readily be constructed by employing equations (11.6), (11.7) and (11.8). Each point on the ellipse is the head of some momentum vector of particle *1*.

But how about particle *2*? It is really necessary to induce the hedgehogs to multiply? That, of course, is one way. But we can also manage with the previous ellipse (see Fig. 30). In the drawing we lay off the vector $\overrightarrow{O'O''}$, equal to the momentum $\mathbf{p}$ of particle O. Then we draw the vector $\overrightarrow{O'Q}$, equal to the momentum $\mathbf{p}_1$ of particle *1*. Next we join points Q and O''. What is the vector $\overrightarrow{QO''}$ equal to? This vector is the one that must be added geometrically to vector $\mathbf{p}_1$ to obtain vector $\mathbf{p}$. It is clear that this is the vector $\mathbf{p}_2$, the momentum of particle *2*. Thus, by constructing point O'' in addition to the ellipse, we can depict the direction and magnitude of the momentum of particle *2*.

How is the ellipse positioned with respect to the sphere (or, in our drawings, with respect to its circular cross section)? Is its left end always to the right of the centre of the sphere as in Fig. 30? No, not always. Everything depends upon the combination of the values of m_1, m_2, m and E (or p). At sufficiently low values of E, the sphere, following the Lorentz transformations, is only slightly shifted to the right and is only slightly stretched, so that point A remains to the left of centre O'! The hedgehog "crawls away" a little and stretches a little, but no one can have any doubt about it being a hedgehog (Fig. 31). As the value of E increases, the ellipsoid stretches more and more, but the distance that it is shifted depends upon the masses m, m_1 and m_2. For example, with $m_1 = m_2 = 0$ (in the decay process $\pi^0 \rightarrow \gamma + \gamma$, for instance), point A will not step over point O' no matter how high the energy of particle O. The decay of the π^0 meson into two photons is always represented as in Fig. 31. But if particles *1* and *2* have a rest mass, sooner or later, with an increase in the energy of particle O, the momenta of particle *1* will be in the position shown in Fig. 32.

This ellipse has no arrows that point backward. Even the particles that were emitted directly

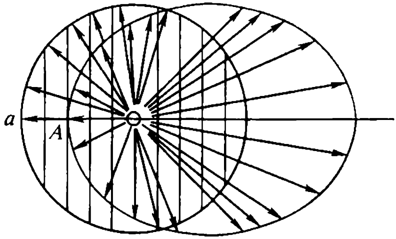

Fig. 31. One arrangement of the two hedgehogs. The meaning of the vertical lines will be explained later.

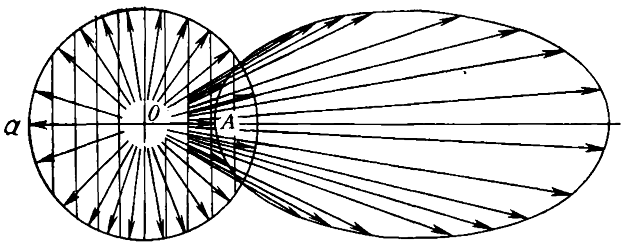

Fig. 32. Another arrangement of the hedgehogs.

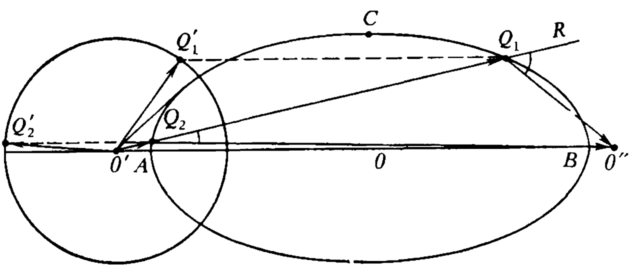

Fig. 33. A problem-solving hedgehog.

backwards (arrow Oa) in the reference frame in which particle O is at rest (the co-moving frame) are found to be travelling forward (arrow OA) in the laboratory frame of reference. Particle O travels so fast in the lab frame (and particle *1* so slow in the co-moving frame of particle O) that, as a result, all the particles *1*, regardless of their direction in the co-moving frame, are turned forward. The fact that the ellipsoid of the decay process $\pi^0 \to \gamma + \gamma$ is depicted by Fig. 31 only means that in this process, however fast the π^0 meson rushed along, we shall always find photons that travelled backward*.

Many other properties of decay processes can be represented clearly and distinctly by drawing a momenta ellipsoid. Let us construct the ellipse for some arbitrary decay $O \to 1 + 2$ when particle O has some definite momentum, specified beforehand (Fig. 33). Now let us do a little work with this ellipse.

Problems

Problem 1. Particle *1* was emitted in decay at the angle 12° with the direction of particle O. What is its momentum?

Solution. We draw a ray from point O' at the angle 12° from the horizontal (Fig. 33). It intersects the ellipse at the two points, Q_1 and Q_2. This means that at an angle of 12° we observe

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* Do not jump to the conclusion that this makes them travel slower. Not at all. Only the magnitude of their momentum decreases, whereas their velocity remains, as before, equal to unity.

both slow particles having a low momentum numerically equal to the line segment $O'Q_2$ and fast particles with a high momentum numerically equal to the segment $O'Q_1$. The same question for the decay $\pi^0 \rightarrow \gamma + \gamma$ (Fig. 34) has the unam-

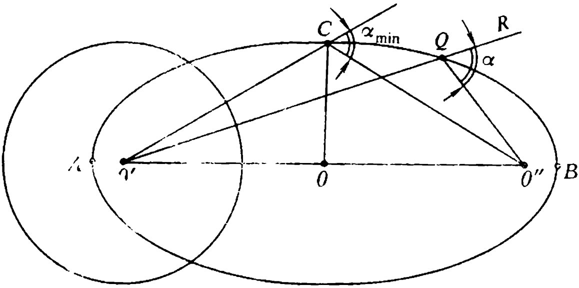

Fig. 34. Decay hedgehog for a π^0 meson in motion.

biguous answer: each angle corresponds to only one momentum.

Problem 2. Following decay, particle *1* is emitted at the angle 12°. What was its direction in the co-moving frame of particle O (see Fig. 33)?

Solution. From point Q_1 we draw a horizontal line up to its intersection with the right-hand side of the circle at point Q_1'. From point Q_2 we draw a horizontal line up to its intersection with the left-hand side of the circle at point Q_2'. (In stretching the circle into an ellipse the right-hand side of the circle became the right half of the ellipse and the left-hand side, the left half.) Vectors $\overrightarrow{O'Q_1'}$ and $\overrightarrow{O'Q_2'}$ indicate the sought-for directions.

Problem 3. What is the greatest angle at which particle *1* is emitted in the decay represented by the ellipse in Fig. 33?

Solution. We draw a tangent to the ellipse from point O' The angle it makes with the p axis is approximately 43°. This is the greatest possible angle of emission of particle *1*.

Problem 4. Under what conditions does particle *1* have a maximum angle of emission?

Solution. When line segment $O'O$ becomes longer than $\overline{AO}$. Since $\overline{O'O}$ $E_1^* p/m$ and $\overline{AO} = Ep^*/m$, this condition is reduced to $E_1^* p \geqslant Ep^*$ or to $p^*/E_1^* \leqslant p/E$. But the ratio of the momentum to the energy is equal to the velocity. Hence, particle *1* has a limiting angle of emission when the velocity of particle O in the lab frame of reference exceeds that of particle *1* in the co-moving frame of particle O:

$$v > v_1^*. \quad (11.9)$$

The higher the velocity, the greater the Lorentz factor, and therefore inequality (11.9) can be replaced by the equivalent (but frequently more convenient) inequality

$$\gamma > \gamma_1^*.$$

The momenta hedgehog is also capable of solving a different type of problem: "How many particles have such and such a momentum? Such and such an angle of emission?" Or "How frequently is such and such a property encountered?" Bear in mind that when we ask, for example, "how frequently do we encounter the emission angle 10°?", we mean an "angle close to 10°" i.e. angles from 9° to 11°, or 9.9° to 10.1°.*

To solve such problems it is necessary to know the properties in the decay of particle *O* when it is *at rest*. It is necessary to know at what angles particle *1* is most frequently emitted when particle *O* is at rest, i.e. the density of the spines at various places on the surface of the round hedgehog. For example, the hedgehog can be isotropic. An isotropic hedgehog has equally dense spines sticking out in all directions. The word "isotropic" means the "same on all sides, or in all directions" If the spines grow more densely on one side of the hedgehog and more sparsely on the other, it will not be an isotropic hedgehog. The decay of a stationary particle is often isotropic. This means that there are equal numbers of spines on equal areas of the sphere's surface, no matter where on the sphere we draw these equal areas.

If the distribution of spines is known for a round hedgehog, we can readily determine how they will be distributed on an elongated hedgehog. A "running" hedgehog (the momenta hedgehog of a moving particle), as is evident, is obtained by the transformation of a "sleeping" hedgehog. We already know how each spine (each momentum) is transformed, therefore we

* The extent of the range of angles is not very important; it is specified on the basis of physical considerations. For example, if the measurement error is ±1°, it is meaningless to take a range of angles less than one degree. On the other hand, if some peak that we are observing has a width of 0.5°, the range should be at least 0.2° as otherwise we simply shall not observe anything.

can imagine how they will be positioned when they are transformed all together, provided we know their positions before transformation.

Let us see how this is done. First of all, let us draw a round hedgehog having equally dense spines pointing in all directions. Do you think it will resemble Fig. 35? No, it will not. A hedgehog is a sphere, not a circle; we only draw it in the form of a circle. Arcs AB and CD are equal

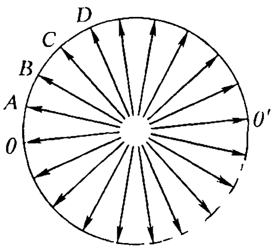

Fig. 35. A pseudoisotropic hedgehog.

in magnitude, but if we revolve them about axis OO', the areas of the spherical zone surfaces they describe differ. But what we want, if the decay is to be isotropic, are equal numbers of spines on equal *areas of zone surfaces*. For this purpose, we must imagine that we cut up the sphere by parallel strokes of a knife into slices of the same thickness. Then there really will be an equal number of momenta, for instance one on an average on each slice. This is so because the area of the spherical surface on each slice, as we were taught in solid geometry, is equal to the circumference of a great circle multiplied by the thickness of the slice (spherical zone). Hence, if the thickness of the slices is the same, the areas of the zone surfaces will also be the same. Consequently, to represent an isotropic decay by arrows on a circle, its diameter should be cut into equal lengths (see Figs. 31

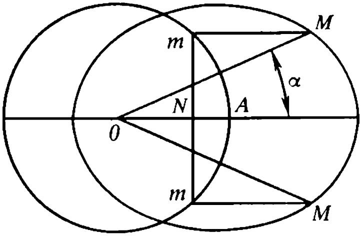

Fig. 36. A diagram for the problem on the number of particles emitted within a given cone.

and 32), instead of cutting its circumference into equal parts, and then one arrow should be drawn into each arc thus obtained (we could just as well draw a hundred arrows, but then it would be impossible to understand anything in the drawing; one arrow per arc is better).

Next we transfer all of these spines according to the previous rule into the ellipse and we obtain a general idea of the density of the spines of a running hedgehog (see Figs. 31 and 32). Can we calculate exactly what percent of all the particles is emitted forward, within the angle α? This is quite simple; we merely projected point M onto the circle (Fig. 36). Then the ratio of the area of the spherical surface of the spherical segment mAm to the whole surface of the sphere is the sought-for percent:

$$\% = \frac{2\pi R \times \overline{NA}}{2\pi R \times 2R} = \frac{\overline{NA}}{2R} \ (\times 100\%).$$

Next, we shall attack a more interesting problem.

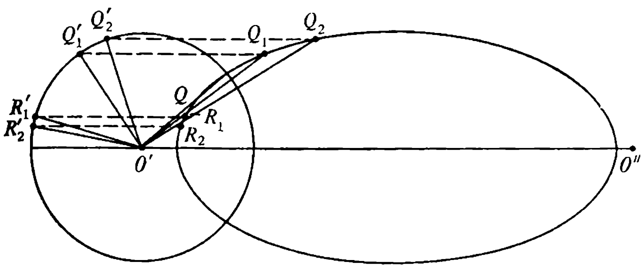

Fig. 37. A diagram for the problem on the properties of the limiting angle.

Problem 5. Prove that the hedgehog's spines grow more densely at the limiting angle.

Solution. We are asked to prove that if particle *1* has a limiting angle, it is most frequently found to be emitted, in the laboratory frame of reference, at an angle close to the limiting one.

We draw a tangent $O'Q$ to the ellipse from point O' (Fig. 37). The angle it makes with the axis of the ellipse is the limiting angle. Next we draw the secant $O'Q_1$ at the small angle β with the tangent, and another secant $O'Q_2$ at the same angle β to the first secant. The solid angles bounded by these closely spaced cones are approximately equal.* But let us determine on the spheri-

* The cones and solid angles referred to here are the ones formed by the revolution of plane figures $QO'Q_1$ and $Q_1O'Q_2$ about axis $O'O''$.

cal surface of which solid angle there are more spines. We project points Q_1, R_1, Q_2 and R_2 horizontally from the ellipse to the circle. The particles emitted into the conical solid formed by the revolution of $QO'Q_1$ are the ones that are emitted into the conical solid $Q_1'O'R_1'$ in the co-moving reference frame of particle O; those emitted into the conical solid $Q_1O'Q_2$ are the ones emitted into the conical solids $R_1'O'R_2'$ and $Q_1'O'Q_2'$. The first conical solid is incomparably wider than the last two simply because the direction of secant $O'R_1Q_1$ is close to the direction of the arc R_1QQ_1 at the point of tangency. We have proved what we had set out to prove: the most frequently encountered particles in decays are those emitted at an angle close to the limiting one.

Problem 6. Prove that if the particle X is heavier than the π meson in a reaction $\pi + p \rightarrow$ $\rightarrow p + X$, then the proton will always have a limiting angle of emission in the lab frame, regardless of the energy of the π meson.

Solution. Let us represent this reaction in the form $\pi + p \rightarrow O \rightarrow p + X$, i.e. we shall assume that first π and p form the particle O with the energy $E_O = E_\pi + m_p$ and the momentum p_π, and then particle O with the mass

$$m_O = \sqrt{E_O^2 - p_\pi^2} = \sqrt{m_\pi^2 + m_p^2 + 2E_\pi m_p}$$

decays to particles with the masses m_p and m_X. The energy of proton p in the co-moving frame of particle O is

$$E_p^* = \frac{m_O^2 + m_p^2 - m_X^2}{2m_O}.$$

If we wish to prove that the proton has a limiting angle of emission we must show (see Problem 4) that the relativistic factor of particle O in the laboratory reference frame exceeds the relativistic factor of the proton in the co-moving frame of particle O. By the definition of the relativistic (Lorentz) factor (see Chapter 4)

$$\gamma_O = \frac{E_O}{m_O} \text{ and } \gamma_p^* = \frac{E_p^*}{m_p}.$$

Next we calculate the difference

$$\gamma_O - \gamma_p^* = \frac{E_O}{m_O} - \frac{E_p^*}{m_p} = \frac{E_O}{m_O} - \frac{m_O^2 + m_p^2 - m_X^2}{2m_O m_p}$$
$$= \frac{2E_O m_p - m_O^2 - m_p^2 + m_X^2}{2m_O m_p}.$$

Into this expression we substitute the values of E_O and m_O^2 and obtain

$$\gamma_O - \gamma_p^*$$
$$= \frac{2(E_\pi + m_p) m_p - (m_\pi^2 + m_p^2 + 2E_\pi m_p) - m_p^2 + m_X^2}{2m_O m_p}.$$

Removing the brackets and adding and subtracting as required we finally obtain

$$\gamma_O - \gamma_p^* = \frac{m_X^2 - m_\pi^2}{2m_O m_p}$$

But, according to the conditions of the problem, $m_X > m_\pi$, hence $\gamma_O > \gamma_p^*$. It follows that the creation of a heavy particle leads to the emission of the proton at an acute angle.

If we compare this with the results of the preceding problem, it becomes clear that in the reaction $\pi + p \rightarrow p + X$, it is best to locate the proton trap at the limiting angle of emission. This is exactly the direction in which the proton is most readily emitted when $m_X > m_\pi$. The magnitude of the limiting angle is determined by the energy of the π meson and the mass of the awaited particle X. We shall return to this discussion again in Chapter 13.

Problem 7. Particle *1* was emitted at the angle 12°. What angle can be expected between the paths of emission of particles *1* and *2* (see Fig. 33)?

Solution. The sought-for angle is equal to either $\angle RQ_1 0''$ or to $\angle Q_1 Q_2 0''$ (two answers).

The Discovery of the π^0 Meson

Problem 8. It is known that π^0 mesons decay to two photons. If the π^0 mesons have the momentum p, are all angles possible between the paths of the emitted photons?

Solution. The momentum ellipse for the decay $\pi^0 \to \gamma + \gamma$ has the form shown in Fig. 34. As a matter of fact (one that you can check!), the radius of the circle is $p^* = m_\pi/2$, the major semiaxis is $\overline{0B} = (E/m_\pi)(m_\pi/2) = E/2$, the vector $\overrightarrow{0'0''}$, as always, is equal to $\mathbf{p}$, so that $\overline{00''} = \overline{0'0} = p/2$. The momentum p is always less than the energy E and therefore $0'$ and $0''$ are certainly within the ellipsoid $(\overline{00''} < \overline{0B})$*. The angle $RQ0''$ is the angle α we are interested in between the paths of the photons. When point Q coincides with point B, $\alpha = 180°$ As point Q

* It can be shown that $0'$ and $0''$ are the focuses of the ellipsoid.

moves to the left, angle α is reduced and at point C it reaches its minimum value. Hence, not any angle (for instance, angle 0°) is possible between the photons.

