SCIENCE FOR EVERYONE

V. N. LANGE

PHYSICAL PARADOXES AND SOPHISMS

MIR

Science
for Everyone

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Physical Paradoxes and Sophisms

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Contents

Preface

Being in doubt, arrive at truth.
Cicero

This book was written for senior schoolchildren and presents a series of physical paradoxes and sophisms differing in theme and complexity. Some of them were known long ago, yet most are published for the first time.

"Sophism" and "paradox" are Greek words. A sophism (σοφισμα) is an argument, though apparently perfectly correct in form, actually contains an error that makes the final deduction absurd. A well-known sophism is "That which you did not lose, you possess. You have not lost horns, hence you possess them."

On the contrary, a paradox (παραδοξοζ) is a statement that seemingly contradicts common sense, yet in fact is true. For example, as a popular Russian saying contends, "it is a fact, however incredible" that when combining velocities with the same direction, the resultant velocity is smaller than their arithmetic sum (this is one of the inferences of the special theory of relativity).

A study of sophisms and paradoxes need not be thought of as a waste of time. Indeed, they were esteemed by such eminent scientists as Gottfried Leibniz, Leonhard Euler, and Albert Einstein. Einstein was very fastidious about his books, and yet he had a shelf full of books on mathematical jokes and puzzles. Maybe it was his early love of original problems that developed his striking ability for nonstandard thinking, which makes any discovery impossible. The study of many paradoxes has played an extraordinary role in the development of contemporary physics.

We hope that this small collection will help its readers avoid making some mistakes. For example, senior schoolchildren and first-year students are often observed, in trying to solve ballistic pendulum problems and the like, finding the system's velocity following an elastic collision by applying the law of conservation of mechanical energy only. Such mistakes will hardly be made again after studying the sophism in Problem 1.25 (a "violation" of the law of energy conservation).

The first section of the collection contains the problems, the second section gives short solutions. The latter are useful to check your own solutions and in the cases when a problem is difficult to solve on one's own.

The first two editions of the book were so popular that they quickly sold out. It has been translated into Bulgarian, Roumanian, German (two editions in GDR), Japanese, and the languages of the peoples of the USSR. Its success stimulated me to compose new paradoxes and sophisms, thus resulting in the present edition. In preparing it, I have omitted some problems, revised the text and solution of others, and added new ones.

Accomplishing his pleasant duty, the author sincerely thanks all those who made comments and remarks on the first and second editions of the book, especially B. Yu. Kogan, who reviewed the third edition. Further advice will also be accepted with gratitude.

The author

Chapter 1

Mechanics

1.1. The Amazing Adventures of a Subway Passenger

Every morning a muscovite goes to work by subway train. Although he starts work at the same time every day, he arrives at the subway station at various times each day. For simplicity we can assume that the time he arrives at the station is quite random.

On the face of it, he thought likely to assume that the first train, after he reaches the platform, goes his way as often as it goes the opposite way. But how was he amazed to find that half as many trains go his way.

He decided to clarify the reason for this mysterious phenomenon and began to go to his work from another station farther away. His new observations made him even more amazed, since at the latter station the case was perfectly different: the number of trains going his way was three times as many as those going the opposite way.

Could you help the passenger to resolve the strange behaviour of subway trains?

1.2. Will the Propeller-Driven Sledge Move?

A model propeller-driven sledge set in motion by a propeller is placed on a conveyer belt. What will the model's velocity be relative to the ground if the conveyer belt and the sledge simultaneously move in opposite directions, i.e. will the sledge be at rest or will it move in either direction?

1.3. What Is a Boat Velocity?

A man standing on a riverbank pulls up a boat by hauling in the rope fastened to its bow with a certain constant velocity $\mathbf{u}_r$. Let us resolve the

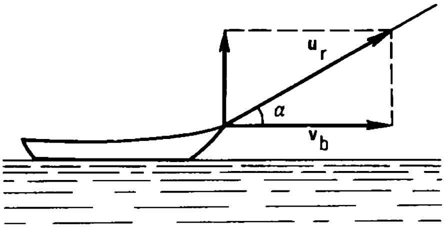

Fig. 1.1

velocity $\mathbf{u}_r$ as shown in Fig. 1.1. The boat's velocity $\mathbf{v}_b$ will be

$$|\mathbf{v}_b| \qquad |\mathbf{u}_r| \cos\alpha.$$

This formula demonstrates that the larger the angle α, i.e. the nearer the boat is to the riverbank, the smaller its velocity. Actually we observe quite the opposite: as the boat approaches the riverbank, its velocity increases, which fact can easily be checked experimentally. Just fasten a thread to a pencil and pull it by the thread like the boat by the rope.

One can prove graphically that the expression derived does not agree with our experiment (Fig.

1.2). Let in a certain time interval τ the boat's bow be displaced from point A into point B with the distance covered equal to AB. If AO is the initial position of the rope and BO is its

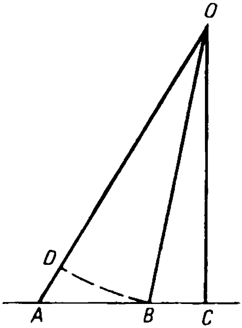

Fig. 1.2

position at the end of the interval τ, then by laying off segment $OD = OB$ on AO, we find by how much the rope must be pulled in (segment AD) for the boat to cover the distance AB.

The figure demonstrates that $AB > AD$, hence

$$|\mathbf{v}_b| > |\mathbf{u}_r|$$

which contradicts the formula

$$|\mathbf{v}_b| \qquad |\mathbf{u}_r| \cos \alpha,$$

since $\cos \alpha < 1$.

What is the reason for the discrepancy between theory and experiment?

1.4. The Strange Outcome of Combining Velocities

Two workmen are lifting a load P using two ropes passing over two fixed blocks (Fig. 1.3).