What, then, is this minimum possible angle equal to? Look at the triangle $O'CO''$ It is an

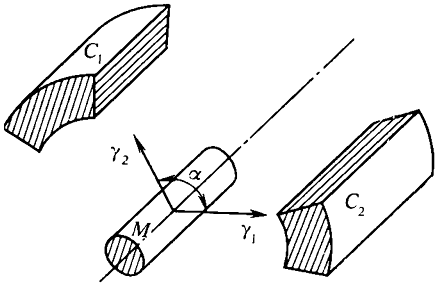

Fig. 38. Discovering the π^0 meson.
The dot-and-dash line is a beam of high-energy photons; the π^0 mesons were produced in target M. C_1 and C_2 are counters that register the photons produced in the decay of the π^0 meson. The plane of the two counters and the target is perpendicular to the axis of the beam.

isosceles triangle. Hence, its exterior angle at the vertex is equal to the sum of the interior angles at the base. Thus, angle $CO'O = \alpha_{min}/2$. Then

$$\tan\frac{\alpha_{min}}{2} = \frac{\overline{CO}}{\overline{OO'}} = \frac{p^*}{\overline{OO'}} = \frac{m_\pi/2}{p/2} = \frac{m_\pi}{p}.$$

Consequently,

$$\alpha_{min} = 2\,\mathrm{arc\,tan}\,\frac{m_\pi}{p}. \qquad (11.10)$$

Try, as well, to derive the formula $\cos(\alpha_{min}/2) = v_\pi$, where v_π is the velocity of the π meson.

The property of photons in π^0 meson decay of not being emitted in too close pairs made it possible to prove that the π^0 meson really exists. Under conditions in which the conjectural π^0 mesons had approximately the same momentum and decayed at the same point, a cone of emitted photons scattered in all directions (Fig. 38). They were registered by two counters. Simultaneous clicks of the two counters indicated that both photons were emitted at the same time, i.e. were produced by a single π^0 meson. When the experimenters began to bring the counters closer together, simultaneous clicks were drastically reduced beginning with a certain angle. Making use of equation (11.10) and the value of this angle, they calculated the mass of the invisible source of the pair of photons. It was found to be close to the mass of the π^+ and π^- mesons. This is how the π^0 meson joined the π^+ and π^- mesons.

A New Method of Solving a System of Equations

Problem 9. Solve the system of equations

$$\sqrt{x^2+m_1^2} \quad \sqrt{y^2+m_2^2}=E, \qquad (11.11)$$

$$x+y=p \qquad (11.12)$$

Solution. If x and y are assumed to be momenta of two particles, *1* and *2*, $\sqrt{x^2+m_1^2}$ and $\sqrt{y^2+m_2^2}$ can be assumed to be their energies. Then equation (11.11) expresses the conservation of energy and equation (11.12) the conservation of momentum in the decay of particle O to two particles emitted toward the same side. The mass of particle O is known: $m = \sqrt{E^2 - p^2}$. There is nothing to prevent us then from drawing the circle and ellipse on the basis of the values of m, m_1, m_2, p and E. Assume that this is the ellipse of Fig. 34. Then $x = \overline{O'A}$ and $y = \overline{AO''}$. This is one solution; the other is: $x = \overline{O'B}$ and $y = \overline{BO}$. As is evident, the solution requires only the extreme points A and B, rather than the whole ellipse. Express the analytical formulas for x and y (the analytical formulas for $\overline{OA} = \overline{OB}$ and $\overline{O'O}$ are known to you). If the ellipse had turned out to be like the one in Fig. 33, there would not have been a positive solution (there $p = x - y$). Making use of this fact, investigate the solution with respect to its positiveness.

This last problem shows that if you know the Lorentz transformations, you can use them to solve purely algebraical problems from high-school mathematics. As a matter of fact, the graphical method is not at all necessary here; the system of equations (11.11) and (11.12) can be solved without it. For this purpose, you carry out the Lorentz transformations on the system (11.11) and (11.12) in the co-moving reference frame of particle O, solve equations (11.1) in this frame, and then carry out a reverse Lorentz transformation.

Using a system of ordinary high-school equations as an example, it becomes evident that in theoretical physics one sometimes manages to solve equations without solving them. For this purpose we assigned physical meanings to the quantities in the system of equations (11.11) and (11.12). Then we recalled the properties of these quantities (in our case, how they change in the Lorentz transformations) and wrote down the answer out of hand on the basis of physical laws that we know.

Try to think up other similar systems of equations that can be solved on the basis of physical considerations. Certain more complex examples of this kind are given near the end of this book.

Chapter 12

The Story of How the π^0 Meson Was Found in Cosmic Rays

This happened way back in 1950. By that time physicists were of the opinion that the third member of the π meson family, the π^0 meson, was somewhere on the run and that it would show up if properly searched for. They guessed that it decays to two photons and that the abundance of electrons and positrons in cosmic rays is due, precisely, to the abundance of photons produced by the π^0 mesons. As the photons pass close to nuclei, they create a great many electron-positron pairs.

Many physicists got down to work on this problem at that time. Having no hope of seeing the elusive π^0 meson with their eyes, they began to seek such features of its decay to two photons that would obviously lead to the conclusion that the source of the photons is the π^0 meson, and no one else. Almost at the same time, several groups reported on the results of their experiments. After this, no more doubt remained concerning the existence of π^0 mesons. We gave an account of the kinematic idea behind one such experiment in Problem 8 of Chapter 11.

Much more interesting, however, is the kinematic idea on which another xperiment was based. It was conducted by three] hysicists. By means of a balloon they elevated photographic plates for registering photons produced by π^0 decays to an altitude of 21 km. They managed to prove the existence of π^0 mesons in cosmic rays by measuring the energy of only separate odd photons. If they had succeeded in registering a pair of photons at the same time, they would be readily convinced of the existence of π^0 mesons. But how much wit and ingenuity was required to discover the π^0 meson by measuring the energy of a single photon in the decay reaction $\pi^0 \rightarrow \rightarrow \gamma + \gamma$, not knowing whether there is a second one somewhere, and without paying any attention to the directions of the observed photon or the invisible π^0 meson.

"Kinematics for the Lightly Equipped"

Let us make an attempt to comprehend this matter. It will be a long story because we have entered a new branch of kinematics that we shall call "kinematics for the lightly equipped" If you are equipped for measuring all the momenta and directions of particles participating in a decay or collision, you have no use for this new branch. But if you are capable of observing only a part of the particles, if you can measure only angles without knowing the energies, or only the energies without knowing the angles of emission (in which cases it is especially difficult to discover something), the laws of kinematics are able to render an incalculable service to physicists. This is the most interesting part of kinematics. There is much here that we still have insufficient knowledge of, but what we do know is always elegant indeed.*

Decay Isotropy

To begin with let us recall that isotropic decay is understood to be decay that is equally frequent in all directions. Decay isotropy, as we now know, is manifested by the occurrence with equal frequency of momenta arrows in layers (or slices) of equal thickness (into which we have divided the momenta sphere) (Fig. 39). Such slices are called spherical zones, as mentioned previously, and we are referring only to their spherical surface.

During the search for the π^0 meson it was already clear that its decay to photons must be isotropic. This led to an interesting conclusion. Let us draw a momenta sphere for the decay of π^0 mesons at rest. All the photons found in a given narrow slice have the same longitudinal component of momentum; it is simply the distance of the slice from the centre of the sphere. Consequently, decay isotropy also means that any longitudinal

* We have already seen how "kinematics for the lightly equipped" functions in the experiment described in Chapter 11 (Problem 8): the π^0 meson was discovered without measuring the energies of the photons.

components of the momenta are encountered with equal frequency (Fig. 40).

What will happen if the momenta of all the π^0 mesons are the same and are equal to p? Then the energy E_1 of one of the photons (*1*) can be found by the Lorentz formula

$$E_1 = \gamma E_1^* + \gamma v p_{\parallel}^*. \tag{12.1}$$

This formula contains the fix d quantities (see

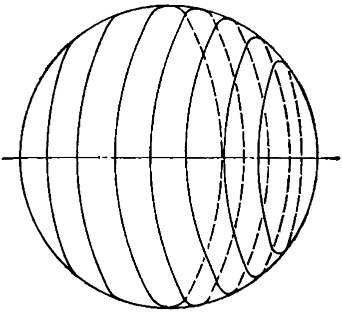

Fig. 39. The concept of isotropy.
How to slice up a sphere into spherical zones of equal spherical surface areas.

Problem 8 of Chapter 11):

$$\gamma = \frac{E}{m_\pi}, \quad E_1^* = \frac{m_\pi}{2} \text{ and } \gamma v = \frac{p}{m_\pi}. \tag{12.2}$$

Only the longitudinal component of the photon's momentum varies from decay to decay. But we already know how it varies: any permissible values are encountered with equal frequency. Hence, any values of the energy E_1 of photon *1* should be encountered with equal frequency. It is a matter of fact that we obtain E_1 by multiplying $p^*_{||}$ by a constant value and adding another constant value. Thus

$$E_1=\frac{E}{2}+\frac{p}{m_\pi}p^*_{||}. \qquad (12.3)$$

The component $p^*_{||}$ varies from $-m_\pi/2$ to $+m_\pi/2$ (the radius of the sphere is $m_\pi/2$), and within these limits there are equal chances for

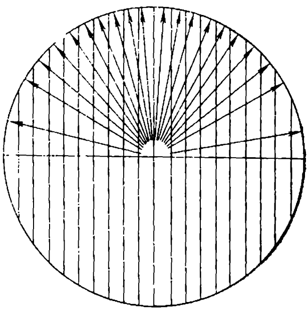

Fig. 40. Isotropic decay.

The momenta hedgehog is sliced into zones, each having the same number of momenta. The diagram shows that in isotropic decay all values of the longitudinal components of the momenta are encountered on an average with equal frequency.

encountering any value of $p^*_{||}$. Hence the energy E_1 varies within the limits from $(E-p)/2$ to $(E+p)/2$ (as is evident if you substitute the extreme values of $p^*_{||}$ into equation (12.3)). Again, within these limits, any value of the energy can be expected with equal probability.

We have proved an important theorem (valid not only for the decay $\pi^0 \rightarrow \gamma+\gamma$): if particles at rest decay isotropically and if their momenta are the same in magnitude, then all possible energies of the particles produced in the decay are equally probable. The limits within which the energy of the photons varies depend on E and p, the energy and momentum of the π^0 mesons.

This suggests the following practical rule for discovering the π^0 meson: if the momentum **p** of the π^0 meson is specified, then we measure the energy of the photons and accumulate as many of such measurements as possible. Next we multiply the energy of the most "energetic" of such photons (its anticipated energy is close to $(E + p)/2)$ by the energy of the "laziest" photon (its energy will be found to be close to $(E - p)/2)$ and we shall obtain $m_\pi^2/4$. From this we find the mass of the π^0 meson. Hence we can discover the π^0 meson by observing only a single photon from each decay reaction and disregarding its direction.

Unfortunately, there are no such ideal conditions in investigating cosmic rays. There we cannot expect all the π^0 mesons to have one and the same energy. What can be done? Let us try to cope with this new difficulty.

We just mentioned that if all the cosmic mesons had the same energy E, we would encounter, among the photons, any values of energy from $(E - p)/2$ to $(E + p)/2$ with equal frequency. If we plot the energy of the photons along the horizontal axis and the number of photons having this energy along the vertical axis, the chances of encountering any energies will be equal. The dependence of the number of photons on their energy is represented in Fig. 41 (the graph reminds one of a soccer goal). If among the cosmic mesons we were to hnd mesons of some other energy E', the dependence of the number of photons they produce on the energy of the photons would be represented by other soccer goals, whose

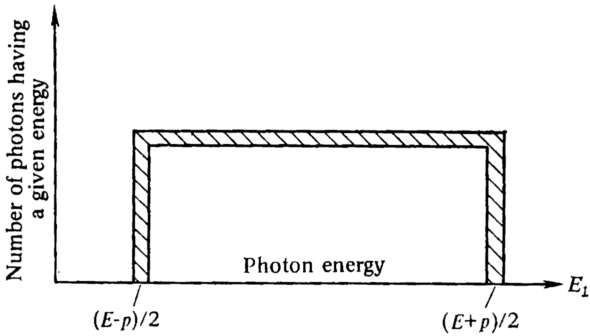

Fig. 41. Dependence of the number of photons on their energy.
All photon energy values from (E $(E+p)/2$ are probable.

goal posts would stand at the points $- p')/2$ and $(E' + p')/2$, etc. All the mesons in cosmic rays can be divided into groups having close values of energy, and each group produces photons with a definite energy range, all the energies within the range being found with equal frequency. We thus obtain a great many soccer goals of various widths and positions, as if each group of π^0 mesons with close values of energy stuck to their own rules and erected their own goals on the cosmic soccer field (Fig. 42).

Of interest is the fact that no matter how many such goals you erect on the axis of photon energies they always have at least one common point. If a forward makes a goal kick to point $E_1 = m_\pi/2$, he will certainly score simultaneously

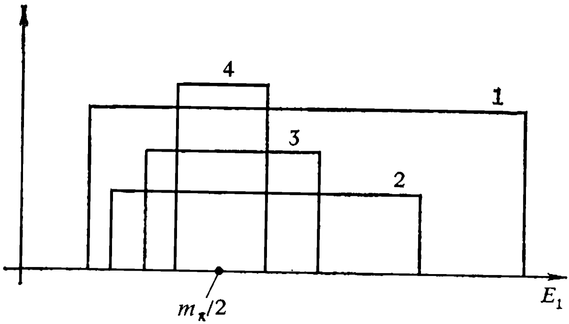

Fig. 42. A case when the energies are different.

Four soccer goals, corresponding to four imaginary groups of π^0 mesons (the areas of the goals are proportional to the population of each group).

in all the goals of all the teams (provided their goalkeepers do not prevent him). The energy $m_\pi/2$, as we have seen, is the geometric mean of the positions of the goal posts of any team. Being a mean it is, of course, between the posts, not outside.

This is a vital fact. We have thereby proved that if π^0 mesons of all possible energies shower from every quarter, among the photons they decay to, ones with the energy $m_\pi/2$ will be found most frequently. Other energies are not encountered so often; it is always possible to specify such momenta to the π^0 mesons that they will not be able to produce photons of the other energies.* And you can be sure that photons with the energy $m_\pi/2$ can be produced by any π^0 mesons, even ones at rest (see equation (9.10)). This, then, is one method for you to use to prove

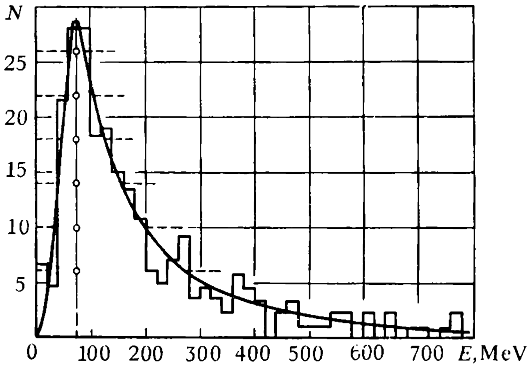

Fig. 43. Detection of π^0 mesons in the stratosphere. The stepped line shows how frequently photons of a specified energy are encountered at an altitude of 21 km. Plotted along the horizontal axis are the energies of the photons in MeV, (equal to one thousandth of a GeV); plotted along the vertical axis are the numbers of times photons of a given energy are encountered. The smooth curve is obtained by smoothing out the stepped line. The maximum was at the value 70 MeV.

that there are π^0 mesons in cosmic rays. It consists in finding the energy that the cosmic pho-

* This can readily be understood; the limits that enclose all the possible energies of photons produced by a π^0 meson of the energy E have a certain interesting property: the higher the meson energy E, the greater the upper limit and the less the lower limit. At $E = m_{\pi^0}$ the upper and lower limits are equal to $m_{\pi^0}/2$. Then, as E increases, the left goal post moves to the left and only to the left, and the right post, only to the right.

tons most frequently have. This was found to be 70 MeV (Fig. 43). This indicates that the mass of their source is 140 MeV, which is exactly the mass of the π^+ or π^- meson.

But there is a more accurate method as well. If, among the cosmic π^0 mesons there were mesons of only four different energies, $E^{(1)}$, $E^{(2)}$, $E^{(3)}$ and $E^{(4)}$, each energy would have its goal, *1*, *2*, *3* and *4*, whose width would be determined by the magnitude of E, and whose area, by the number of mesons having this energy E. If we count how many times we encounter a photon with one or another energy, we will not obtain four curves, *1*, *2*, *3* and *4*, but will obtain their sum in the form of a stepped pyramid of ancient Mexico (Fig. 44). If there are a great many soccer goals, rather than four, the pyramid is converted into a hill, having the outline of a continuous smooth curve with its summit at the point $E_1 = m_\pi/2$, but we are concerned now only with the bottom ends, or feet, of this hill. We know that the smooth rise to the summit is simply the steeply trimmed edges of the goals that we piled up on one another. The positions of these bottom ends are related to $m_\pi/2$ by the equation

$$\sqrt{\text{left end} \times \text{right end}} = \frac{m_\pi}{2} \qquad (12.4)$$

As we can see, the mass of the π meson is again expressed in terms that are dear to the heart of true soccer fans. After counting up the frequency with which we encounter various cosmic photon energies, we plot a curve on the basis of these

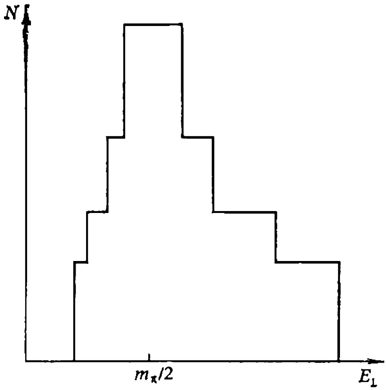

Fig. 44. Overall photon distribution in the four groups of π^0 mesons.

The steepness of the ascent for this pyramid depends upon the number of π^0 mesons with the corresponding energy.

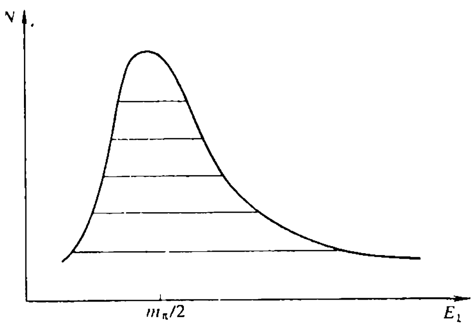

Fig. 45. Photon spectrum obtained in the limit by increasing the number of groups.

The steepness of the ascent for this hill depends upon the nu of π^0 mesons with the corresponding energy.

data (frequency as a function of the energy). Then we slice the "hill" we obtain by several horizontal lines (Fig. 45), measure their intersections with the slopes of the "hill", multiplying their coordinates together pairwise, take the square roots of the products, and then calculate their average over the whole cross section. This will yield a more precise value of one half of the mass of the source of photons. It is more precise because the calculations include *all the photon energies*, not only the most frequently encountered ones.

What other facts can we discover by making a topographical survey of this hill? Recall that the hill was obtained from a stepped pyramid. Each step was formed by π^0 mesons of a definite energy (or, to be more exact, by photons produced by π^0 mesons of a definite energy). Some steps were high, indicating that there were many π^0 mesons with that energy. Other steps were low, indicating that there were not many π^0 mesons with that energy. Consequently, from the smoothed pyramid (i.e. hill) we can determine from the steepness of the slope how many π^0 mesons of a certain energy we had. As a matter of fact, by measuring the steepness of the hill at equal intervals, the investigators found out how many π^0 mesons and with what energies appear in cosmic rays (Fig. 46). The energy was calculated by drawing the horizontal median of the hill. The arithmetic mean of the coordinates yielded one half of the energy of the π meson, just as the geometric mean had yielded one half of its mass.

This experiment is almost forgotten today. The age of accelerators has begun, and exotic adventures in high-altitude cosmic-ray recording stations, ascents in balloons and similar methods of research no longer interest physicists. They require controllable and reproducible conditions for their experiments. But the discovery that we

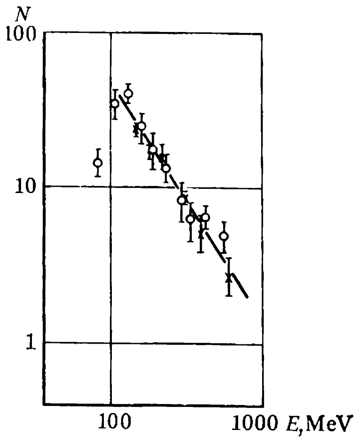

Fig. 46. Energy spectrum of π^0 mesons in the stratosphere. Taken from the same experiment in the stratosphere, the curve shows how many π^0 mesons of any specified energy are found at an altitude of 21 km. The circles are the results obtained in measuring the steepness of the slopes (see Fig. 43) with their errors. The energies E of the π^0 mesons are plotted along the horizontal axis in MeV; the number of cases N, along the vertical axis.

have narrated here is a triumph of kinematics, one that yielded an elegant and mathematically pure result. The accomplishments of kinematics may, of course, appear modest on the background of brilliant new physical theories, advanced every now and then, that penetrate deep into the dynamics of interaction, making an attempt to solve the cardinal problems of physics. Physicists do not care much for absolutely correct theories and the onclusions they lead to. Their attention is attracted by controversial problems, inexplicable facts, limits beyond which established laws are no longer valid. This is exactly where the still unknown is concealed. But, in a study of the history of science, gems are found now and then that fill the heart of a scientist with pride for his science. Such is the reasoning of Nicolas Léonard Sadi Carnot on heat engines or the discovery and explanation of the effect named after Rudolf Ludwig Mössbauer. Such also is the discovery of the π^0 meson in cosmic rays.

The Colour of the Atom in Motion

All that we have just said about π^0 mesons and photons also pertains to atoms and the light they radiate. There this phenomenon—the dependence of the energy of the photon on the direction it is emitted with respect to the motion of the emitter—has been known for a long time. But there we speak of the frequency of the light rather than the energy of the photon, and the phenomenon is called the Doppler effect, after Christian Johann Doppler. When light from the travelling atom is radiated in the direction of travel of the atom, its frequency in a stationary reference frame seems higher; when in the opposite direction, it seems lower. The colour of a travelling atom (if we employ the language of Chapter 10) is not the same as that of a stationary one, and it is different in front than in back. In front it is of a violet colour and in back it is red. A baboon has the same colour scheme, but hardly for the same reason.

The Lorentz transformation formula (12.1) for photons is

$$E_1 = \gamma E_1^* + \gamma v p_{\parallel}^*,$$

which is the formula for the Doppler effect. The energy E_1^* of photons emitted by an atom at rest is equal to their momentum p^*. Moreover, the energy of a photon is related to the frequency of the light by the equation E_1^* $p^* = h\nu^*$. When a photon is emitted (in the co-moving reference frame of the atom) at the angle θ^* to the direction of motion, $p_{\parallel}^* = p^* \cos\theta^* = h\nu^* \cos\theta^*$. The equation for the frequency of the light visible in the laboratory frame of reference takes the form

$$\nu = \nu^* (\gamma + \gamma v \cos\theta^*).$$

Recalling that γ here is the relativistic (Lorentz) factor, we find that in front ($\theta^* = 0$) the atom will seem to be emitting light of the frequency

$$\nu_{max} = \nu^* \sqrt{\frac{1+v}{1-v}}, \tag{12.5}$$

and in back, of the frequency

$$\nu_{min} = \nu^* \sqrt{\frac{1-v}{1+v}}. \tag{12.6}$$

Assume that we have before us an incandescent gas whose atoms (in their co-moving frames of reference) emit light of a single frequency ν^*, but travel in all possible dir ctions with the same velocity v. Then, instead of the frequency ν^*, we shall see light of all frequencies from ν_{min} to ν_{max}, with all the frequencies being equally common (remember the soccer goals). If, however, the gas contains atoms travelling at all velocities, we shall most often encounter light of the frequency ν^*, because at any velocities $\nu^* = \sqrt{\nu_{min} \times \nu_{max}}$.

But in an ordinary incandescent gas the spread of frequencies is negligible. The velocity v of the gas atoms is so much less than unity, that practically even in the depths of the sun, where the temperature T is about ten million degrees, the velocity v equals 0.0016*, which corresponds to the same shift in frequency. For the colour of the gas to spread, due to the motion of the atoms, by one octave to the right and to the left ($\nu_{max} : \nu^* \;\; \nu_{min} = 2 \;\; 1 : 1/2$), the temperature T of the gas should reach 4×10^{13} degrees!

But what is beyond the power of an incandescent gas is easily accomplished by the γ quanta from the cosmic-ray π^0 mesons. High velocities are reached in a gas as a result of the chaotic exchange of impacts with other atoms. This is a very inefficient way of gaining velocity. The closeness of the velocities of cosmic-ray particles to unity proves that the mechanism of their acceleration was an entirely different one, and that a space accelerator is in operation somewhere in the depths of the universe. Otherwise, the frequency of the photons in cosmic rays (see

* This figure was obtained from the equation $\frac{mv^2}{2} = \frac{3}{2}kT$, where $k = 8.62 \times 10^{-14}$ GeV/deg.

Fig. 43) would not have a spread of several octaves.

Nevertheless, a Doppler effect of octave span for visible light is also well known to astronomers. But this is not the spread in the spectrum in both directions, discussed above, but a shift in only one. In the spectra of all distant stars and nebulas, all the known lines are shifted toward the red end of the spectrum. These stars are evidently recessing from us at velocities comparable to that of light. We look from in back at the light radiated by the atoms of these stars ($\theta^* = 180°$) in which case equation (12.6) is valid. The source of energy of the accelerator that accelerates, not protons, but whole galaxies to a velocity $v \approx 1$ is something that we shall not discuss here.

The $\Lambda\eta$ Resonance

But let us return to the earth from the depths of space. We shall visit the city of Dubna near Moscow to see how "kinematics for the lightly equipped" suggested, to the physicists working there, the possibility of the existence of a new, previously unknown, resonance particle, the resonance between the Λ^0 hyperon and the η^0 (eta-zero) meson. Like the π^0 meson, this η^0 meson can decay to two photons, and it is better to deal with this matter now, before the properties of a decay to two photons has been effaced from our memories.

The following experiment was conducted at the High-Energy Physics Laboratory in Dubna. A chamber, filled with liquid propane was irradiated by high-energy π mesons and, among the great many reactions that occurred, only cases were selected in which a Λ^0 hyperon and at least one photon were produced. When such photographs were found, the energy and direction of the Λ^0 hyperon and photon were measured, and the effective mass $m_{\Lambda\gamma}$ of the $\Lambda\gamma$ system was calculated. After collecting over a hundred cases with Λ and γ particles, and counting which values of $m^2_{\Lambda\gamma}$ are most frequently encountered, the physicists discovered the following interesting feature. In the range $1.7\ \text{GeV}^2 \leqslant m^2_{\Lambda\gamma} \leqslant 1.9\ \text{GeV}^2$, the values of $m^2_{\Lambda\gamma}$ are encountered too often, more often than they should be according to the laws of chance. It was simplest, of course, to conclude that a resonance particle had been discovered in the $\Lambda\gamma$ system. But another possibility could not be ruled out: that the observed phenomenon was actually a resonance of the Λ hyperon with some other particle, for instance, a π^0 meson, and that we observe only one of the two photons that a π^0 meson decays to:

$$X \to \Lambda^0 + \pi^0 \\ \qquad\qquad \lfloor\!\to \gamma_1 + \gamma_2. \qquad (12.7)$$

The particles Λ^0 and π^0 could have been created, of course, without forming a resonance, but then the effective mass $m_{\Lambda\gamma}$ could have any value whatsoever. If, however, reaction (12.7) was occurring, the effective mass of the Λ^0 hyperon and one of the photons could vary within only restricted limits. Moreover, the less the mass of particle X differs from the sum of the masses of particles Λ^0 and π^0, the more restricted the limits would be. Maybe, the physicists reasoned, this is precisely why so many values of $m^2_{\Lambda\gamma}$ were found in the range from 1.7 to 1.9 GeV^2.

To convince ourselves that such reasoning is correct, we must determine within what limits the effective mass of the $\Lambda\gamma$ system in reaction (12.7) can vary if the mass of particle X is specified. This should not be difficult, we have just discussed the kinematics of $\pi^0 \to \gamma_1 + \gamma_2$ decays in all their minutest details.

The effective mass of the $\Lambda\gamma_1$ pair is uniquely related, in the co-moving reference frame of particle X, to the energy of the remaining photon γ_2 as follows:

$$m^2_{\Lambda\gamma_1} = \text{(according to the definition of effective mass)}$$

$$= (E_\Lambda + E_{\gamma_1})^2 - (\mathbf{p}_\Lambda + \mathbf{p}_{\gamma_1})^2 = \text{(according to the conservation laws)}$$

$$= (m_X - E_{\gamma_2})^2 - (0 \quad \mathbf{p}_{\gamma_2})^2 = \text{(according to the laws of algebra)}$$

$$= m_X^2 - 2m_X E_{\gamma_2} + E^2_{\gamma_2} - p^2_{\gamma_2} = (E = p \text{ for the photon})$$

$$= m_X^2 - 2m_X E_{\gamma_2}. \qquad (12.8)$$

Hence, the minimum (maximum) value is obtained for $m_{\Lambda\gamma_1}$ when, in the co-moving reference frame of particle X, the energy of the photon γ_2 is maximal (minimal). And when is the energy of the photon produced by the π^0 meson maximal (minimal)? This was discussed in deriving equation (12.3): the energy of the photon is restricted by the limits $(E - p)/2$ and $(E + p)/2$, where E and p are the energy and momentum of the π^0

meson. In the co-moving reference frame of particle X, these quantities have quite definite values because X decays to two particles, Λ^0 and π^0, and the energies of the products of the decay of a particle at rest are fixed values. Thus, we can continue our mathematical operations with equation (12.8):

$$m^2_{\Lambda\gamma_{\min}^{\max}} = m_X^2 - 2m_X E_{\gamma_2 \,{\min \atop \max}} = m_X^2 - 2m_X \frac{E_\pi \mp p_\pi}{2}$$

$$= m_X (m_X - E_\pi \pm p_\pi).$$

But, according to the conservation laws,

$$m_X - E_\pi \quad E_\Lambda \text{ and } p_\Lambda \quad p_\pi,$$

and then, finally,

$$m^2_{\Lambda\gamma_{\min}^{\max}} = m_X (E_\Lambda \pm p_\Lambda). \tag{12.9}$$

This is exactly the equation we were seeking; it answers the question about the limits within which the square of the effective mass of the $\Lambda\gamma$ pair can vary if γ is produced in the decay $\pi^0 \to \gamma + \gamma$ and the mass of the $\Lambda\pi^0$ pair is specified. But we do not know this mass and cannot check, as yet, whether we will obtain the values 1.7 and 1.9. Maybe we can try to solve the reverse problem: what should the mass m_X be in order for the maximum value to be 1.9 GeV^2 and the minimum, 1.7 GeV^2? In other words, we are to find m_X from the system of equations

$$\left.\begin{aligned} m_X (E_\Lambda + p_\Lambda) &\quad 1.9, \\ m_X (E_\Lambda \quad p_\Lambda) &\quad 1.7. \end{aligned}\right\} \tag{12.10}$$

As a matter of fact, even more can be determined from the system of equations (12.10), because there are two equations, not one. Let us try to determine, besides m_X, m_π as well. The latter is the mass of the particle that decayed to two photons. For this purpose we write equations (12.10) in their general form:

$$\left.\begin{aligned} m_X(E_\Lambda + p_\Lambda) &= m^2_{\Lambda\gamma\,\mathrm{max}}, \\ m_X(E_\Lambda - p_\Lambda) &= m^2_{\Lambda\gamma\,\mathrm{min}}. \end{aligned}\right\} \qquad (12.11)$$

First we multiply one equation by the other and obtain

$$m_X^2(E_\Lambda^2 - p_\Lambda^2) = m^2_{\Lambda\gamma\,\mathrm{max}} \times m^2_{\Lambda\gamma\,\mathrm{min}}$$

or

$$m_X^2 m_\Lambda^2 = m^2_{\Lambda\gamma\,\mathrm{max}} \times m^2_{\Lambda\gamma\,\mathrm{min}}, \qquad (12.12)$$

from which

$$m_X^2 = \frac{m^2_{\Lambda\gamma\,\mathrm{max}} \times m^2_{\Lambda\gamma\,\mathrm{min}}}{m_\Lambda^2} = \frac{1.9 \times 1.7}{1.115^2} = 2.6\ \mathrm{GeV}^2.$$

Now we can readily find m_π as well. For this purpose we add the two equations (12.11):

$$2m_X E_\Lambda = m^2_{\Lambda\gamma\,\mathrm{max}} + m^2_{\Lambda\gamma\,\mathrm{min}}$$

and recall equation (9.8) for E_Λ, the energy of one of the two particles to which particle X decays:

$$2m_X E_\Lambda = m_X^2 + m_\Lambda^2 - m_\pi^2.$$

Then we obtain

$$m_\pi^2 = m_X^2 \quad m_\Lambda^2 - m^2_{\Lambda\gamma\,\mathrm{max}} - m^2_{\Lambda\gamma\,\mathrm{min}}. \qquad (12.13)$$

If we substitute the value of m_X^2 we have just found, as well as the values of m_Λ^2, $m^2_{\Lambda\gamma\,\mathrm{max}}$ and $m^2_{\Lambda\gamma\,\mathrm{min}}$, we obtain $m_\pi^2 = 0.24\ \mathrm{GeV}^2$, i.e. the mass of the π^0 meson turns out to be equal to 0.49 GeV. What a strange π^0 meson we have found; almost four times as heavy as the ordinary ones! Obviously, it is no π^0 meson at all, but some other kind of particle that also decays to two photons.

We do know of such a particle; it is the η^0 meson with the mass 0.55 GeV. It follows that if the surplus of $m^2_{\Lambda\gamma}$ values in the range from 1.7 to 1.9 GeV^2 is due to a two-stage process of the type (12.7), the particle X should have the mass $\sqrt{2.6} = 1.61$ GeV, and the particle that we thought was a π^0 meson is actually an η^0 meson.

The obtained assessments of the masses are crude and approximate because the limits themselves (1.7 and 1.9 GeV^2) were estimated by eye. In any case, the conducted calculations indicated that maybe a particle, as yet undiscovered, may exist with a mass slightly exceeding the sum of the masses of the Λ^0 and η^0 particles, which decays to a Λ^0 hyperon and an η^0 meson. This is no rigorous proof, the evidence is too circumstantial. But the supposition was advanced and investigators began their search in earnest, abiding by all the rules. A year and a half later, after conducting new experiments, American physicists confirmed the existence of a resonance with the mass 1.675 GeV, which decays to a Λ^0 hyperon and an η^0 meson. Research on this resonance is being continued. It is thought that this is the first member of a whole family of resonances of baryons with the η^0 meson, and that it will prove possible to discover the sigma-eta ($\Sigma\eta$) and maybe even the xi-eta ($\Xi\eta$) resonances.

Chapter 13

2 + 3 = 23

"Forward, forward, on with my tale!" Hedgehogs, soccer goals and photons are matters of the past. Has not the time come to attack a much more difficult problem? There is a very common type of decay that we have not touched upon so far. This is a decay to three particles.

It is known, for instance, that K^0, ω^0 and η^0 mesons decay to three π mesons. The commonly known neutron decays to a proton, an electron and an antineutrino. After 10^{-6} s a mu-plus meson decays to a positron and a neutrino-antineutrino pair. There are even more complex cases. One of the resonances decays to a pair of particles, the π and ω^0 meson, and the ω^0 meson decays to three π mesons. There are many more reactions of this type.