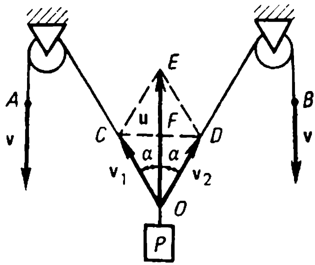

Fig. 1.3

Let us calculate the load's velocity at the moment, when the ropes make an angle 2α and the workmen are pulling the rope at A and B at equal speeds, i.e. $|\mathbf{v}_1| = |\mathbf{v}_2| = |\mathbf{v}|$.

Guided by the parallelogram rule we find the speed $|\mathbf{u}|$ of the load to be

$$|\mathbf{u}| = 2OE = 2OC \cos\alpha = 2|\mathbf{v}| \cos\alpha.$$

Let us analyze the result. Suppose now the load is placed fairly low, with the blocks placed not too far away. Then the angle α is close to $0°$ so that its cosine can, to a high degree of accuracy, be approximated to unity and the above formula yields

$$|\mathbf{u}| \approx 2|\mathbf{v}|.$$

The absurdity of this expression is obvious: a load cannot rise at a velocity exceeding that of a rope hauling in! This means that our reasoning is faulty. What is wrong?

1.5. What Is the Average Velocity?

A motorcyclist moved from point *A* to point *B* at 60 km/h. He returned along the same route at 40 km/h. Find the average velocity of the motorcyclist for the whole travelling time neglecting only the time he stopped at point *B*.

1.6. Less Haste, Less Time Waste!

Suppose we want to find the initial velocity of a stone thrown vertically, which after 4 s has risen 6 m.

The initial velocity of a body uniformly accelerated in a straight-line motion in this case is

$$v_0 = \frac{2s - at^2}{2t}.$$

Calculate v_0 for our situation by assuming the acceleration due to gravity —10 m/s² (the minus sign indicates that the acceleration is in the opposite direction to the body's motion):

$$v_0 = \frac{2 \times 6\ \text{m} + 10\ \text{m/s}^2 \times 16\ \text{s}^2}{2 \times 4\ \text{s}} = 21.5\ \text{m/s}.$$

How must the initial velocity be changed in order for the stone to reach the same height (6 m) in half the time? The need for an increase in the velocity seems to be obvious, but do not be hasty!

Suppose the stone reaches 6 m in 2 s rather than 4 s, then we obtain

$$v_0' = \frac{2\times 6\ \text{m} + 10\ \text{m/s}^2 \times 4\ \text{s}^2}{2\times 2\text{s}} = 13\ \text{m/s}.$$

Indeed, this appears to be something like the proverb: "More haste, less speed!".

1.7. "Notwithstanding" the Law of Inertia

The first law of mechanics can be formulated as: a body remains at rest or is in a state of uniform motion in a straight line unless forced by another body to change its state.

Why then do we often observe passengers in a slowing down train leaning backwards and not forwards, as is required by the law of inertia?

1.8. The Weight of a Diesel Locomotive Equals That of the Train

Were it not for the friction between the driving wheels and the rails then the engine would not be able to pull its train at all. According to Newton's third law the tractive force developed under a uniform motion equals the friction force between the driving wheels and the rails, viz.:

$$|\mathbf{F}_{\text{trac}}| = |\mathbf{F}_{\text{fr}}| = k_1 |\mathbf{P}_1|,$$

where k_1 is the friction coefficient for the engine wheels all of which, we assume for simplicity, are the driving wheels, and $\mathbf{P}_1$ is the engine's weight.

Based on Newton's third law the tractive force must be equal to the force against which the loco produces work, i.e. under a uniform motion it is equal to the friction force of the wheels of the train against the rails:

$$|\mathbf{F}_{trac}| = k_2 |\mathbf{P}_2|,$$

where $\mathbf{P}_2$ is the train weight. Comparing both expressions yields

$$k_1 |\mathbf{P}_1| = k_2 |\mathbf{P}_2|$$

By cancelling out $k_1 = k_2$ (steel-against-steel friction) we arrive at an evident absurdity:

$$|\mathbf{P}_1| = |\mathbf{P}_2|,$$

i.e. the engine weighs as much as the train!?

1.9. Why Are the Ends of the Shafts Lying in Radial Bearings Tapered?

The friction force is given by the friction coefficient, that is dependent on the surfaces in contact

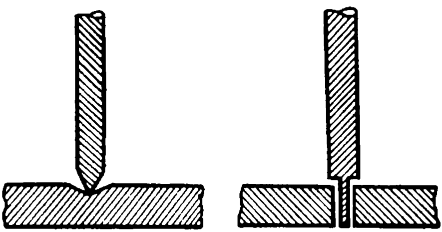

Fig. 1.4

and the normal pressure force, but it is practically independent of the area of the surfaces in contact. Why then are the ends of shafts in radial bearings tapered, while those fixed in plain bearings are designed to be as thin as possible (Fig. 1.4)? In some textbooks these measures are claimed to be taken to reduce friction.

1.10. The Wear on the Cylinder Walls of an Internal-Combustion Engine

An inspection of an internal-combustion engine that has run for a long time shows that the walls of the cylinders are worn most at *A* and *B*, which

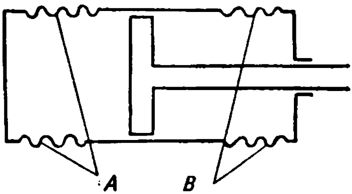

Fig. 1.5

is where the piston stops and its motion reverses in direction (Fig. 1.5).

This seems to contradict common sense, which would imply that the wear would be most pronounced where the piston's velocity is at its maximum. The forces of liquid friction are directly proportional to velocity and even (at high velocities) to its square. What is the matter?

1.11. Rolling Friction Must Vanish

We begin this problem indirectly. Suppose that a rectangular bar with height b and width a (the thickness is not essential) is lying on a horizontal plane. At a height h we apply a horizontal force **F**. Simultaneously, a friction force **Q** equal in magnitude to **F** appears if the latter does not exceed the maximum friction force at rest: $|\mathbf{Q}_{max}| = k\,|\mathbf{P}|$ (Fig. 1.6a).