But we shall not specify a definite type of decay. Assume, simply, that particle *O* decays to three other particles: *1*, *2* and *3*. Assume, in addition, that it is at rest. What do the conservation laws have to say about this decay process? Let us write them out:

$$E_1 + E_2 + E_3 = m, \tag{13.1}$$

$$\mathbf{p}_1 = \mathbf{p}_2 + \mathbf{p}_3 = 0^*. \tag{13.2}$$

---

* It is evident from this equation that all three vectors, $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ are coplanar (they lie in a single plane). Recall that in the decay to two particles, $\mathbf{p}_1$ and $\mathbf{p}_2$ lay on one straight line (see equation (9.2)).

To this we must add the invariant relations

$$E_1^2 - p_1^2 = m_1^2, \quad E_2^2 - p_2^2 = m_2^2 \quad \text{and} \quad E_3^2 - p_3^2 = m_3^2 \tag{13.3}$$

and investigate the properties of the solution of the system of equations we have obtained, like we did in decay to two particles.

Then we found that the energy of particle *1* was fully determined by the masses of particles *O*, *1* and *2*:

$$E_1 = \frac{m^2 + m_1^2 - m_2^2}{2m}, \tag{13.4}$$

and then found out that there is no restriction on the direction of particle *1*, as well as many more details of the decay reaction.

What can prevent us from pretending that a decay to three particles is simply a decay to two? We assume that the pair of particles *2* and *3* is a single particle with the energy $E_2 + E_3$ and the momentum $\mathbf{p}_2 + \mathbf{p}_3$. We shall denote it by *23*, joining *2* and *3*. But, for it to really pass as a particle, we must furnish it with a mass as well. For this mass we take the invariant

$$m_{23} = \sqrt{(E_2 + E_3)^2 - (\mathbf{p}_2 + \mathbf{p}_3)^2}, \tag{13.5}$$

because if particle *23* was real and did decay to particles *2* and *3*, its mass would be exactly what we have written.

Thus, we have the particle *O* that decays at rest to the particles *1* and *23*, with the masses m_1 and m_{23}. Making use of equation (13.4) we can write that the energy of particle *1* is

$$E_1 = \frac{m^2 + m_1^2 - m_{23}^2}{2m}. \tag{13.6}$$

Now we can easily draw the momenta hedgehog, draw the ellipsoid and, in a word, solve all the problems given in Chapters 9, 10 and 11. There is, however, one important "but" Previously, the mass of the second particle emitted in the decay had a fixed value. In this case, m_{23} is a variable quantity. It depends upon how particles *2* and *3* are emitted, i.e. their directions and energies. Hence E_1 will also differ in the various decays $O \rightarrow 1 + 23$. Hence, it is of no use to draw the momenta hedgehog. In a decay of a particle to three, their energies are not constant and are not determined by only the masses of the particles. These energies may vary from case to case, from one observed decay to another.

The Discovery of the Neutrino

Kinematics is the maidservant of physics. It strives to apply each new observation and each new conclusion as soon as possible to render aid to either theoretical or experimental research. Even the conclusion that we have just come to played a vital role in the physics of its time. The fact, as such, is quite elementary: the decay of a particle at rest to two particles differs from the decay to three in that in the former each particle produced always has the same energy, whereas in the latter, the energy of each particle varies. But this fact served as the basis for the discovery of the most inconceivable of all elementary particles, the neutrino.

This happened in the following way. The β decay of nuclei had been observed for a long time. Sometimes, one or another nucleus, by itself, emitted electrons (β radiation is a flux of electrons or positrons), and was thereby transformed into another nucleus with a charge one unit higher or lower. This was evidently a reaction resembling the γ decay we discussed in Chap. 10:

$$N^* \rightarrow N + e^-. \tag{13.7}$$

If this is so then all the electrons produced in the given type of decay would always have one

Fig. 47. Typical energy spectrum for electrons in a β decay.

The energies of the electrons produced in the decay are plotted along the horizontal axis; the numbers of times these energies are found, along the vertical axis.

and the same energy. But it was found that the energies of the electrons emitted in β decays varied drastically from case to case. If you observe β decays of a single type of nucleus for a sufficiently long time, you find electrons with all possible values of energy, from very low energies to quite substantial values (Fig. 47).

If this phenomenon had been disclosed more recently, after the discovery of many resonances, its explication would not have posed any questions. It is quite clear, we would contend, that a β decay produces at least one more invisible particle. We would add that it is something like the π^0 meson but lighter (lighter because the difference in mass between N* and N is very small). There would be no doubt but that the actual reaction is

$$N^* \rightarrow N + e^- + X^0 \quad (13.8)$$

and that the mass of particle X^0 can be determined by measuring the momentum vectors of N and e^-, and calculating the missing mass. At the beginning of the thirties, however, when no other elementary particles besides electrons and protons were even thought of, such an idea would have seemed much too bold. "What!" almost any physicist would exclaim at that time. "You want to introduce a new particle only in order to have it carry away the deficit in energy and momentum?! A particle serving only to save the conservation laws?! A particle that has not manifested itself in any other manner?! Is it impossible to try first to find a simpler cause: for example, that we have found a process in which energy is not conserved?" At that time it was easier for many physicists to believe that energy is not always conserved than in the existence of new particles. When Wolfgang Pauli proposed that the decay was according to the mode (13.8), his idea was regarded as the highest possible flight of fancy. The feasibility of the nonconservation of energy in a β decay seemed to a great many scientists, due to the frame of mind and level of knowledge prevailing in those days, not to be so senseless. This possibility was discussed even by Niels Bohr. It was mentioned among the possible sources of energy of the stars by the outstanding Soviet physicist Lev Davidovich Landau.

Several experiments were conducted in the twenties and thirties to check the energy balance in collisions of elementary particles, and in some experiments the balance was found to be violated! Then the future members of the USSR Academy of Sciences, A. I. Alikhanyan, A. I. Alikhanov and L. A. Artsimovich conducted an experiment to check whether the balance of momentum was violated in the decays. What strange people these physicists are! They permit themselves to doubt the soundness of such a simple law, so understandable to philosophers, fitting so easily into various philosophical conceptions, as the law of conservation of energy! Nothing comes from nothing; why complicate matters unnecessarily? What kind of additional experiments do they lack now?!

I wish to digress somewhat from the decays to three particles, and dwell in more detail on the different approaches of physicists and half-baked dogmatists to the conservation of energy. It would seem, at first hand, that the latter are right. As a matter of fact, what could be simpler? Energy is conserved everywhere, high and low; how can this be doubted? If somebody does doubt this obvious fact, let him build a perpetual motion machine! But, up-to-date science actually cannot explain *why* energy should be conserved; this is beyond its powers. "So far it has been conserved" is the only reason that we can offer. Theoretically, the conservation of energy is derived from the invariability of physical laws with time. But this last is also an experimental fact, i.e. a law of the type: "so far it has been so". Physicists understand this and regard the depth of their knowledge on the conservation of energy with due humility. Some philosophers, however, pretend that they know something about the conservation of energy that is unknown to physicists. They make this something a matter of principle, proving that energy must be conserved because..., followed by elegant, ideological generalizations. They make everything look as if they are aware of something incomprehensible to ordinary people. They do not understand that if one fine day some experiment may indicate that somewhere energy is not conserved, or is conserved to limited accuracy, they, and the rest of us, will just have to swallow the pill and change our world outlook so that it agrees with the nonconservation of energy.

It does not follow that physicists believe less in the law of conservation of energy than the philosophers do. Not at all. But some simply have blind faith and others take things with a grain of salt. Or, without joking, they have faith that goes hand in hand with true knowledge.

But let us return to the β decay. We need only add that Pauli's proposal, based on the belief that energy and momentum are conserved, as well as on his vast intellectual valour, turned out to be absolutely correct. Not so long ago, when nobody had any more doubt of its existence, the neutrino was noted in a more direct way (and not on the basis of only kinematic considerations). Today, physicists readily deal with two kinds of neutrino and two kinds of antineutrino.*

We have digressed so far from our line of reasoning that it will be necessary to remind you of what we started with and then proceed. We wanted to represent a decay to three particles as a decay to two. This presented no difficulty, but we found that one of the two particles (*23*) now has a variable mass and, as a result, the energy of particle *1* is also variable.

Let us see within what limits this energy may vary. Recall the equation that we derived:

$$E_1 = \frac{m^2 + m_1^2 - m_{23}^2}{2m},$$

where

$$m_{23}^2 = (E_2 + E_3)^2 - (\mathbf{p}_2 + \mathbf{p}_3)^2.$$

When does E_1 reach its maximum value? Obviously, when m_{23} becomes minimal (the other quantities in equation (13.6) are constant). The minimum value of m_{23} equals $m_2 + m_3$ because, as we know, particle *23* must further decay to particles *2* and *3*, and at $m_{23} < m_2 + m_3$ the energy will be insufficient for this decay. At $m_{23} = m_2 + m_3$ the decay $23 \rightarrow 2 + 3$ is still feasible. True, in this case, particles *2* and *3* are deprived of the kinetic energy required for them to be emitted anywhere. In the co-moving reference frame of particle *23* they seem to be stationary, but in any other frame of reference they are emitted side by side, without departing from

* Still another neutrino, already predicted theoretically, is being sought at present. It is called the tau neutrino (ν_τ).

each other. Thus

$$E_{1\,\max} = \frac{m^2 + m_1^2 - (m_2 + m_3)^2}{2m}. \qquad (13.9)$$

As to the minimum value of E_1, it cannot, in any case, be less than m_1. Check whether when

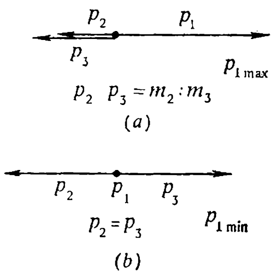

Fig. 48. Decay to three particles.
(*a*) Configuration for which the momentum of particle *1* reaches its maximum value; (*b*) configuration for which it equals zero

$m_{23} = m - m_1$, E_1 is exactly equal to m_1 and particle *1* is stationary. In this case, particles *2* and *3* are emitted in opposite directions with the same momentum (Fig. 48). Thus

$$m_1 \leqslant E_1 \leqslant E_{1\max}. \qquad (13.10)$$

The particular values of energy within this range that are more frequently encountered and those encountered less frequently depend, not upon the conservation laws, but upon the specific habits of these types of particles (particles *0*, *1*, *2* and *3*). But, whatever their habits, the energy will not jump out of the limits of equation (13.10).

The limiting energy of each of the particles depends upon the masses of all the particles participating in the decay. These may include invisible ones, whose presence we can only presume. But, from the maximum energy observed in the decay, we can estimate the mass of the

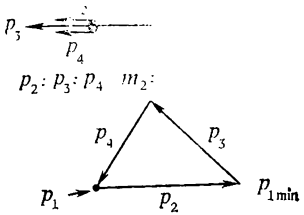

Fig. 49. Decay to four particles.

Shown above is the configuration of the four particles for which the momentum of particle *1* reaches its maximum value. Shown below is a case in which the momentum of particle *1* equals zero.

invisible particle. (Thus, for instance, it was established from the maximum energy of the electrons in a β decay that the mass of the neutrino is very small. It is supposed that the mass of the neutrino, like that of the photon, equals zero.)

Here again our already familiar hedgehogs appear. Equation (13.9), as you can see, no longer has variable quantities as there were in equation (13.5). Consequently, we have the right to draw a momenta sphere and, elongating it according to the Lorentz transformations, draw the ellipsoid alongside. They will help us answer questions of the following kind. What is the *maximum* momentum of particle *1* in the decay $O \to 1 + 2 + 3$, if the momentum of particle O is given and particle *1* is observed at a certain angle? Another question might be: what is the limiting angle of emission of particle *1* under these conditions? This does not necessarily concern the creation of three particles. It is clear that with any number of emitted particles, particle *1* has the highest energy when all the other particles merge into one having the mass $m_2 + m_3 + \ldots + m_n$ (Fig. 49). (It has minimum energy when particle *1* is at rest.) Then we can construct both a sphere and ellipsoid. The surface of the ellipsoid will correspond to all values of energy of particle *1*, whereas the internal part of the ellipsoid corresponds to all other energy values (in decay to two particles, the internal part of the ellipse had no particular meaning).

The Missing Mass Spectrometer

At this point we are very likely to be capable of understanding the idea of an experiment proposed in 1965 at the European Council for Nuclear Research. An attempt was made in this experiment to find what charged resonance particles are produced by energetic π^- mesons. Assume that a π^- meson collides with a proton. Their interaction sometimes results in the proton being recoiled off to one side with the creation of several light particles. As you must know by now, anything may be created if its creation is not forbidden. Therefore, sometimes one particle, say another π^- meson is produced; sometimes two, π^- and π^0 mesons; and sometimes three or four. They do not have to be π mesons; resonances are also produced, either by themselves or accompanied by other particles. If we denote all that is produced by X^- (where the minus sign concerns the total charge), all such reactions can be conditionally written out as the single reaction:

$$\pi^- + p \rightarrow p + X^- \qquad (13.11)$$

We should note, however, that unlike π^- and p, the rest mass of particle X^- is not specified beforehand. It may vary from case to case, from one π^--p collision to another, and equals

$$\sqrt{(E_1 + E_2 + \ldots)^2 - (\mathbf{p}_1 + \mathbf{p}_2 + \ldots)^2},$$

in which the parentheses include the energies and momenta of all the particles produced together with the proton. These values alternate and vary very capriciously; whatever the square root is equal to is the mass of "particle" X. Thus, in the reaction $\pi^- + p \rightarrow p + X^-$, particle X will have, as is said, a whole mass spectrum, i.e. a whole set of m_X values, like the quantity m_{23} in equation (13.5) had a whole spectrum of values. But some values will be encountered appreciably more often than others. This will occur when the fictitious "particle" happens to be a real particle or a real resonance. The mass m_X of particles and resonances is a fixed value (sometimes with great precision, and sometimes less accurately), and in this case the energies and momenta of the particles, into which they might decay, are distributed so that

$$\sqrt{(E_1 + E_2 + \ldots)^2 - (\mathbf{p}_1 + \mathbf{p}_2 + \ldots)^2}$$

will be equal to m_X. Hence, by noting which values of m_X are encountered noticeably more frequently than others, we can find the masses of real particles and resonances X produced in the reaction mode $\pi^- + p \rightarrow p + X^-$.

All this is not new to us; this is precisely the procedure that was resorted to to discover the resonances (see Chap. 8). The novel idea of the experiment being described is that the investigators wanted to manage without measuring m_X. They contended that by noting only one value, the angle at which the proton glances off, it is also possible to discover new resonances. It was found that if you establish which angles of recoil of the proton occur most frequently, you can find the mass of particle X that is most often encountered. Our aim in the following is to find out why this is so.

It would be most simple to return to Problems 5 and 6 from Chap. 11, because all that is required was mentioned there. But it is of more advantage to recall everything in its proper order. We can begin with Problem 6. It follows from this problem that if particle X is heavier than the π meson, then the protons have a limiting angle of emission: in the laboratory reference frame the proton cannot be emitted backwards. What does the limiting deflection of the proton depend upon? It is restricted by the tangent to the momenta ellipse. Hence, it depends on the shape and position of the elliptical hedgehog, i.e. in the final analysis, on the rest mass of the initial $\pi^- p$ system and the masses of the two particles, p and X, that are produced. If the energy of the π^- meson is specified beforehand, then the mass m_0 of the $\pi^- p$ system will also be constant. The shape and position of the ellipse are thereby uniquely determined by the mass of particle X. This means that there is an unambiguous relation between the maximum angle of emission of the proton and the mass of particle X. Thus we can indirectly measure the mass of this particle by measuring limiting angles.

"Fine!" you object, "but how do we know that the proton was emitted at the limiting angle? It leaves the point of collision at any angle it likes, not necessarily at the maximum permissible angle."

"Right! But recall the property that we cleared up in Problem 5 of Chap. 11. The limiting angle is the favourite angle of emission of particles. Their paths accumulate mostly about the limiting direction. Nothing extraordinary; this just happens to be a property of momenta ellipses (see the problem for more details). Consequently, though the direction of the proton is not conditioned in any way, in practice there are many more emitted at the limiting angle than at any other angles."

It seems that we have finally understood why there is a close relationship between the angles at which protons are emitted and the mass of particle X. This relation is not unambiguous; we cannot maintain that in each case that we know the angle of the proton recoil, we thereby find the mass of particle X. But we can contend that very often the proton is emitted so that its direction enables m_X to be estimated. There exists, as they say, a correlation between the direction of the proton and m_X; not a complete relation, but a relation nevertheless.*

* We find many correlations in everyday life. The height of a youth is not uniquely related to his age: you cannot determine his height from his age. But height and age are

By now we should have no difficulty in understanding the idea of the above-mentioned experiment. We measure the angles at which the proton recoils and, after investigating some tens of thousands of such collisions occurring in the process $\pi^- + p \rightarrow p + X^-$, we count up how many times we run across one or another angle. Certain angles will be encountered so frequently that their occurrence cannot be explained by the laws of chance. Next we select ellipses for which the slope of the tangent is equal to these angles. From this we find the values of m_X which correspond to these ellipses. In this manner we obtain the masses of all the particles and resonances (of negative charge) that are produced together with the proton.

It is not a simple matter to conduct this experiment. To obtain a reliable result you must register an immense number of events of the type $\pi^- + p \rightarrow p + X^-$, determine in each case the direction of the incident π^- meson, make sure that precisely a proton is ejected and that it is ejected precisely at the instant the π^- meson hits the target, measure the angle of the proton recoil, etc. This procedure requires entirely new electronic apparatus, joined directly to a computer. The reward for all this work is a princely one: you gain the capacity to discover a whole series of resonances automatically, without the touch of a human hand.