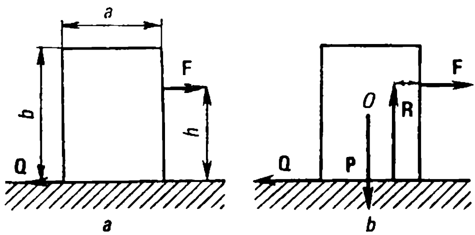

Fig. 1.6

Since **F** and **Q** do not lie along the same straight line, they produce a moment $|\mathbf{F}|\,h$ which tends to turn over the bar clockwise. The larger the force $|\mathbf{F}|$ and the higher up the bar it is applied to, the larger is the overturning moment.

If only the force couple $|\mathbf{F}|$ and $|\mathbf{Q}|$ existed, the bar would turn over whatever force **F**. In fact, a force of a certain magnitude must be applied to overturn the bar. Hence, a moment exists that prevents the bar from overturning. The origin of this moment can be understood quite easily.

The moment of couple of forces **F** and **Q** acts so as to raise the left edge of the bar and press the right edge to the plane. As a result, reaction force **R** applied vertically upwards to bar base and equal in magnitude to the force of gravity **P** now will no longer pass through both the centre of the bottom face and bar's centre of gravity.

Instead it will pass through a point somewhat to the right (Fig. 1.6*b*). The larger the modulus of **F**, the larger the overturning moment and the further to the right must be displaced **R** for the bar not to be turned over. Depending on the relationship between the magnitudes of *a*, *b*, *h* and *F*, two cases are possible:

(1) The magnitude of force **F** will attain the value of $|\mathbf{Q}_{max}| = k|\mathbf{P}|$ earlier than **R** leaves the limits of support's contour. Then the bar will move along the plane without turning over.

(2) Support reaction reaches the right-hand boundary of the bar bottom before $|\mathbf{F}|$ equals $k|\mathbf{P}|$. Then the moment of the force couple **R** and **P** will not compensate for the moment of the couple **F** and **Q** and the bar will be overturned.

Among other things, this suggests an unsophisticated technique for determining the friction coefficient between the bar and the surface it stands on.

Let us apply force $|\mathbf{F}|$ slightly exceeding $k|\mathbf{P}|$ at the bottom face of the bar. The bar will then be set in uniform motion. Let us now gradually raise the point where force **F** is applied (you can try this easily using a shoe-box). At a certain height the bar begins to turn over without slipping.

Let us write down the "boundary conditions" which will describe transition from one mode to another, i.e. the balance of forces and moments, which will be defined with respect to an axis passing perpendicular to the plane of the figure and through bar's centre of gravity, and considered as positive if the bar is rotated clockwise and negative if the bar is rotated anticlockwise:

$$|\mathbf{R}| - |\mathbf{P}| = 0,$$

$$|\mathbf{Q}| - |\mathbf{F}| = 0, \ (\mathbf{F} = k\mathbf{P}),$$

$$|\mathbf{F}| \ \left(h - \frac{b}{2}\right) + |\mathbf{Q}| \cdot \frac{b}{2} - |\mathbf{R}| \frac{a}{2} - |\mathbf{P}| \times 0 = 0.$$

Hence the friction coefficient is

$$k = a/2h.$$

This expression shows that the experiment will not succeed with every type of bar: the friction coefficient can only be found in this way if the bar's height b is such that

$$b > a/2k.$$

For example, for a cube where $a/b = 1$ the friction coefficient cannot be found by the "overturning technique" since in the majority of real cases the friction coefficient is smaller than 1/2. However, this experiment can be realized with a rectangular bar whose faces are appropriately positioned.

Now let us formulate the sophism. Suppose that there is a ball and not a bar on a horizontal plane. It has only one point of contact with the plane, therefore the support reaction force and the force of gravity of the body will always pass through this point. This suggests that the moment of couple of forces R and P (or the sum of the moments of these forces with respect to the contact point) vanishes. Hence, even if very small, any force exerted onto the ball must set it in rotation, In other words, the rolling friction coefficient must always vanish! Yet in reality this coefficient, though much less than that of sliding friction, is nevertheless greater than zero.

Where is the fault in our reasoning?

1.12. What Is the Force Exerted by a Table's Legs?

Figure 1.7 shows a table resting on a tilted plane. Let the force of gravity **P**, applied to the centre of gravity C of the table, be replaced by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ parallel to **P** and passing through

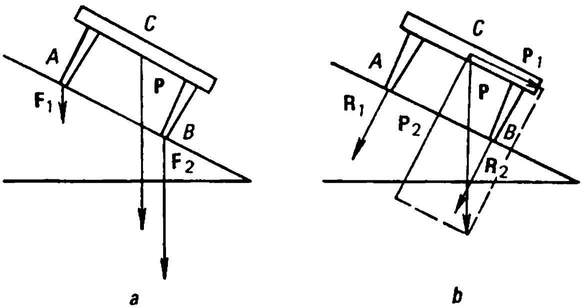

Fig. 1.7

the ends of the table's legs, i.e. points A and B (Fig. 1.7*a*). The moduli of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ must sum to $|\mathbf{P}|$ and their ratio must be inversely proportional to the distances from A and B to the line showing the direction of force **P** (the latter statement presumes that the table does not rotate with respect to the axis passing through its centre of gravity perpendicular to the plane of the figure). Decomposing, at points *A* and *B*, forces $\mathbf{F}_1$ and $\mathbf{F}_2$ into components (not shown in the figure) perpendicular and parallel to the tilted plane, we make sure that the pressure forces exerted at *A* and *B* by table's legs on the tilted plane prove to be different.

However, one can resolve the forces as shown in Fig. 1.7*b*: at first decompose the force of gravity $\mathbf{P}$ into the components $\mathbf{P}_1$ and $\mathbf{P}_2$. The component $\mathbf{P}_1$ acts so as to set the table in motion along the tilted plane and, since the table is at rest, it is compensated for by friction force. By decomposing $\mathbf{P}_2$ into the components $\mathbf{R}_1$ and $\mathbf{R}_2$ which pass through points *A* and *B*, we see that these forces (the forces of the pressure of the table's legs on the tilted plane) are equal.

Thus, the pressure exerted by the table's legs appears to be dependent not only on the force of gravity of the table, but also on the way of decomposing the forces which conclusion contradicts both common sense and experience. Hence, one of our arguments is faulty.

Which one?

1.13. A Mysterious Lever

Let a lever (Fig. 1.8) be balanced by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. A force $\mathbf{F}_3$ applied to the lever end along its length is commonly believed not to disturb the equilibrium. However, we can "prove" this to be wrong!

Let us take the resultant of forces $\mathbf{F}_2$ and $\mathbf{F}_3$ (call it $\mathbf{R}_1$) and the force $\mathbf{F}_1$, extend it to intersect at point *C* and add them. We then get a force

$\mathbf{R}_2$, which is the resultant of all three forces: $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$.

The figure depicts that the extension of force $\mathbf{R}_2$, and hence its moment relative to the lever's

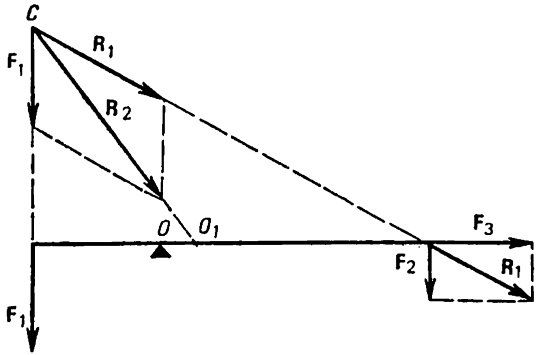

Fig. 1.8

pivot O, does not vanish. Therefore, the lever must rotate clockwise.

Is this conclusion true?

1.14. A Troublesome Reel

People who are engaged in needlework know about the strange behaviour of reels of cotton that have rolled under a piece of furniture. When you try to retrieve the reel by pulling at the thread keeping it horizontal, the reel obediently rolls out of its "hide-out". Yet try pulling it by a thread at an angle and you will see the peculiar phenomenon of the reel hiding even deeper. Why does the reel behave in this odd way?

Note. If you try an experiment, you'd better use a nearly full reel and the angle should not be too small.

1.15. Was Aristotle Right?

The famous Greek philosopher Aristotle who lived in the 4th century B.C. (384-322) is known, not without reason, as The Father of Science. His contribution to the development of natural science including physics is tremendous. However, Aristotle's views and deductions do not coincide with those accepted now. Let us take one of his arguments as an example.

He argued that a stone falls with a certain velocity. If we fix another stone on top of the original one, then the upper stone will push the lower one, so the lower one will fall faster.

Meanwhile, it has now been strictly established that all bodies, irrespective of their mass, fall with the same acceleration, i.e. in a given interval of time their velocities increase by the same value.

What is then the error Aristotle made?

1.16. Will a Bar Move?

Consider two bars with masses M_1 and M_2 resting on a horizontal perfectly smooth surface (Fig. 1.9). Let us apply a force **F** to the left bar so that it acts through this bar on the right one. According to Newton's third law the second bar will react upon the first one with an equal and opposite force —**F**. Since there is no friction (the surface is perfectly smooth), the resultant force **R** acting upon the left bar is balanced by the

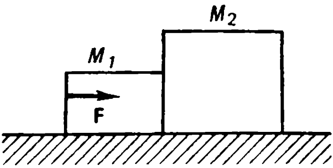

Fig. 1.9

sum of the applied force **F** and the reaction force —**F** of the second bar:

$$\mathbf{R} = \mathbf{F} + (-\mathbf{F}) = 0.$$

Whence the acceleration of the left bar is

$$\mathbf{a}_1 = \frac{\mathbf{R}}{M_1} = 0.$$

Thus, however large the force **F**, it will never shift M_1?!

1.17. What Force Is Applied to a Body?

A force is applied to a body 2 kg in mass and increases its velocity from 10 to 20 m/s in 5 s in a distance of 30 m. What is the magnitude of this force? Assume friction is negligibly small and the directions of both the force and the displacement are identical.

At first sight this is a common problem which, as is often the case, can be solved in several ways, For example, it can be solved on the basis of dynamic considerations (in other words, starting from Newton's second law):

$$F = ma = m\frac{v_2 - v_1}{t}.$$

Putting in the numerical values we get

$$F = 2\ \text{kg}\ \frac{20\ \text{m/s} - 10\ \text{m/s}}{5\ \text{s}} = 4\ \text{N}.$$

The principle of energy conservation can also be used. By equating the work done by the force to the increase in the body's kinetic energy we have

$$Fs = \frac{mv_2^2}{2} - \frac{mv_1^2}{2},$$

whence

$$F = \frac{m}{2s}(v_2^2 - v_1^2).$$

On substituting in the numerical values, we find that

$$F = \frac{2\ \text{kg}}{2 \times 30\ \text{m}}(400\ \text{m}^2/\text{s}^2 - 100\ \text{m}^2/\text{s}^2) = 10\ \text{N}.$$

Why does this problem have two different solutions? Is it possible that the two different answers are "right" simultaneously?

1.18. Two Hand-Carts

Newton's second law states that equal forces impart equal accelerations to bodies of equal masses. Why then does the hand-cart shown in Fig.