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correlated: in most cases, taller boys are older. There are correlations between the time of the year and the temperature; between the age and the vocabulary of a child, etc.

But more details of this experiment are beyond the scope of what we are dealing with here.

I had hoped to end this chapter here, but remembered that I am in debt to the reader. It was shown in Chap. 5 that the condition

$$m \geqslant m_1 + m_2 \qquad\qquad + m_n$$

must be complied with for a decay to n particles *1*, *2*, etc., to occur. But the proof that this is a sufficient condition was postponed. Right now is the most suitable time to provide such a proof.

Of advantage for this purpose is the method of mathematical induction. A decay to two particles when $m \geqslant m_1 + m_2$ is permitted by all conservation laws because (see Fig. 22) with this condition you can always construct a triangle having the projections of the sides equal to m_1 and m_2, with the sum of the sides being equal to m. Hence, at $n = 2$ the sufficiency of the condition has been proved.

For $n = 3$ assume that the condition

$$m \geqslant m_1 + m_2 + m_3$$

is complied with. Then we can find a value m_{23} that will satisfy the two inequalities:

$$m \geqslant m_1 + m_{23}$$

and

$$m_{23} \geqslant m_2 + m_3,$$

(it is sufficient, for example to take $m_{23} = m_2 + m_3$). But under these conditions, as we have just proved, the following decays are allowed:

0 → *1* + *23*

23 → *2* + *3*.

But this constitutes the decay $O \rightarrow 1 + 2 + 3$. This proves the sufficiency of the condition for $n = 3$.

I leave you to complete the proof yourselves.

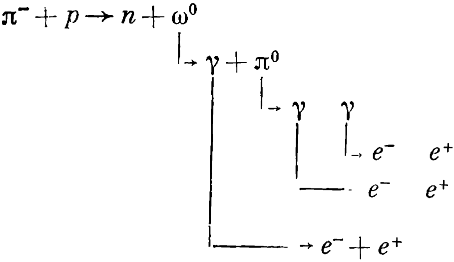
Chapter 14

Three-Photon Cone

An elegant kinematic problem was formulated and solved by the physicists of the Moscow Institute of Theoretical and Experimental Physics. At their disposal they had a bubble chamber filled with a mixture of liquid propane and xenon. It is much easier to note the creation of a high-energy photon in such a chamber than in one filled with hydrogen (you recall that we find out that a photon has been created by observing its transformation into a e^+e^- pair close to the nucleus; the higher the nuclear charge, the more frequently such transformations occur; the nuclear charge of xenon is many times higher than that of hydrogen).

It follows that the physicists had the opportunity of observing high-energy photons. It was to be used to prove that the ω^0 meson can decay, not only to three π mesons ($\omega^0 \rightarrow \pi^+ + \pi^- + \pi^0$), but to three photons ($\omega^0 \rightarrow \gamma + \gamma + \gamma$) as well. It was expected that the fast π^- mesons, ejected from the accelerator, would collide with protons in the chamber (each propane molecule has eight protons) and convert them into neutrons. At this, the π^- mesons would themselves be converted into ω^0 mesons, which sometimes decay on the spot to a π^0 meson and a photon. This

π^0 meson, practically on the same spot and without fail, decays to two more photons. Each of the photons, after travelling some centimetres invisibly, create a e^-e^+ pair (electron and positron) near some xenon nucleus that it happens to pass by chance. This newly produced pair is what can be observed in the chamber. Thus, they were to investigate the process



(14.1)

The outward appearance of this process is as follows: we see the track of the π^- meson, which suddenly breaks off and then at a distance we see three two-prong e^-e^+ forks, whose sharp corners point back to the spot where the initial track of the π meson was interrupted (Fig. 50). The directions of all the photons can be quite accurately measured in such photographs. They coincide with vectors whose tails are at the end, or break-off, point and whose heads are at the branching points of the forks. The directions of the photons were determined to an accuracy within 1°

This cannot, unfortunately, be said of the energies of the photons. For various reasons, the energies of photons can be only very approximate-

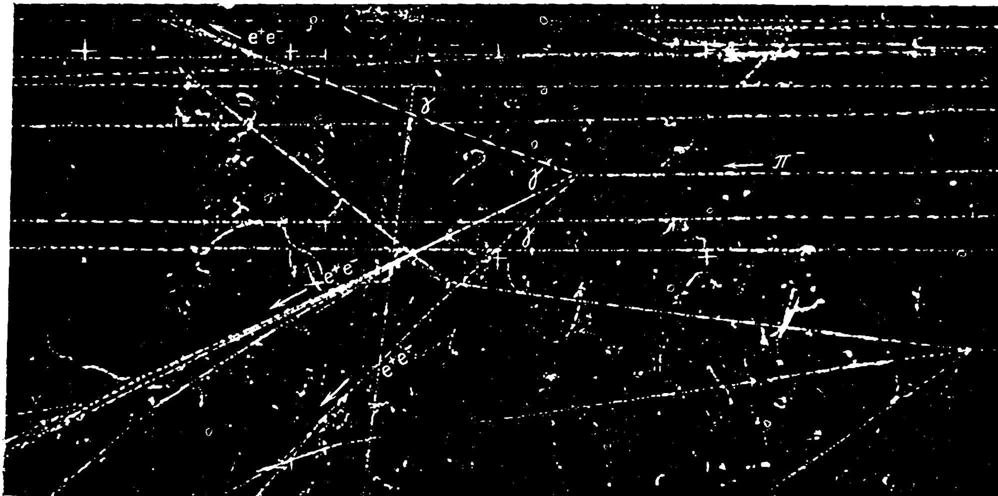


Fig. 50. Investigating the process $\pi^- + p \rightarrow n + \omega^0$ (one of the photographs).
The prongs of certain $e+, -$ forks merge into a single track. The second photograph, in which the same case of decay was shot from another point, is not given here. A complete three-dimensional picture of an event can be reconstructed only from two simultaneous photographs.

ly determined in a chamber filled with a heavy liquid. This being so, there is no possibility of calculating the invariant mass of the three photons to make sure that it equals the mass of the ω^0 meson.

This confronted the investigators with a difficult problem. Without seeing either the neutron or the ω^0 meson, and knowing only the directions of the photons, they were obliged to prove that the three photons in such photographs originate from the ω^0 mesons. Just note how much ingenuity they displayed in coping with their task.

Do you recall how the π^0 meson was discovered (Chap. 11)? The reaction used then resembled reaction (14.1):

$$\pi^- + p \rightarrow n + \pi^0 \quad (\pi^0 \rightarrow 2\gamma)$$

Under conditions in which all the sought-for π^0 mesons had one and the same velocity calculated beforehand, the investigators counted the number of pairs of photons emitted with a definite angle α between their paths. It was found that there are no photon pairs emitted at an angle α between them less than a certain definite angle. The velocity of the π^0 mesons could be estimated from the magnitude of this limiting angle:

$$\cos \frac{\alpha_{min}}{2} = v. \qquad (14.2)$$

This velocity coincided with the previously calculated value, thereby proving the existence of the π^0 meson.

The process

$$\pi^- + p \rightarrow n + \omega^0$$
$$\hookrightarrow 3\gamma$$

was a considerably more complex one (it was necessary to deal somehow with three photons).

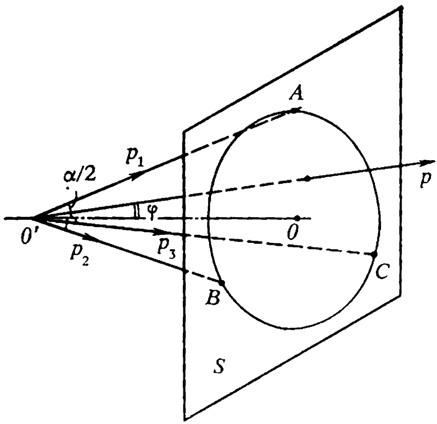


Fig. 51. A three-photon cone.

But the investigators succeeded in showing what quantity was to be measured in the experiment so that equation (14.2) is complied with as previously. Such a quantity was the apex angle of cone constructed by using the directions of the photons as generatrices, or elements.

Assume that the three photons to which the ω^0 meson decays have the momenta $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$. Assume, further, that they are directed in space in any manner (Fig. 51). Lay off a line segment of any length along the arrow $\mathbf{p}_1$; then lay off line segment of the same length along arrows $\mathbf{p}_2$ and $\mathbf{p}_3$. Pass the plane S through the ends A, B and C of the line segments. Next draw a circle with its centre at point O through the three points, A, B and C. After joining the points on the circle with point O' the site of decay of the ω^0 meson, we obtain a cone. It will be a right circular cone because line OO' is perpendicular to plane S (try to prove this). This is the cone we have in mind when we speak of the cone constructed by using the directions of the photons as generatrices. It is obvious that whatever the directions of the photons, the cone can always be constructed (though sometimes, it is true, it may degenerate into an ordinary plane). What the investigators proposed was to measure the apex angle of this cone on all the photographs in which three photons are seen. When this was done, the smallest angle turned out to be related to the velocity of the ω^0 meson by the same equation (14.2).

Let us prove this. We draw the axis OO' of the cone. Vector $\mathbf{p}$ represents the momentum of the ω^0 meson. It does not, of course, have to coincide with the axis of the cone, but it must coincide with the total momentum of the three photons

$$\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = \mathbf{p}. \tag{14.3}$$

Let us project the four momenta on the axis of the cone. Recall that the length of the projection of any line segment on some axis is equal to the length of the line segment multiplied by the cosine of the angle between the segment and the axis. By definition the axis of a right circular cone is a straight line that makes the same angle with all the generatrices of the cone. We denote this angle by $\alpha/2$ (having in mind the fact that the apex angle of the cone is equal to α). The momentum $\mathbf{p}$ makes a certain angle φ with the cone's axis. Now we can proceed to project the momenta. The length of the projection of momentum $\mathbf{p}_1$ on the cone axis equals $p_1 \cos(\alpha/2)$, just as the projections of $\mathbf{p}_2$ and $\mathbf{p}_3$ are equal to $p_2 \cos(\alpha/2)$ and $p_3 \cos(\alpha/2)$. The projection of momentum $\mathbf{p}$ equals $p \cos\varphi$.

The sum of the projections of the vectors $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ on any axis should equal the projection of their sum, vector $\mathbf{p}$, on the same axis, i.e.,

$$(p_1 + p_2 + p_3) \cos\frac{\alpha}{2} = p \cos\varphi.$$

The quantity in parentheses is the arithmetical, not vector, sum of the momenta of the three photons. But the momentum of a photon is equal to its energy. Hence the sum in parentheses is simply the energy of the ω^0 meson:

$$E \cos\frac{\alpha}{2} = p \cos\varphi.$$

We divide both sides of this equation by E and recall that p/E is the velocity of the ω^0 meson:

$$\cos\frac{\alpha}{2} = v \cos\varphi. \tag{14.4}$$

We have thus derived a simple equation that shows how the apex angle of the three-photon cone depends upon the directions of the photons and the velocity of their source. Now let us assume that all the ω^0 mesons have the same velocity, but are emitted in all possible directions and decay to three photons haphazardly. The angle φ between the axis of the cone and the direction of the ω^0 meson may have any value and, with it, the apex angle α of the cone may vary arbitrarily. But the cosine of any angle cannot exceed unity. Hence, the maximum value of cos $(\alpha/2)$ that we can find on all possible photographs of these decays will not exceed v:

$$\left(\cos\frac{\alpha}{2}\right)_{\max} \leqslant v. \tag{14.5}$$

As is evident, this equation really does resemble equation (14.2) (do not forget that the maximum value of the cosine of an angle corresponds to the minimum angle). It remains to show that the sign "$\leqslant$" can be replaced by the sign "$=$". For this purpose it is sufficient to cite an example in which

$$\cos\frac{\alpha}{2} = v.$$

This is easy. Assume that the ω^0 meson is emitted forward upon being produced, in the same direction that the π meson was travelling. Assume, further, that its decay to three photons occurred in a plane perpendicular to its motion. Such a decay yields, in the laboratory frame of reference, the required cone (Fig. 52). We can see here that the momentum of each photon, for instance $\mathbf{p}_1^* = \overrightarrow{O'A'}$ is transformed into the momentum $\mathbf{p}_1 = \overrightarrow{O'A}$. Let us write down the Lorentz transformations. Here they look especially simple because they contain no components with the longitudinal projection of the momentum. Thus

the longitudinal projection of $OO' = p_{1\parallel} = \gamma v p_1^*$ (without the member $\gamma p_{1\parallel}^*$);

and the energy (or momentum) $O'A = p_1 = \gamma p_1^*$ (without the member $\gamma v p_{1\parallel}^*$).

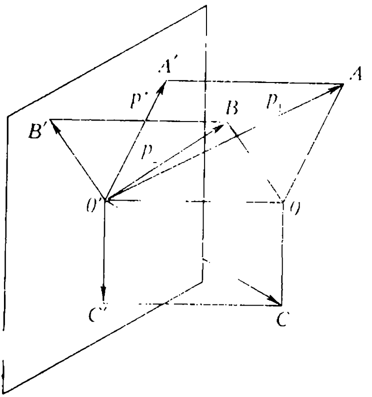


Fig. 52. The decay $\omega^0 \to \gamma + \gamma + \gamma$.

The case when the decay plane $A'B'C'$ of the ω^0 meson is perpendicular to the direction $O'O$ of motion of the ω^0 meson. Then the momenta of the three photons make the same angle with axis $O'O$ or, what is the same, the momentum of the ω^0 meson is the axis of the cone (all three momenta of the photons are the same in the diagram, but this is not necessary).

From the triangle OAO' we can write

$$\cos \angle AO'O = \frac{\overline{OO'}}{\overline{O'A}} = \frac{\gamma v p_1^*}{\gamma p_1^*} = v.$$

Thus, the momentum of any of the three photons makes the same angle, arc cos v with the direction of travel of the ω^0 meson. Hence they lie on the surface of the cone for which

$$\cos\frac{\alpha}{2}=v,$$

which was to be proved.

What use can we make of equation (14.5)? It can enable us to find an $\omega^0 \rightarrow 3\gamma$ decay only if all the ω^0 mesons being produced have the same velocity and it is known beforehand. But in our process:

$$\pi^- + p \rightarrow n + \omega^0,$$

in the laboratory reference frame, this is not at all the case. The momenta hedgehog of the ω^0 meson is stretched out forward, and the momenta of the ω^0 meson differ in different directions. But we know of a frame of reference in which the momentum of the ω^0 meson will be one and the same, regardless of its direction. If we imagine the process

$$\pi^- + p \rightarrow n + \omega^0$$

as occurring in two stages: first the particles π^- and p merge into a single particle O after which O decays to n and ω^0, then in the co-moving reference frame of particle O, the momenta hedgehog of the ω^0 meson becomes round as a sphere (see Chap. 9), which is all that we need.

We have now prepared all that is required to understand the course of the experiment that proved the existence of the decay $\omega^0 \rightarrow 3\gamma$. The investigators took a great number of photographs similar to Fig. 50. In each of them the vectors representing the directions of the photons were projected on a plane perpendicular to the track of the π^- meson (Fig. 53) and the angles β, between the photons and the track of the π^- meson, were measured. With the momentum of the π^- meson being known, the velocity v_0 of the fictitious particle O could be readily determined.

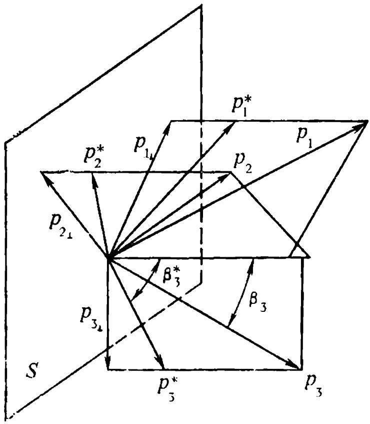


Fig. 53. Transformation of the photon momenta from the laboratory frame of reference to the co-moving frame of particle O.

The horizontal line shows the direction of the $\pi-$ meson; S is a plane perpendicular to this line; following Lorentz transformations, momenta $\mathbf{p}_1^*$, $\mathbf{p}_2^*$ and $\mathbf{p}_3^*$ remain in planes passing through each of the momentum vectors $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ and the direction of the $\pi-$ meson. It remains to pass a cone through momentum vectors $\mathbf{p}_1^*$ $\mathbf{p}_2^*$ and $\mathbf{p}_3^*$, and to measure its apex angle.

Then the angles β were recalculated to obtain the angles β^*, the angles at which the photons are emitted in the co-moving reference frame of particle O. The required equations are quite simple; we merely write the Lorentz transformations from the laboratory frame to the co-moving frame:

$$p_1^* \cos \beta^* = \gamma_O p_1 \cos \beta - \gamma_O v_O p_1,$$
$$p_1^* = \gamma_O p_1 - \gamma_O v_O p_1 \cos \beta$$

and divide one equation by the other. The momenta p_1 and p_1^* in both frames of reference cancel out, and only the relation between the angles remains:

$$\cos \beta^* = \frac{\cos \beta - v_O}{1 - v_O \cos \beta}.$$

This is the equation that was used to determine the directions of the photons in the co-moving frame of particle O. Then the investigators constructed a cone with three generatrices having precisely these directions and calculated its apex angle.*

It was found that the apex angles of the cone are mainly large ones, and that apex angles less than the value 2 arc cos v are quite rare. To make sure that this did not happen by chance, the experiment was repeated several times, irradiating the chamber with π^- mesons of various energies. Different calculated velocity values were obtained for the ω^0 mesons, depending upon the energy of the π^- meson. (Incidentally, we can also calculate the velocity without any trouble: assigning a value of energy to the π meson, we can find the mass of particle O, as was done in Chap. 5. Then, according to the rule of Chap. 9,

* Try by yourself to solve the following problem: three generatrices of a cone make the angles β_1^*, β_2^* and β_3^* with a certain straight line. Planes passed through this straight line and the generatrices make the angles α_1, α_2 and α_3 with one another. Find the apex angle of the cone.

we find the energy of the ω^0 meson in the comoving reference frame of particle O. Knowing the energy, we can readily find the momentum and the velocity.) It was also found that each time, in each experiment, the angles α end abruptly at the required value.