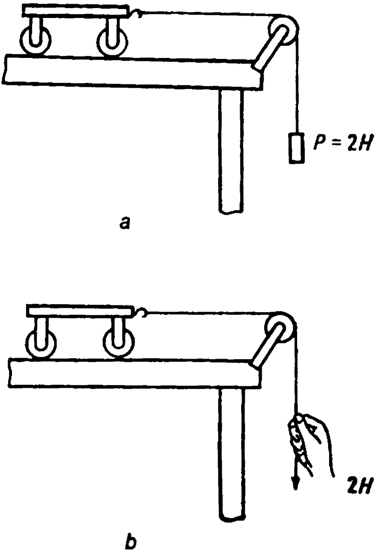

Fig. 1.10

1.10*a* pick up speed slower than the hand-cart shown in Fig. 1.10*b*, while their masses are the same?

1.19. What Is the Acceleration of the Centre of Gravity?

Three identical balls M_1, M_2 and M_3 are suspended on weightless springs I and II one beneath the other so that the distances AB and BC are equal (Fig. 1.11) and the system's centre of gravity matches that of ball M_2. If the thread fastened to ball M_1 is cut, then the whole system begins to fall due to gravity.

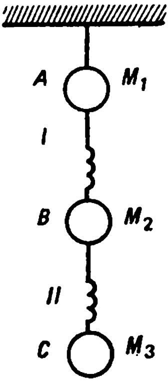

Fig. 1.11

The acceleration of the system's centre of gravity (it is also called the centre of mass or the centre of inertia) is found by dividing the sum of the external forces acting on the system by its mass:

$$\mathbf{a} = \frac{M_1\mathbf{g} + M_2\mathbf{g} + M_3\mathbf{g}}{M_1 + M_2 + M_3} = \frac{3M\mathbf{g}}{3M} = \mathbf{g}.$$

However, the arguments given below seem to disprove this conclusion.

In reality, spring I pulls up ball M_2 stronger than spring II pulls it down, since their tensions before and after the thread is cut are $|\mathbf{f}_{\mathrm{I}}| = 2M|\mathbf{g}|$ and $|\mathbf{f}_{\mathrm{II}}| = M|\mathbf{g}|$, respectively. Hence M_2 (i.e. the system's centre of gravity) must fall with an acceleration smaller than g.

How can this contradiction be resolved?

1.20. A Swift Cyclist

A cyclist is able to develop a thrust of 100 N without any special effort. If the friction force is a constant equal to 50 N and the mass of the cyclist and his bicycle is 100 kg, then the acceleration

$$a = \frac{100\ \text{N} - 50\ \text{N}}{100\ \text{kg}} = 0.5\ \text{m/s}^2.$$

Given this acceleration the velocity after 20 min of cycling will be

$$v = 0.5\ \text{m/s}^2 \times 1200\ \text{s} = 600\ \ \text{m/s}.$$

Why, this is the velocity of rifle's bullet!

1.21. Following an Example by Münchhausen

We heartily laugh when we read how the boastful Baron Münchhausen drags himself and his horse out of a swamp by pulling his hair up. But does not a cyclist mounting a pavement act in the same way? Indeed when the front wheel of his bicycle approaches pavement he pulls up the handle-bars, the front of the bicycle rising, and the cyclist can move from the road to pavement without dashing.

Why does the cyclist succeed where Münchhausen fails?

1.22. The Enigma of Universal Gravitation Forces

The law of gravitation can be written

$$F = \gamma \frac{m_1 m_2}{R^2}.$$

By analyzing this relationship we can easily arrive at some interesting conclusions: as the distance between the bodies tends to zero the force of their mutual attraction must rise without limit to infinity.

Why then can we lift up, without much effort, one body from the surface of another body (e.g., a stone from the Earth) or stand up after sitting on a chair?

1.23. Which High Tides are Higher?

High and low tides are well-known to be due to the attraction of water by both the Sun and the Moon. The Sun is 390 times farther from the Earth than the Moon, while the Sun's mass is 27×10^6 times that of the Moon, so that every object on Earth is attracted to the Sun $\frac{27 \times 10^6}{390^2} =$ 180 times stronger than they are to the Moon.

Therefore high tides due to the Sun should be higher than ones due to the Moon. However, in fact the tides due to the Moon are higher.

How can this paradox be explained?

1.24. In What Way Is Work Dependent on Force and Distance?

When two quantities A and B are directly proportional, the relationship is expressed thus

$$A = kB,$$

where k is called the proportionality factor.

The amount of work A produced by a force F along a distance S is proportional to both the force and the distance. Hence, two equalities must be satisfied, i.e.

$$A = k_1F \qquad (1)$$

and

$$A = k_2S. \qquad (2)$$

By multiplying these equalities term by term we get

$$A^2 = k_1k_2FS. \qquad (3)$$

Let us denote the product k_1k_2 as k_3^2. Then equality (3) can be rewritten

$$A^2 = k_3^2FS.$$

By taking the root of both sides of the equation we get

$$A = k_3\sqrt{FS}, \qquad (4)$$

i.e. the work is proportional to the square root of the product of the force and the distance covered.

But this is not the whole of it! Another manipulation is possible. Let us divide Eq. (2) by Eq. (1), viz.,

$$1 = \frac{k_2 S}{k_1 F}.$$

By denoting the ratio k_2/k_1 as k_4 we get

$$F = k_4 S \tag{5}$$

which means that the force gets larger, the greater the distance covered due to its effect.

How could you explain all these absurdities?

1.25. A "Violation" of the Law of the Conservation of Energy

The following argument seems to prove a violation of the law of the energy conservation.

Suppose that a resting hand-cart of mass m is hit by and retains a projectile of the same mass. The projectile had been flying horizontally before the collision at a velocity v in the same direction as the hand-cart. As a result of this impact the hand-cart and projectile will together set in motion at an initial velocity which can be found from the law of the conservation of momentum

$$v_1 = \frac{mv}{2m} = \frac{v}{2}.$$

Hence, the kinetic energy of the hand-cart and projectile together is

$$W_1 = \frac{2m\left(\frac{v}{2}\right)^2}{2} = \frac{mv^2}{4},$$

while before the collision the projectile had a kinetic energy of

$$W = \frac{mv^2}{2}$$

i.e. twice as large. Thus, after the collision half the energy has vanished altogether.