This is how a new channel of ω^0-meson decay to lighter particles was revealed.

Chapter 15

". . With a Faint Wave of the Hand. . "*

This is an exceptional chapter. In it the laws of kinematics are made use of for an unexpected purpose: to improve the quality of a physical instrument.

There is a building in Dubna that houses the synchrophasotron, or proton synchrotron, in an enormous room. When you enter this room you are dazzled. So much equipment: chambers, electromagnets, wiring, pipelines, blocks of concrete, protective grids, light signals, tracks, cranes and hoists, etc. This is the instrumentation laboratory. This is the place where experiments are conducted for investigating the properties of elementary particles. Sometimes a year or even two is spent in assembling apparatus in this room to obtain an installation that the physicists hope will double the accuracy in measuring the mass or the probability of decay of some particle. Such

* From the historical poem "Poltava" by A. S. Pushkin 1799-1837) —*Tr.*

is experimental high-energy physics, the foundation of today's knowledge of the building bricks of the universe.

Let us discuss one such installation. It was used in 1967 for observing the decay of mesons and resonances to photons (γ quanta). These decays had not been studied in sufficient detail at that time. Decays to charged mesons were known better by far. Charged mesons leave a track, which substantially simplifies their investigation. The photon is quite another matter. Special conditions are required for it to leave a track: the medium it is travelling through must be filled with a heavy substance (i.e. a substance whose atoms have sufficiently heavy nuclei). The heavier the nuclei, the more frequently the photon, passing near to them, forms electron + positron pairs, and the tracks of these pairs are visible (see Fig. 50).

But here, as we already know, another difficulty arises: though the photons become visible (by the e^+e^- pairs they create), their energy cannot be measured with any fair accuracy. All these circumstances drastically hinder the investigation of the photon decays of resonances.

In Chap. 14 we told about one of the methods used to avoid this difficulty. The following describes another method.

A target, consisting of a vessel with liquid hydrogen, was placed in the path of the π^--meson beam. The π^- mesons, colliding with the hydrogen nuclei, create new particles. Most often, of course, the π^- meson simply recoils off to one side:

$\pi^- + p \to \pi^- + p$,

but sometimes the so-called charge exchange occurs:

$$\pi^- + p \rightarrow \pi^0 + n.$$

All of these processes have been investigated in some detail, but it will be of interest to see how often, for instance, the process

$$\pi^- + p \rightarrow \eta^0 + n$$

occurs. Here the η^0 meson (whose mass, as you recall, is about 0.55 GeV) instantly decays, either to the triplet π^+, π^- and π^0 mesons, or to a pair of photons. The former decay was already well known, but how often the latter occurs is what was to be found out. It was also to advantage to find out whether there are any other particles that decay to two photons. These photons are emitted from almost the same spot their parent particle was produced and, after being ejected from the target, go off in the air. How can we manage to observe them? Placed for this purpose (Fig. 54), at a distance of almost two metres from the target, were two spark chambers SC_1 and SC_2 (arranged symmetrically on both sides of the beam of π^- mesons). These chambers are devices in which each photon triggers off a series of sparks enabling one to determine where exactly the photon passed through. The chambers had to be installed where most photons were expected. It was known from other experiments that in similar reactions (for example, in the reaction $\pi^- + p \rightarrow n + \omega^0$, described in the preceding chapter) mesons are most often emitted straight forward. Knowing this, it is possible to calculate the energy that the η^0 mesons most frequently have in this experiment (we solved such problems in Chap. 11). After this we are ready to draw the momenta hedgehog for the decay $\eta^0 \rightarrow \gamma + \gamma$ (Fig. 55) and install the chambers along the directions $0'C'$ and $0'C''$. These are

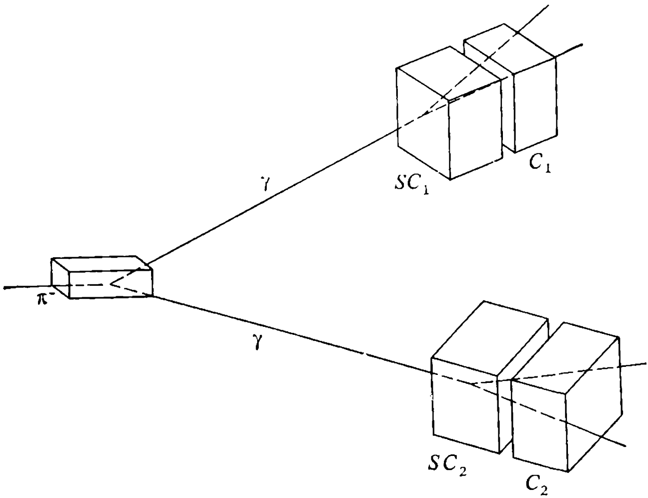


Fig. 54. Principle of the installation.

the directions of photons emitted with the same energy ($\overline{0'C'} = \overline{C'0''}$). The chambers are quite wide, so that pairs of photons with other energies will also pass through them (for instance, with the energies $\overline{0'Q}$ and $\overline{Q0''}$), but it can be proved that most of all the photons passing through the chambers in this arrangement will be ones with equal energies.

Thus, spark chambers enable photons to be registered and their direction to be observed. But this is, of course, insufficient. We must also know the energies of the photons. Otherwise, how can we prove that they were produced in the decay of

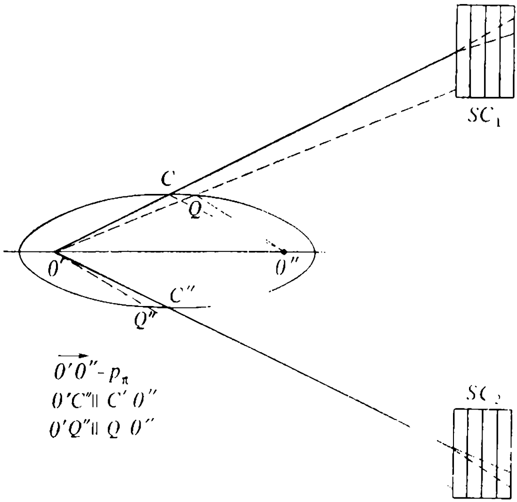


Fig. 55. Momenta ellipse of photons produced by the decay $\eta^0 \to \gamma + \gamma$.

Decay occurs at point O'. Two pairs of photons are shown; one pair with the same energies and the other with different energies.

an η^0 meson, rather than a π^0 meson or an ω^0 meson? For this purpose, thick glass slabs C_1 and C_2, Cherenkov counters, were placed into the chambers. These slabs are of a special glass that begins to glow when electron pairs produced by a photon pass through them. The greater the energy of the photon, the brighter this glow, or radiation. We determine the energy of the photon by measuring the intensity of radiation.

In the experiment, each time both counters flash, the spark chambers are switched on and the direction of the photon causing radiation is determined. After measuring the energy and direction of the two photons, it is necessary to clear up their origin. You already know how this is done. Adding the energies E_1 and E_2 of the two photons, we obtain the energy E of the particle that produced them:

$$E = E_1 + E_2.$$

Adding together the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ (the magnitude of the photon's momentum is equal to its energy and the directions of the photons are known), we obtain the momentum $\mathbf{p}$ of the particle that produced them:

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2.$$

After subtracting the square of the momentum from the square of the energy, we obtain the square of the mass of this particle:

$$M^2 = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2. \qquad (15.1)$$

Then we must examine the masses to find which values are especially numerous. These will be the masses of particles from which pairs of photons are created. One value that is to be expected with high frequency is $M = 0.135$ GeV. This belongs to the π^0 meson, the main source of photons in such collisions. Also repeated a great many times is the value 0.55 GeV, which belongs to the sought-for η^0 mesons. If some other value of the mass is found much too often, that will be fine. It will mean that we have discovered some additional, so far unknown, source of photon pairs.

Equation (15.1) can be simplified. Look at Fig. 56. The line segment $\overrightarrow{AB}$ is vector $\mathbf{p}_1$. The

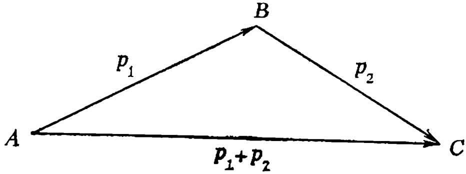


Fig. 56. Diagram for calculating the effective mass of a pair of photons.

length $\overline{AB}$ is equal to the energy E_1 (such are properties of any photon). Emerging from point B in the required direction is vector $\mathbf{p}_2$; it is represented by arrow BC. Here also $\overline{BC} = E_2$. But vector $\overrightarrow{AC}$ indicates the magnitude and direction of the vector $\mathbf{p}_1 + \mathbf{p}_2$. On the other hand, $E_1 + E_2$ is the sum of the two sides $\overline{AB} + \overline{BC}$. Hence, the value we are interested in is

$$M^2 = (\overline{AB} \quad \overline{BC})^2 - \overline{AC}^2.$$

Known in the triangle ABC are the sides AB and BC and the angle B between them (if the vectors $\mathbf{p}_1$ and $\mathbf{p}_2$ are known, the angle between them is also known), and we are to determine $(\overline{AB} + \overline{BC})^2 - \overline{AC}^2$. Recall the law of cosines from trigonometry:

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - 2 \times \overline{AB} \times \overline{BC} \times \cos B.$$

Substituting, we obtain

$$M^2 = 2 \times \overline{AB} \times \overline{BC}\,(1 + \cos B)$$
$$= 4 \times \overline{AB} \times \overline{BC} \cos^2 (B/2).$$

Hence the mass is

$$M = 2 \cos \frac{B}{2} \sqrt{E_1 E_2}. \qquad (15.2)$$

The more accurately an instrument operates, the higher its quality. Our instrument is designed for seeking particles that decay to two photons. The question is: what is its accuracy? If, for example, along with the η^0 meson, some other meson of close mass is produced as well, can our instrument detect its existence? It is evident from equation (15.2) that it all depends on the accuracy with which the energy of the photons and the angle between them are determined. As far as the angles are concerned, things are fine: they can be measured with high accuracy. The energy of the photons is an entirely different matter: here errors of 10% or even more are quite feasible. If, for instance, the photon by itself has an energy of 2 GeV, the instrument may indicate 2.2 and 1.8 GeV or any other close value. With what error will this permit us to determine the mass of the meson? Also to about 10%. The scintillators will trap many pairs of photons with close energies, and at $E_1 \approx E_2$, the mass

$$M = 2 \sqrt{E_1^2} \cos \frac{B}{2} \approx 2E_1 \cos \frac{B}{2}.$$

Since B is known to high accuracy, the relative deviation of M from the true value will aproximately coincide (in order of magnitude) with the relative errors in the measurement of the energy.

It turns out that such an installation is not particularly good; its resolution (capacity to differentiate between particles of close masses) is not higher than the errors with which energy can be measured. If, for instance, there are two particles η and η' with the masses 0.55 and 0.60 GeV, that decay to two photons each, instead of the first mass, 0.55 GeV, the calculation by equation (15.2) yields a whole set of values from about 0.5 to 0.6 GeV, not because particles with such masses exist, but simply because the energy cannot be more precisely measured. In exactly the same way, instead of 0.6 GeV (the mass of the imaginary particle η'), we shall obtain a great many values in the vicinity of 0.55 to 0.65. These two ranges of values, 0.50 to 0.60 and 0.55 to 0.65, considerably overlap, and we shall not succeed in detecting the existence of two mesons. They will have the appearance of a single meson with a strongly blurred mass.

Let us see how, without doing anything to the instrument, we can improve its resolving power.

The laws of kinematics allow equation (15.2) to be replaced by another equation. This proves sufficient to substantially enhance the quality of our instrument.

The idea that the quality of an instrument can be improved by a stroke of a pen, seems, on the face of it, to be absurd. Is it not all the same to the instrument that we make certain manipulations with a pen on a piece of paper? The instrument has done its work: it has delivered the prescribed quantities, energies and angles, with the accuracy it is capable of. No matter how we manipulate these figures, this cannot improve the instrument. This is the line of reasoning of anybody that hears about this matter.

But let us try the following simple procedure. Instead of the mass of the η^0 meson, let us calculate its Lorentz factor. You may remember (see equation (4.5)) that this factor is the name of the ratio of the energy of a particle to its mass:

$$\gamma = \frac{E}{M}.$$

And, lo and behold, it turns out the Lorentz factor of a particle can be more accurately determined than its mass with the same accuracy of measurement of energy. Not always, of course, but this is so for our experiment in which the chambers are specially aimed at pairs of photons with close energies.

As a matter of fact, the energy of the η meson is equal to the sum of the energies of the two photons, and the mass is expressed in terms of the energy by equation (15.2). Therefore, the Lorentz factor is

$$\gamma = \frac{E_1 + E_2}{2\sqrt{E_1 E_2}\cos(B/2)}.$$

Taking the factor depending upon the angle out of brackets and dividing $E_1 + E_2$ termwise by $\sqrt{E_1 E_2}$, we obtain

$$\gamma = \frac{1}{2\cos(B/2)}\left(\sqrt{\frac{E_1}{E_2}} + \sqrt{\frac{E_2}{E_1}}\right). \qquad (15.3)$$

The factor depending upon the angle is determined with sufficiently high accuracy and we shall pay no attention to it. Let us clear up the accuracy with which we can determine the sum $\sqrt{E_1/E_2} + \sqrt{E_2/E_1}$. Assume that our scintillators measured E_1 and E_2 with some error. Then $\sqrt{E_1/E_2}$ is also inaccurate, with an error of several percent (for example, we obtain a value that is 15% greater than the actual one). But γ also includes the addend $\sqrt{E_2/E_1}$. It is also obtained with a deviation, but in the other direction, since it is equal to $1/\sqrt{E_1/E_2}$, and if the denominator of a fraction increases, the fraction decreases. When the denominator is close to unity, then the fraction is reduced by about the same factor that the denominator is increased. Hence, if E_1 and E_2 are approximately equal, the amount that $\sqrt{E_1/E_2}$ deviates to one side is approximately the amount that $\sqrt{E_2/E_1}$ deviates to the other. But then the sum $\sqrt{E_1/E_2} + \sqrt{E_2/E_1}$ is obtained with an incomparably less deviation from its true value than each of its addends taken separately. The deviations of the addends cancel each other! The closer E_1 is to E_2, the more exact this compensation; the farther apart they are, the poorer this compensation is, but it is always present. Consequently, γ is always obtained much more exact, even with high errors of the instruments, than M, where there is no trace of such cancellation.

But of what good is the Lorentz factor to us? To identify a particle, we must know M and not γ. As we know, it is the mass, rather than factor γ that differentiates one kind of particles from another.

Sometimes, however, under certain definite conditions, particles can be identified by their factor γ. In our case, for instance, if we seek not simply η^0 mesons, but η^0 mesons that were produced in the reaction $\pi^- + p \rightarrow n + \eta^0$, we can suggest a frame of reference in which all the η^0 mesons created in this reaction have the same Lorentz factor. This is no special privilege of the η meson alone. The ω meson from the reaction $\pi^- + p \rightarrow n + \omega^0$ in the preceding chapter had the same property. This is always the case when the energy of the π meson is specified at the beginning and only two particles are left at the end of the reaction. Let us assume, as before, that first the particles π^- and p merge and produce the O particle, and then that this particle decays to n and η^0. If the energy of all the mesons is one and the same, the mass of particle O will be the same in all collisions:

$$M_O = \sqrt{(E_\pi + m_p)^2 - p_\pi^2} = \sqrt{m_\pi^2 + m_p^2 + 2m_p E_\pi}.$$

Then in the co-moving frame of imaginary particle O, all mesons, in whatever direction they are emitted, will have the same energy. The pertinent equation was derived back in Chap. 9:

$$E_\eta^* = \frac{M_O^2 + m_\eta^2 - m_n^2}{2M_O}.$$

But if the η mesons have fixed energy, their Lorentz factor is also a fixed value:

$$\gamma = \frac{E_\eta^*}{m_\eta} = \frac{M_O^2 + m_\eta^2 - m_n^2}{2M_O m_\eta}.$$

It is unambiguously determined by the mass m_η. Each value of the mass of a particle that produces a photon pair corresponds to only a single value of factor γ. In this experiment, therefore, the factor γ is in no way inferior to the mass for identifying a particle. Since its value is known with higher accuracy than the mass, it can serve this purpose even better. Two particles with close masses, that cannot be distinguished by means of equation (15.2), can be readily differentiated by means of equation (15.3). Expressed as a percentage, the difference in quantity γ approximately coincides with the difference in masses, but factor γ can be calculated from the energies with higher accuracy than the mass can be. As we have seen, the resolving power of the instrument really is improved by a mere stroke of the pen, provided, of course, that the pen is guided by the laws of kinematics.

One question remains that is not quite clear. The Lorentz factor was fixed in the co-moving frame of particle O, whereas the experimental installation operates in the laboratory frame of reference. The question is: can its readings be used to determine γ in the co-moving frame of particle O? Yes, they can; only the Lorentz transformations are required. They are to be carried out as was described at the end of the preceding chapter. If we imagine that a plane is passed through the direction of the incident π^- meson and the line of emission of one of the photons then, after the Lorentz transformations (in the co-moving reference frame of particle O), the momentum of the photon remains in this plane, but the angle at which the photon is emitted will seem to be different. Instead of β it becomes β*, and

$$\cos\beta^* = \frac{\cos\beta - v_O}{1 - v_O\cos\beta},$$

where v_0 is the relative velocity of the two frames of reference, i.e. the ratio of the momentum of the π meson to the total energy of the π meson and proton. The equations for the energy of photons in the co-moving frame of particle *O* are known to us. For example,

$$E_1^* = \frac{E_O}{M_O} E_1 - \frac{p_O}{M_O} E_1 \cos\beta_1.$$

In short, the co-moving reference frame of imaginary particle *O* is no less convenient than the laboratory frame, and factor γ in this frame is constant and can stand us in good stead.

Strictly speaking, the tale of the pen tip ends he e. We found that *such* a way of improving the quality of our instruments is also feasible. We shall not deal here with the applications of this idea in practice. We agreed at the beginning that we would not concern ourselves with anything in this book except kinematics.

But let us think over this matter. What, in fact, enabled us to raise the resolving power of our instrument without even touching it? Why at first did it seem that its resolving power could be no better than the error with which it could measure the energy of the photons, and then it became evident that the resolving power could be improved? Maybe it is a matter of the equations we used? First we multiplied the energies and this made the error in *M* as large as that of the energies. After that we divided, took the square root and added the reciprocal values, thereby drastically reducing the error.

No, this is not th point. It is good thing to have such an equation, but that is not enough. We are dealing here with quite a different matter. We simply remembered something we had not thought about previously. We finally took into account the fact that the η meson is only *one of the two* particles being produced. At first we had forgotten about this aspect of the reaction. A meson is created, it immediately decays, we collect the decay products and re-establish its mass: this was our initial incentive. *How* it is produced, together with one or with many particles, made no difference to us. Later, only after we recalled that in the co-moving frame of a particle decaying into two, the descendants should have a *definite* factor γ, only then did we acquire the possibility of advantageously utilizing the welcome properties of the combination

$$\sqrt{\frac{E_1}{E_2}}+\sqrt{\frac{E_2}{E_1}}.$$

This signifies that we improved the quality of the instrument by making use of new, previously unknown, information. *An improvement of quality in exchange for new information* is how this can be called.

The concept of information has been so deeply instilled into science in recent years that it is difficult to picture a time, within the living memory of scientists, when this word did not have the connotations it has today. The word existed, of course, in this sense as well, but its implications were different and more meagre than they are today. Information meant what they gave you in an inquiry office or what you read in the gossip column of your newspaper. With the advent of cybernetics, the meaning of this word expanded on a gigantic scale, became much more profound, developed and extended in all ways and directions. Somehow it immediately became a part of the system of popular concepts of the investigator; it seemed that many vital aspects of scientific activity were contained in this single word. Some of these aspects, maybe for want of this connotation, previously escaped the attention of physicists. Finally, somebody used the word information in this sense and, at once, it was found to be indispensable. Whether we are formulating a problem, attempting to reach conclusions from an experiment, assessing the reliability of the final results of numerous observations and calculations, everywhere, maybe not explicitly, but we make use of the concept of information. Sometimes in a quantitative sense, as in a statistical problem, but more often simply qualitatively. Many physicists do not even realize how deeply and with what comfort the basic idea of information in the physical sense has settled in their minds. This idea is: *without new information, you cannot reach a new conclusion.*

It is clear then that the problem of acquiring new knowledge of nature is one of vital importance to the scientist. This is no abstract problem, but one confronting him from day to day. In insufficiently developed branches of science, or still obscure realms of highly developed branches, new information is not always required to gain new knowledge. Here, they frequently do not know what to do with the old information; there is a lack of ways of assimilating the information they already have. Sometimes, in these cases, a new mind, after romping in a field of long-known facts, suddenly comes up with an armful of information that was not noticed by his predecessors. Consequently, in such branches of science, the concept of information often has no practical (a scientist would say heuristic) significance. There it simply has nothing to do with the matter; there they just cannot cope with the abundance of data.

This concept of information does not particularly worry a high-ranking theoretical physicist. His principal source of new information and new ideas to work on is not outside. It is inside; it is his own mind, and the scantest information may be enough for him to develop a triumphant theory. (He cannot, of course, do without any information at all. He bases himself on the facts like an automobile on the ground, but the direction and rate of motion are determined, nevertheless, by his internal forces.) But a physicist that bases his research on experiments, one in whose science the techniques of processing information are most thoroughly developed, one who is a practised hand at conducting an experiment with a limited number of external influences, is one to whom the concept of an ample supply of information is certainly to his liking. The unapprehended and unformulated by anybody concept that *without new information there can be no new conclusions* is at work with might and main throughout all of physics. Unnoticeably, without making a show of itself, but hard at work.

Amusing, is it not, that our knowledge, or our information of the world, does not want to come into existence out of nothing? This sounds even trivial: if we have extracted all the consequences from a certain situation, for additional conclusions we obviously require new data. But, in the first place, there remains the possibility of brainwork in the form of mental construction, the free fight of imagination. In certain fields of activity, this is all in the day's work. A physicist, however, conducts experiments; he requires tested facts and theories. This is what leads to the necessity of this nonmaterial something called information. In the second place, in real everyday life we never know whether *all the conclusions* have been reached. It sometimes seems that new knowledge has come into existence out of nothing, when actually it was simply a conclusion that nobody had yet arrived at. Physics, on the other hand, has reached such a level of development that in some fields all conceivable inferences, at the present state of the art, are drawn from each new experiment. Here, beyond any doubt, new conclusions require new information. It need not necessarily be experimental; it can also be theoretical: something is understood that we had not previously surmised; something is recalled that we had previously forgotten. As a result, the problem begins to take on new hues, suggests new ideas, etc. Take our experience in investigating the reaction $\pi^- + p \rightarrow n + \eta^0$ as an example. Along with the experimental information on the layout of the instrument and the accuracy with which the energies could be measured, we had theoretical information on the constancy of the γ factor of the η meson. This last was supplied, not by the instrument, but by the science of kinematics and the laws of conservation of energy and momentum. One form of information is of equal value to another form. This last piece of information is precisely what enabled us to improve the resolving power of our apparatus. Equal value of the two forms of information is precisely what we are obliged to for the feasibility of improving the quality of our instrument without even touching it. We could cite dozens of examples of the close unity between the two forms of information: theoretical and experimental.

Chapter 16

To Our Regret, the Last Chapter

Our first attempt (in Chap. 13) to master the kinematics of a decay to three particles was unsuccessful. We tried to represent it as a decay to two particles, but very little resulted from that endeavour. Therefore, in Chap. 14, without further ado, we resorted to a problem that did not require a knowledge of the energies and momenta of the particles. But there is no way of getting around an honest and complete analysis of a system of three particles, which, in fact, is an analysis of the system of equations

$$E_1 + E_2 + E_3 = m, \tag{16.1}$$

$$\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0, \tag{16.2}$$

$$E_1^2 - p_1^2 = m_1^2, \quad E_2^2 - p_2^2 = m_2^2 \text{ and } E_3^2 - p_3^2 = m_3^2. \tag{16.3}$$

Giving preference, as before, to geometric language, we shall now try to graphically represent the conditions (16.1), (16.2) and (16.3).

New Geometry of a Triangle

Let us draw an equilateral triangle *ABC* of altitude *m* (Fig. 57). Then, from an arbitrary point *0* within the triangle, we draw straight lines to the three vertices *A*, *B* and *C*. The distances of point *0* from the sides of the triangle we denote by E_1, E_2 and E_3. The area of triangle *ABC* is equal to the sum of the areas of triangles *0AB*, *0BC* and *0CA*. Thus

$$\frac{1}{2} m \times \overline{AC} = \frac{1}{2} E_1 \times \overline{BC} + \frac{1}{2} E_2 \times \overline{CA}$$

$$+ \frac{1}{2} E_3 \times \overline{AB}.$$

Cancelling out like factors, we obtain

$$E_1 + E_2 + E_3 = m, \qquad (16.4)$$

which is simply the well-known theorem stating that the sum of the distances from any point in an equilateral triangle to its sides is constant.

Before us is a finished law of the conservation of energy. This means that whatever the case of decay we observe, within the triangle *ABC* we can always find a point whose distances from the sides yield the energies of the particles in the observed decay. All these points fill a certain region within the triangle. On the contrary, any point within this region represents a conceivable, allowable case of decay.

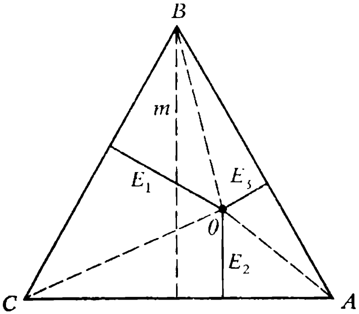
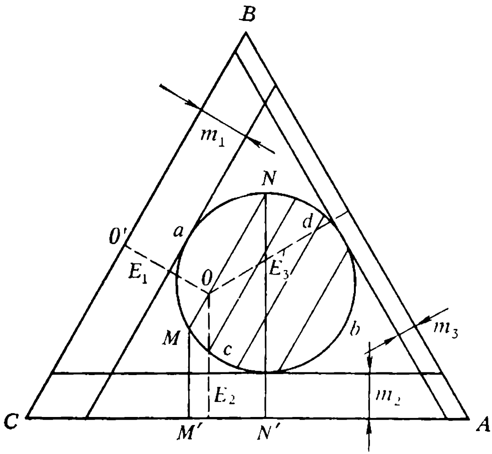


Fig. 57. The law of conservation of energy.



$MM' = E_{2\,min}$ at $E_1 = 00'$,
$NN' = E_{2\,max}$ at $E_1 = 00'$

Fig. 58. Diagram for deriving the equation of the boundary for a Dalitz plot.

Let us now learn to draw the boundaries of this region. (We cannot expect the region to seize the whole triangle, because we did not take into account the restrictions imposed by equation (16.2).) This is a good problem on the Lorentz transformation.

What does drawing a boundary signify? What properties do points on the boundary line possess? Assume, for instance, that the boundary has the shape shown in Fig. 58. Let us next draw several line segments parallel to side BC of the triangle. Let us take any one of them, say MN (where M and N are the points of its intersection with the boundary). For points along line segment MN, their distances from side AC will vary, whereas their distances from side BC are constant. These points correspond to decays in which particle *1* always has the same energy and particle *2* has different energies. At point M the energy of particle *2* is the minimum, and at point N the maximum of the possible values. If we knew the positions of these points (their distances from side AC) for all the line segments $MN \parallel BC$, we could draw the whole boundary of the region. Hence, the problem is reduced to finding the maximum and minimum possible values of E_2 for a fixed value of E_1.

But if the energy of particle *1* equals E_1 and its momentum is $\mathbf{p}_1$, then the energy $m - E_1$ and momentum $-\mathbf{p}_1$ (Fig. 59) fall to the lot of the fictitious particle *23*. The invariant mass of this particle is found to be

$$m_{23}^2 = (m - E_1)^2 - p_1^2 = m^2 - 2mE_1 + m_1^2 \qquad (16.5)$$

(so far this is the same as equation (13.6)).

Now assume that particle *23* decays to particles *2* and *3*. The energy of particle *2* in the co-moving reference frame of particle *3* is

$$E_2^* = \frac{m_{23}^2 + m_2^2 - m_3^2}{2m_{23}} \tag{16.6}$$

(this, as you may recall, is equation (9.9)),

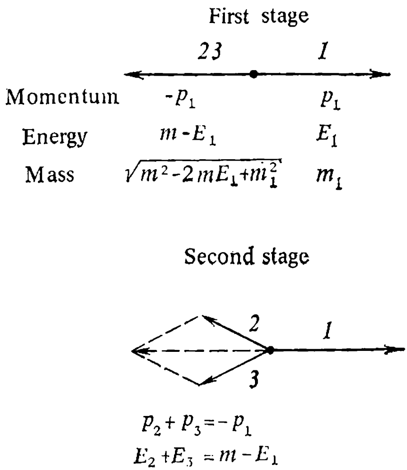


Fig. 59. Decay to particles *1*, *2* and *3* in the co-moving reference frame of particle *O*.

and its momentum $\mathbf{p}_2^*$ is also thereby determined. Important here for us is that both E_2^* and p_2^* are determined entirely unambiguously by the energy E_1: the quantity E_1 is actually present in equation (16.6) because this equation is based on equation (16.5).

Now let us return to the co-moving reference frame of particle *O*. The energy of particle *2* is

$$E_2 = \gamma E_2^* + \gamma v p_{2\parallel}^* . \tag{16.7}$$

This is a Lorentz transformation. The equation includes the relativistic factor γ, i.e. the ratio of the energy of particle *23* to its mass:

$$\gamma = \frac{m - E_1}{\sqrt{m^2 - 2mE_1 + m_1^2}} \tag{16.8}$$

and γv is the ratio of the magnitude of the momentum of particle *23* to its mass:

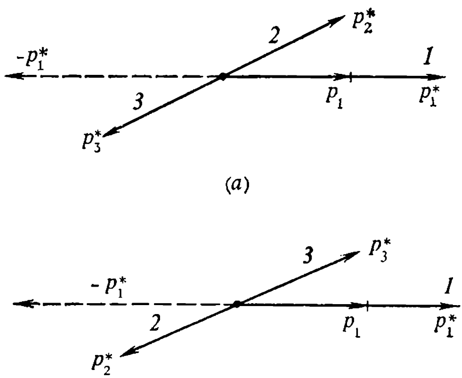
$$\gamma v = \frac{p_1}{\sqrt{m^2 - 2mE_1 + m_1^2}} . \tag{16.9}$$

Besides, equation (16.7) includes the quantity $p_{2\parallel}^*$, which is the projection of the momentum p_2^* of particle *2* (in he co-moving reference frame of particle *23*) along the direction of particle *23*. The momentum p_2^* itself is a fixed value, but its projection can have any value. Depending upon the direction of particle *2* in the co-moving frame of particle *23*, the momentum $p_{2\parallel}^*$ becomes either larger or smaller. But E_2 in equation (16.7) varies together with $p_{2\parallel}^*$. The maximum value of E_2 is obtained when particle *2* travels in the same direction that the momentum $-\mathbf{p}_1$ has; the minimum value when it travels in the opposite direction (Fig. 60).

In both of these cases, the projection of the momentum coincides in length with the momentum p_2^* itself. Thus

$$E_{2\,\max} = \gamma E_2^* + \gamma v p_2^*, \tag{16.10}$$

$$E_{2\,\min} = \gamma E_2^* - \gamma v p_2^* . \tag{16.11}$$



$$p_2^* = p_3^* = \sqrt{(E_2^*)^2 - m_2^2} \quad \text{where}$$

$$E_2^* = \frac{m_{23}^2 + m_2^2 - m_3^2}{2m_{23}}$$

(b)

Fig. 60. Decay to particles *2* and *3* in the co-moving frame of particle *23*.

Also shown is the momentum of particle *1* in the co-moving frame of particle *23*. (*a*) The energy of particle *2* in the co-moving frame of particle *O* is close to its minimum value (in Fig. 58 the point representing such events turns out to be in the vicinity of point *M* for case (*a*) and of point *N* for case (*b*)). (*b*) The energy of particle *2* is close to the maximum possible for the given energy value of particle *1*

Now we know how to draw the boundary of the region of decays to three particles. At the same time, we found that in boundary decays (those of a point on the boundary line), all three particles are emitted along a single line: particle *2* travels either in the same direction as particle *1*, or in the opposite direction (Fig. 61). In either case, particle *3* cannot go off to one side. The triangles that can always be constructed from the momenta $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ are converted into line segments on the boundary line.

Nevertheless, what line do equations (16.10) and (16.11) represent?*

It has no special name in mathematics. But in physics it is called a Dalitz plot, after Richard Henry Dalitz who first drew one. Its shape depends upon the values of m_1, m_2, m_3 and m. If $m_1 = m_2 = m_3 = 0$, the plot becomes an equi-

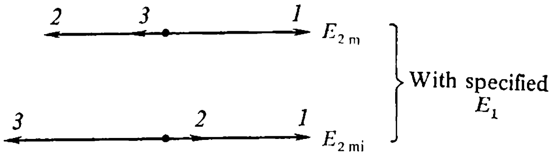


Fig. 61. Extreme energy values of particle *2*.

lateral triangle (Fig. 62*a*). When m is only slightly greater than $m_1 + m_2 + m_3$, the line almost becomes an ellipse (Fig. 62*b*). If, in addition, $m_1 = m_2 = m_3$, the line almost becomes a circle.

Physicists have a great liking for Dalitz plots. It is convenient to represent the results of the analysis of a photograph in which a decay to three particles is observed as a point. One photograph, one point; another photograph, another point, so that a thousand photographs provide a thousand points. The results of the observations of a great many decays of the same type are all within a figure (plot) drawn beforehand. The results

* The portion *adb* (in Fig. 58) corresponds to equation (16.10), and the portion *acb*, to equation (16.11).

of a whole experiment are represented in a single picture. In almost each issue of any thick physics journal that publishes papers on three-particle systems we find these freckled Dalitz plots.

But what good are these plots? The point is that these plots have one exceptionally useful property. The density of the points at any part of the plot is proportional to the frequency with which decays occur, whose representative points are within this part. For example, if inside a small square *2* (Fig. 63) there are five times as many points as within an identical square *1* at a different part of the plot, then the probability of observing triplets of particles with energies in the region of square *2* is five times greater than the probability that a decay produces particles with energies in the region of square *1*. We see in the Dalitz plot which energies are encountered more and which less frequently*. Moreover, the frequency with which we find particles with certain energies is closely associated with the manner that these particles (*0*, *1*, *2* and *3*) interact with one another. There are very many theories of such interaction. Each theory submits its version on the frequency with which one or another energy of a particle is to be found. By employing a Dalitz plot, theoretical physicists select the most suitable of the many theories of interaction for the case on hand.