Can you say where it has gone?

1.26. A Mysterious Disappearance of Energy

By lifting up a bucket of coal to a third floor stove we increase the potential energy of the coal by about 800 J (the force of gravity on the coal is about 80 N and it is raised by about 10 m). Where will this additional potential energy go to when this coal is burnt in stove?

1.27. The Paradox of Rocket Engines

Modern liquid-propellant rocket engines develop a thrust of about 2000 N, with a kilogram of the propellant and oxidizer mixture being burnt per 1 s. Hence, at the minimum velocity needed for launching an Earth artificial satellite (orbital velocity is about 8 km/s) the power developed per kilogram of burnt mixture must be

$$N = Fv = 2000 \text{ N} \times 8000 \text{ m/s} = 16$$
$$\times 10^6 \text{ J/s} = 16{,}000 \text{ kW} = 16 \text{ MW}$$

Meanwhile, the heat of combustion of the commonly used mixture of kerosene and nitric acid is about 6300 kJ/kg (about 1500 kcal/kg), i.e. burning one kilogram of the mixture per second ensures "only" 6300 kW or 2.5 times less than the above value.

How can you explain that at orbital velocity the propellant yields 2.5 times more energy than it is supposed to?

1.28. Where Is the Energy Source?

In order to lift a body above the ground work must be done to increase the body's potential energy. This work comes from different sources. For example, an elevator draws its energy from the main power network, an aircraft takes off utilizing the energy developed when fuel is oxidized (burnt) in its engine.

But what is the source of the energy used to raise stratospheric and meteorological sounding balloons having no engines?

1.29. A Hoop and a Hill

By rolling down a hill of height H a hoop's potential energy decreases by mgH. If friction is negligibly small, then its kinetic energy will be increased by the same value, i.e.

$$mgH = \frac{mv^2}{2}.$$

Whence the final velocity of the hoop is

$$v = \sqrt{2gH}.$$

If H is set to be 4.9 m, find

$$v = \sqrt{2 \times 9.8\ \mathrm{m/s^2} \times 4.9\ \mathrm{m}} = 9.8\ \mathrm{m/s}.$$

However, in practice the velocity of a hoop that has rolled down such a hill is about 7 m/s, i.e. about one and a half less. Such a large discrepancy cannot be explained as being due to friction. Then what is the real reason?

1.30. Brickwork Paradox

In order to build a cornice a bricklayer lays bricks one onto another so that part of each brick juts out over the one beneath. It is interesting to know how much the top brick can overhang the lower one without using cement, mortar, lime or other binder?

At first sight it would seem that it is not too much, about half a brick. However, in reality, with sufficient bricks the top brick can overhang the lowest one by as much as is desired!

Try to prove this.

1.31. What Is True?

The following expressions can be used to calculate centripetal acceleration:

$$a = \frac{v^2}{R} \text{ and } a = \omega^2 R.$$

The first equation shows that centripetal acceleration is inversely proportional to the distance between the moving point and the rotation axis, while the second equation suggests the opposite, that is acceleration is directly proportional to rotation radius. But surely only one equation can be true!

1.32. Is This Engine Possible?

Suppose that water flows down the bent tube shown in Fig. 1.12. Since it moves along an arc a centripetal force acting from the side of tube

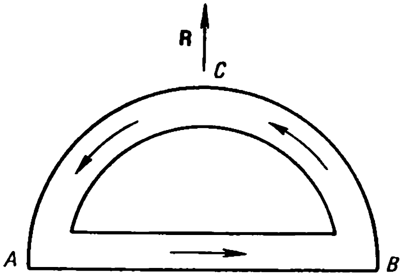

Fig. 1.12

walls onto water arises. In turn, according to Newton's third law there must exist an equal and opposite in direction force, which is sometimes called the centrifugal force which acts from the side of the water onto tube walls. In the figure this force is **R**.

Will the system be set in motion due to **R**?

1.33. Where Will a Car Overturn after a Sudden Turn?

The more suddenly a car, motorcycle, or bicycle has to turn, the larger the centripetal force will be needed to keep it upright and, unfortunately, the more likely the vehicle is to overturn. It can be stated for sure that the larger the centripetal force during a turn, the more probable is an accident.

However, does not it seem strange that the direction in which the car overturns is always opposite to the direction of the centripetal force, i.e. by steering abruptly to the left a car will usually overturn on its right side and vice versa? How can you explain the contradiction?

1.34. A Simple Derivation of the Pendulum Formula

In physics textbooks the formula for the period of swing of a pendulum is given without proof. Meanwhile it is possible to suggest a simple derivation of the dependence of oscillation period on the pendulum's length and the acceleration due to gravity. This derivation does not require any advanced mathematics and we now present it.

At small deflection angles (and it is only for such conditions that the conventional pendulum formula is valid) the arc AB (see Fig. 1.13) can be approximated by the chord AB. Given the isosceles triangle AOB it is possible to write

$$AB = 2OB \cos \alpha = 2l \cos \alpha.$$

The pendulum's motion over this distance can be considered to be a uniformly accelerated one, since the projection P_1 of the force of gravity $\mathbf{P}$ onto the direction of the pendulum's motion, i.e. along AB, is

$$P_1 = |\mathbf{P}| \cos \alpha = m |\mathbf{g}| \cos \alpha.$$

Hence, the modulus of the pendulum's acceleration along AB will be

$$|\mathbf{a}| = |\mathbf{g}| \cos \alpha.$$

Under a uniformly accelerated motion time, distance, and acceleration are related

$$t = \sqrt{\frac{2s}{a}}.$$

By substituting into this equation the acceleration value for motion along AB and its length

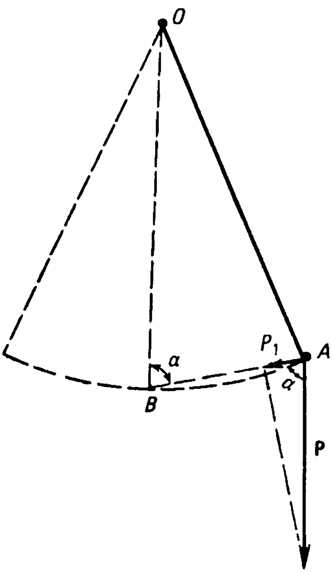

Fig. 1.13

and taking into account that the pendulum period is four times longer than the time it takes to travel along AB, we get

$$T = 8\sqrt{\frac{l}{g}}.$$

Why then does the formula in the textbooks contain the factor 2π, i.e. about 6.28, instead of 8?

1.35. Conical Pendulum

A centrifugal machine has a disc with a wire arc fastened on its axis. From the midpoint of the

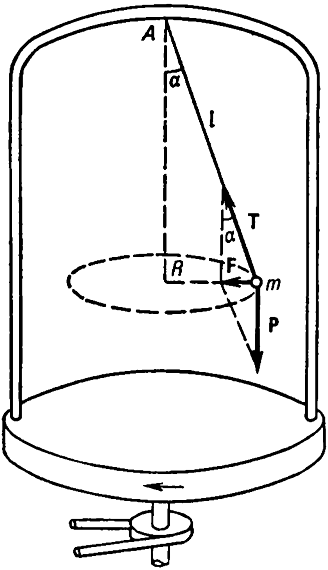

Fig. 1.14

arc a small ball of mass m is suspended on a thread of length l. When disc is at rest the thread falls plumb along the machine axis. When the machine is rotated, the thread with the ball on its end begins to describe a cone in space (whence the name conical pendulum) making an angle α with the vertical as shown in Fig. 1.14. Let us find the angle when the angular velocity of the rotation is ω.

The ball is, evidently, acted upon by only two forces, i.e. the thread tension force **T** and the force of gravity $\mathbf{P} = m\mathbf{g}$. Their resultant **F** is a centripetal force. Since the ball moves within the horizontal plane, the forces **F** and **P** form a right angle. Therefore

$$|\mathbf{F}| = |\mathbf{P}| \tan\alpha = m\,|\mathbf{g}| \tan\alpha.$$

By Newton's second law the modulus of the resultant **F** can be expressed via the centripetal acceleration of the ball $a = \omega^2 R = \omega^2 l \sin\alpha$ as follows

$$|\mathbf{F}| = m\omega^2 l \sin\alpha.$$

By equating both expressions for the modulus of **F** we have

$$m\omega^2 l \sin\alpha = m\,|\mathbf{g}| \tan\alpha$$

or, after reducing by $m \sin\alpha$:

$$\omega^2 l = \frac{|\mathbf{g}|}{\cos\alpha}.$$

This expression rearranges to

$$\cos\alpha = \frac{|\mathbf{g}|}{\omega^2 l}.$$

Assume the thread is 0.2 m long, while the angular rotation velocity of the centripetal machine is 3.5 rad/s. Then

$$\cos\alpha = \frac{9.8\ \text{m/s}^2}{3.5^2\ \text{s}^{-2} \times 0.2\ \text{m}} = 4.$$

However, mathematics only allows the value of the functions up to unity!

What's the matter? Why does physics "conflict" with mathematics?

1.36. Are Transversal Waves Feasible in Liquids?

A textbook once stated that "longitudinal waves can propagate in solids, liquids and gases because changes in their volumes produce elastic forces. Changes in shape in gases and liquids produce no elastic forces, therefore elastic transversal waves cannot propagate in them".

Meanwhile, somewhat earlier in the same textbook it was stated that "If you throw a stone into a pond, you will observe circular *transversal* waves propagating from the point where the stone entered the water".

Thus, the author of the textbook contradicts himself first giving an example of transversal waves in a liquid and then denying they can exist.

Which of his two assertions is true?

1.37. Do We Hear Interference in This Experiment?

A textbook describes the following experiment. When a ringing tuning fork is slowly rotated around its longitudinal axis and placed closely to your ear you will distinctly hear the sound getting periodically louder and softer. This textbook states that the observed effect is due to interference of the waves coming from the tuning fork's two prongs. Let us consider whether this is true.

For the sound to be softer due to interference of the waves coming from different prongs, the oscillations must arrive with a path difference of half a wavelength. For example, if the tuning fork oscillates at a frequency of 440 Hz and the velocity of sound is 340 m/s, such a path difference can be produced along the distance of about 0.4 m, while the separation between the tuning fork's prongs is only about 2-3 cm.

Does this mean that the phenomenon observed has nothing to do with interference?

1.38. Why Is the Sound Intensified?

Generally, the sound produced by a tuning fork is so weak that it can only be heard in its immediate vicinity. However, if the tuning fork is fastened to a resonator, i.e. a rectangular wooden box, its sound will be audible in quite a large room.

Where does the "extra" energy come from? Have we not encountered here a violation of the law of the conservation of energy?

1.39. Will the Buggy Move?

Let a buggy of the shape shown in Fig. 1.15 be filled with water or other liquid—the heavier, the better—e.g. mercury. The average pressure on the right- and left-hand walls is the same since it is due only to the height of the liquid column and its density. However, the right-hand wall is larger in area, therefore the pressure exer-

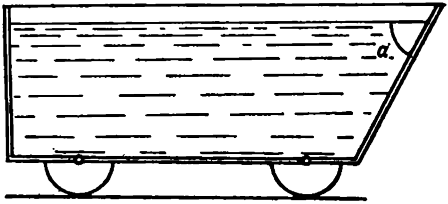

Fig. 1.15

ted upon it is larger. This is why the buggy seems to move to the right.

But isn't it a perpetual motion engine?

1.40. Why Are Submarines Uncomfortable?

People who have read *A Thousand Leagues beneath the Sea* by Jules Verne probably remember how the comfortable and spacious rooms aboard the submarine *Nautilus* delighted Professor Aronax when he was brought on board her.