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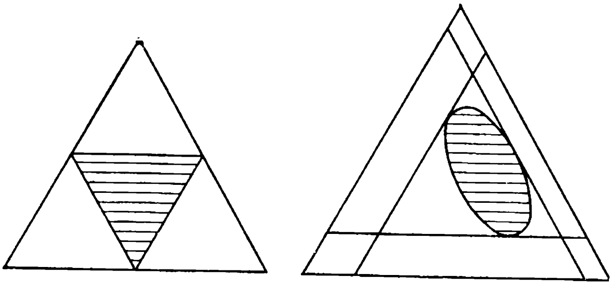
* This does not concern only energies. Knowing the energies, we can readily calculate the angles between the paths of the particles (from the momentum triangle in Fig. 9) or the invariant masses of pairs of particles (from equation (16.5)).



(a) (b)

Fig. 62. Dalitz diagrams.

(*a*) For decays to three photons. (*b*) For decays to three heavy particles (the outer triangle should have been drawn much larger than the inner one).

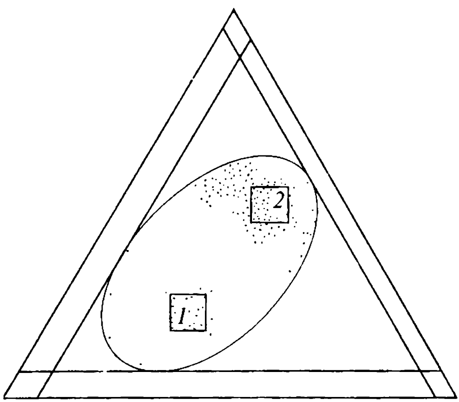


Fig. 63. A^- Dalitz plot.

Spin of the ω^0 Meson

In Chap. 8 we mentioned the discovery of the ω^0 meson. In investigating the triplet $\pi^+\pi^-\pi^0$, physicists found that too many triplets have an invariant mass m close to the figure 0.787 GeV, and understood that they were witnessing the decay of a hitherto unknown particle. This raised the question about its properties. Here the Dalitz plot stood them in good stead. The energies of all the triplets of π mesons with a mass close to 0.787 GeV were transformed to the co-moving reference frame of the conjectural particle ω^0. This was done in the following way. Adding together the energies E_1, E_2 and E_3 they obtained E_ω; adding together the momenta $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$, they obtained $\mathbf{p}_\omega$. Projecting $\mathbf{p}_1$ onto $\mathbf{p}_\omega$, they found $p_{1\parallel}$. Then they wrote $E_1^* = \gamma E_1 - \gamma v p_{1\parallel}$, where $\gamma = E_\omega/m_\omega$ and $\gamma v = p_\omega/m_\omega$. The final equation was of the form

$$E_1^* = \frac{E_\omega E_1 - p_\omega p_{1\parallel}}{m_\omega}.$$

The same was done with E_2^* and E_3^*. Decays of the ω^0 mesons to π mesons with such energies (E_1^*, E_2^* and E_3^*) were represented by points in a Dalitz plot (Fig. 64). On the plot, of course, were cases, not only of the decay $\omega^0 \to \pi^+ + \pi^- + \pi^0$, but simply triplets of π mesons whose invariant mass was, by chance, close to 0.787 GeV. It was not possible to separate any kind from the others. But just look at the interesting way in which the points are distributed. They are dense in the centre of the plot, and less and less towards the periphery.

Just by looking at this plot, a physicist immediately understands that the ω^0 meson is a particle with spin. This means that it can be imagined as a rotating sphere or top. The direction of the axis of the top is called the spin vector. The inter-

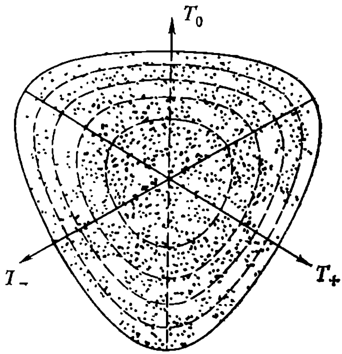


Fig. 64. Cases of the decay $\omega^0 \rightarrow \pi^+ + \pi^- + \pi^0$ marked on a Dalitz plot.
T_+, T_- and T_0 are the kinetic energies of the π^+, π^- nd π^0 mesons.

action of these spinning tops with external fields depends upon the relative directions of the spin vector and the field. In short, a particle with spin "feels", in this sense, the direction of the field. There are also particles without spin; their interaction with the external field is independent of the direction of the field.

It is of vital importance to know which particles have spin and which do not. This determines their closest relatives, how they affect particles in their vicinity, and much more. But the presence or absence of spin is not at all simple to determine, especially when, like the ω^0 meson, the particle has a lifetime of 10^{-22} s. Nevertheless, it was successfully shown that the ω^0 meson is a particle having spin. We shall try to give a general idea of how this was achieved.

The proof was based on a rule that governs the decay of particles with spin to π me ons. This rule

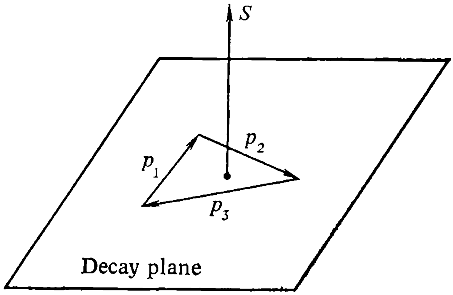


Fig. 65. The decay $\omega^0 \rightarrow \pi^+ + \pi^- + \pi^0$.
The planes in which the momenta of the three π mesons lie should predominantly be perpendicular to the spin vector of the ω^0 meson.

states that *if the pattern of the directions of the outgoing π mesons is such that it determines some vector in space, then the most frequently observed directional patterns are ones in which this vector is along the axis of rotation, i.e. along the spin vector of the initial particle.* Imagine, now, the decay of an ω^0 meson at rest. The momenta of the three π mesons form a triangle, because the equation $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$ is represented in the form of a triangle (Fig. 65). The plane of this triangle determines the direction of the vector in space: the vector is perpendicular to the plane. Hence, as the rule states, the vector perpendicular to the plane of the decay should have a preferred orientation along the axis of rotation of the ω^0 meson. Or, more simply, the decay plane of three π mesons should most frequently be perpendicular to the axis of rotation of the ω^0 meson. If, however, the ω^0 meson had no axis of rotation (being a particle without spin), there would not be any preferred decay plane.

This seems to be impossible to prove. The axis of rotation is invisible, it can have any direction. How, then, can we be sure that some one decay plane of the ω^0 meson is preferable to all the others?

Nevertheless, there is a way out of our dilemma. We must find whether among the decays there are ones in which the momenta triangle is stretched out into a single line. These are "maximally obtuse" triangles with one angle equal to 180° The vertices of such triangles no longer determine the decay plane. They lie in a straight line, and any number of planes can be passed through a straight line. Consequently, among them there can be no preferred decay plane. But there *should be* one if the particle that decays has spin. Hence, in the decay of particles with spin, no "maximally obtuse" triangles, stretched out to a single straight line, should be observed. The more the triangle is stretched out, the more seldom such triangles should be found in the decays.

The decay of a spinless particle is quite a different matter. Here the position of the decay plane is in no way stipulated. It is even of no consequence whether it exists at all. Here "obtuse" and "acute" decays may be encountered with equal frequency.

But look again at the Dalitz plot (Fig. 64). You recall that here a place was allotted for cases in which all the particles go off in a single straight line. The points corresponding to these cases are on the boundary line. The sparsely populated boundary zone of the decay region is a clear indication that the ω^0 meson cannot tolerate having its descendants, the π mesons, move away from the plane that it had foreordained. "Your plane is predestined," it seems to tell them, "so please be so kind as to travel in such a way that it is quite clear in which direction my head was held during my lifetime."

The more instable the plane bearing on the heads of their momenta like on a tripod, the fewer such cases. Our ω^0 meson is obviously a particle having spin.

But our attentive reader is already on the alert. It becomes clearer and clearer to him that something is wrong here.

The Deception is Exposed

"Aha, really?" you interrupt me, "You said that the pattern of the directions of outgoing π mesons should determine some vector in space?"

"Yes."

"And that this vector should be perpendicular to the decay plane?"

"Yes."

"And that since the π mesons travelling along a single straight line have no decay plane, there is no vector in space that could coincide with the direction of the spin vector?"

"Suppose that I did."

"But that is not true! They also have an allotted direction. It is the straight line itself, the one they are travelling along. They can simply leave along the axis of rotation of the ω^0 meson and your rule will have been complied with. Conse-

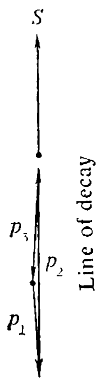


Fig. 66. The decay $\omega^0 \rightarrow \pi^+ + \pi^- + \pi^0$.
It would seem that the three π mesons could be emitted along the direction of spin of the ω^0 meson. But an analysis of Fig. 64 indicates that this does not occur. The spin vectors S in Figs. 65 and 66 behave differently when they are reflected by a mirror.

quently, nothing hinders the creation of such stretched-out triplets, provided they are mainly oriented along the axis, rather than crosswise."

"You are right," I am obliged to acknowledge, "and I am glad that you have found me out. I just wanted to simplify the true state of affairs. I shall have to tell the truth. As a matter of fact, if the three mesons form a plane, the "special" vector is perpendicular to it. If they form a straight line, then the direction of it is singled out (Fig. 66). But there are two types of vectors. One type, ordinary vectors, joining two points in space, are reversed in direction after being reflected by a mirror (with the mirror perpendicular to the vector). The other kind, which determine the position of a plane in space, do not change, in essence, after reflection because a plane has no up or down directions.

"The almost complete absence of points in the Dalitz plot that represent decays with the special line of travel of particles proved that the ω^0 meson is not simply a particle having spin, but one with definite properties with respect to reflection by a mirror."

"What properties, specifically?"

"That is something that I am unable to explain with sufficient simplicity so as to be understood by the general reader."

If, however, the reader is interested in this question, he should turn to more serious books on high-energy physics. The time has come for us to end our discussion.

But, in winding up, I cannot help looking back and reminding the reader that all that is known about the ω^0 meson, its mass, lifetime and spin properties, was found out without ever actually seeing either the ω^0 meson itself or even a single photograph on which it is positively known that its decay occurred. It has never been known whether we are dealing with the ω^0 meson or some imitation of it, but, nevertheless, it has been studied in no less detail than other particles.

In conclusion, let me suggest several problems.

1. Prove that if two particles are travelling at velocities that are the same in magnitude and direction, their velocities will be the same in any other frame of reference.

2. A similar problem: if it is found that in some frame of reference the ratio of the momenta of two particles is equal to the ratio of their

masses, this will be true for any other frame of reference.

3. Making use of the Lorentz transformations, the results of the preceding problem and Figs. 48 and 49, solve the system of equations

$$\sqrt{x^2+m_1^2}+\sqrt{y^2+m_2^2}+\sqrt{z^2+m_3^2}=E,$$

$$x+y+z=p,$$

$$x \quad y = m_1 \quad m_2.$$

This problem can be readily generalized for cases with any number of unknowns.

4. How does the Dalitz plot look for the case: $m_2 = m_3 = 0$ and $m_1 \neq 0$?

5. We have never completely solved the system of equations (11.1), though it reduces to a quadratic equation. Find the solution and write an equation expressing the momentum of particle *1* in terms of its angle of emission. This equation readily yields one for determining the limiting angle of emission of particle *1*.

Conclusions

I picture elementary particle physics as an Alpine country dominated by two towering mountains. Their foothills, interlacing and entangled with one another, cover the whole country, and their peaks are lost in clouds and mist.

One mountain consists of the means of detection and the results of observation. It consists of experimental apparatus: instruments as huge as the Luzhniki Sports Facilities Complex in Moscow and semiconductor scaling circuits. It consists of the search for particles by the most commonplace means, such as boiling, sparking and photography; and by such an unusual one as Vavilov-Cherenkov radiation. It consists of investigating and handling hundreds of thousands and even millions of photographs in hope of finding the one confirming an idea that has occurred to some theoretical physicist. It consists of a tide of new particles and resonances; of the discovery of infringements of the most indisputable, in our opinion, physical laws. Such is one peak. It is quite difficult for the layman to comprehend this tumult of Twentieth Century engineering and science.

The other mountain consists of theoretical views and computations. It consists of theories whose very premises make the hair of a man of common sense stand on end; it consists of calculations in which an exact solution sometimes covers dozens of pages and before which any electronic computer might shirk. It is the ability to manipulate infinities and meaningless operations so as to finally obtain four or five reliable digits. It consists of a language that is richer or, in any case, more laconic and exact than all the world languages taken together. A language in which some popular concept, such as "a virtual particle" cannot be explained by any human words whatsoever; it has no meaning outside the equation in which it appears. There are designations that are introduced at first for the sake of convenience, and subsequently turn out to be cardinal words of this language. This mountain also consists of the consequences of theories that seem to be incredible or doubtful even to those who advanced them, but, nevertheless, these consequences are later confirmed by experiments with unexpected precision. And finally, it consists of a science in which, since a certain time, the requirement that structure be elegant has almost become the most basic one and sometimes displaces such old-fashioned criteria as logicality, reliability and faithfulness to experimental results in every detail. Ask a theoretical physicist what is behind the laws of nature, logic or beauty, in the sense of elegant ideas and theories. The answer will be: *beauty*. As a matter of fact, Saint Augustine knew that "beauty is the shining essence of truth"

Such is the second peak. It is difficult for a person standing a long way off from all this to absorb even the most scanty husks of the world outlook of a theoretical physicist.

But the interest displayed in this "roof of the world" is so great that from time to time attempts are made to dispel the mist wreathing the peaks and to demonstrate to the astonished onlookers the beautiful mountainous landspace, or the structure of amazing elegance erected by the efforts of its inhabitants.

I am skeptical about such attempts. They are incapable, in any case, of providing the layman with an idea of the real difficulties that face the inhabitants of this country, of what really thrills them, of their true joys. Because it is impossible to understand the joy of a child bathing in the sea if you never did so as a child, if you yourself have never been toppled over by a merry wave, or if you have never sunk your feet into the sandy bottom. It is as impossible for the man of our time to understand the first farmer, the first man to plow and sow his field to the best of his knowledge, to reap his harvest when the time came and thresh the grain with his own hands, and at last inhale the fragrance of freshly baked bread. In exactly the same way, the reading of books on science for the layman, which treat of things that the reader cannot do and test for himself, only creates the illusion of true understanding. Maybe it enriches his vocabulary and in some way extends his mental outlook, but in doing so it unnecessarily deludes the reader into thinking that he has attained true comprehension when this is entirely out of the question.

Such a classic writer of Soviet books in this field as Yakov Perelman employed an entirely different approach. His was an explanation with figures and formulas, carried through to the very end. He did not attempt to cover too wide a range of phenomena, but the problems on which he wanted to throw some light were clarified just enough for the reader to then readily grasp an understanding of something similar.

The present author has tried to follow Perelman's example. It seemed to him that there is a field of elementary particle physics that can be fully mastered by anyone having a sixth-form (high-school) education. One that could be mastered to the extent that the reader can derive new equations by himself, analyze special cases of already derived equations and understand the course of reasoning behind many physical discoveries. Between the two chief cloud-wreathed mountains of our Alpine country I tried to find a narrow canyon, hidden among the foothills, along which we could pass into the very centre of the country. The idea was as follows: let a helicopter deliver us to the entrance of the canyon, i.e. we accept as initial facts the equations of Einstein, the Lorentz transformations and the capacity of particles to decay and scatter in collisions. But going farther, along the trail through the canyon, we pushed forward by ourselves, surmounting all the difficulties that are overcome by a student majoring in physics. He travels faster, of course, because he has high-speed apparatus, his mathematical knowledge. We, with our school algebra, advanced much slower, but on our own; we went through on foot rather than flying. Conscientiously, step by step, we passed through the whole canyon with its branching ravines. Our journey ended in the very heart of the land of elementary particles. We did not scale either of the peaks of which the country is so deservedly proud, but our expedition was a conscientious one.

This ends our planned route. Now it all depends on you whether you decide to return from this remote corner to the civilized world, or to change over from tourism to mountain-climbing, and begin your ascent to the very summits in a search for new riddles and new unheard-of adventures.

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To the Reader

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PROBABILITIES OF THE QUANTUM WORLD

DANIEL DANIN

This absorbing account of the "quantum revolution" in physics introduces the reader to the fascinating concepts of probability and relativity—and to the men and women whose successes and disappointments in their work made it possible.

The term "quantum" first appeared at the very beginning of the century, on December 14th 1900. Its reluctant coiner, Max Planck, considered it "only a working hypothesis" In a few years time Einstein was to declare, "the quantum exists!" Yet the development of quantum mechanics was not a steady series of triumphs: as perhaps never before in the history of science, dramatically new ideas were paralleled by dramatic events in the lives of those who created the new physics.

The author (a well-known Soviet writer on science) presents an intriguing story that draws on the priceless taped interviews with the pioneers of this revolution that are preserved in Copenhagen; the book will engage the interest and emotions of the scientific and general reader alike.

SCIENCE FOR EVERYONE

G. KOPYLOV, D.Sc. (Phys.-Math.)

ELEMENTARY KINEMATICS OF ELEMENTARY PARTICLES

This book tells a fascinating story of one of the basic goals of physics today: the discovery of the primary building blocks of matter. This field of science is called particle, or high-energy, physics, and is one of the frontiers of present-day physical research.

How is it possible to detect particles a hundred thousand times smaller than the atom, which itself is as many times smaller than an apple as the apple is smaller than the earth? How can we follow the motion of particles that have a velocity almost that of light? How can we measure the lifetime of these particles when it is of the order of 0.0000000000000000000001 second? What kind of a clock can we use? How can we investigate the properties of these astonishing and elusive bits of matter?

All these and many other questions are comprehensively answered in this book written for the layman by the late Dr. Gertsen Kopylov, who was a prominent scientist, well known in the world of particle physics. This book was written to be understood even by readers having only a secondary school education and it requires a knowledge of only elementary algebra and geometry. Nevertheless, in the author's treatment, the material is in no way oversimplified or distorted.

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