A spacious dining room, a library as large, a saloon, comfortable cabins, broad passage-ways, and a colossal machine room.

This is quite unlike modern submarines two thirds or even three quarters of whose internal space is occupied by equipment. Crew members may not even have their own hammocks, generally sharing them with crewman on watch. The cramped conditions aboard submarines literally fetter every movement. This is why only the most hardy people are chosen to serve on submarines.

Why then are more spacious submarines not constructed? The reason does not appear to be either to save living space or to ensure Spartan living on a warship, since surface warships (battleships and cruisers) have spacious mess-rooms and, in any case, each crewman has his own permanent place for lodging and recreation.

What prevents submarine rooms from being made more spacious?

1.41. Has Water to Press onto Vessel's Bottom?

Only a few people know that until the end of his life Galileo Galilei (1564-1642) had doubts as to the existence of atmospheric pressure. The honour of discovering is due to Evangelista Torricelli (1608-1647), Galileo's outstanding disciple.

Galileo supported his belief with the following argument. An imaginary volume of water (or any other liquid) inside a larger volume is acted upon by two opposing forces, i.e. the force of gravity and the buoyancy force. According to Archimedes' principle these forces are balanced, therefore the volume is at equilibrium, i.e. it does not float or sink. *We could say that water in water is weightless.* But how can something weightless exert pressure onto lower layers?

Analogously, air in air, Galileo went on, "being itself weightless"[1], cannot exert pressure on

[1] We put this phrase into quotation marks, since we have not used the expression literally: nor did Galileo have doubts about the air ponderosity. Moreover, he was the first to determine its density (1637). Yet, however strange it may sound, Galileo did not believe in the existence of atmospheric pressure.

lower layers and, finally, on the Earth's surface.

What is the fault in Galileo's arguments?

1.42. Hydrostatic Paradox

Figure 1.16 depicts two vessels shaped like right truncated cones. Each vessel has a mass m of

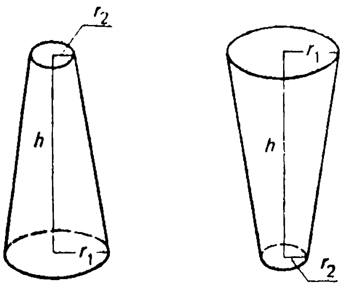

Fig. 1.16

400 g, a height h of 30 cm, but different base areas, $S_1 = 200$ cm² and $S_2 = 50$ cm². The bottom of the first vessel is the larger base, the bottom of the second one—the smaller base.

Let us fill both vessels with water. Since the liquid levels in both vessels have the same height, the pressures p exerted on the bottom will certainly be the same and equal to

$$p = Dgh = 10^3 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times 0.3 \text{ m}$$
$$= 2940 \text{ N/m}^2 = 2.94 \text{ kPa}.$$

Now let us calculate the moduli of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ due to the water pressure on the bottoms of both vessels:

$$|\mathbf{F}_1| = pS_1 = 2.94 \times 10^3 \text{ N/m}^2 \times 2 \times 10^{-2} \text{ m}^2 = 58.8 \text{ N},$$

$$|\mathbf{F}_2| = pS_2 = 2.94 \times 10^3 \text{ N/m}^2 \times 5 \times 10^{-3} \text{ m}^2 = 14.7 \text{ N}.$$

Since the weight of each vessel is $|\mathbf{P}| = m|\mathbf{g}| = 0.4 \text{ kg} \times 9.8 \text{ m/s}^2 = 3.92 \text{ N}$, we would seem to have found that the first vessel exerts pressure on its support with a force whose absolute magnitude is

$$|\mathbf{R}_1| = |\mathbf{F}_1| + |\mathbf{P}| = 58.8 \text{ N} + 3.92 \text{ N} \approx 62.7 \text{ N},$$

while the second vessel exerts a force of

$$|\mathbf{R}_2| = |\mathbf{F}_2| + |\mathbf{P}| = 14.7 \text{ N} \times 3.92 \text{ N} \approx 18.6 \text{ N},$$

i.e. about 3.5 times less.

Thus, if we put the vessels on the weighing scales, the first vessel should overbalance the second, though they are identical (except that one vessel is an upside down version of the second),

The volume of a truncated pyramid is

$$V = \frac{\pi}{3}(r_1^2 + r_1 r_2 + r_2^2)\, h = \frac{1}{3}(S_1 + \sqrt{S_1 S_2} + S_2)\, h$$

$$= \frac{1}{3}(200 \text{ cm}^2 + \sqrt{200 \text{ cm}^2 \times 50 \text{ cm}^2} + 50 \text{ cm}^2) \times 30 \text{ cm} = 3500 \text{ cm}^3$$

and so the weight of water in both vessels should be

$$|\mathbf{P}_0| = VDg = 3.5 \times 10^{-3}\ \text{m}^3 \times 10^3\ \text{kg/m}^3 \times 9.8\ \text{m/s}^2 = 34.3\ \text{N}$$

while the weight of each vessel together with water should be

$$|\mathbf{P}_0| + |\mathbf{P}| = 34.3\ \text{N} + 3.92\ \text{N} \approx 38.2\ \text{N}.$$

Thus, the first vessel presses down on its support with 62.7 N — 38.2 N = 24.5 N more force than it should, while the second vessel exerts 38.2 N — 18.6 N = 19.6 N less force.

That's sheer nonsense, as if an object's weight (here that of a water-filled vessel) changes if rotated 180° around a horizontal axis?!

This contradiction to common sense was called hydrostatic paradox. Although some historians have ascribed its discovery to the French physicist, mathematician, and philosopher Blaise Pascal (1623-1662), it was in fact detected and properly explained by the Dutch scientist Simon Stevin (1548-1620), who became famous for his work on mathematics, mechanics, and engineering. The misunderstanding is due to the fact that Pascal was unaware of Stevin's work, and constructed and described the device to demonstrate the hydrostatic paradox.

What is the origin of the paradox?

1.43. A Physicist's Error

"Errare humanum est" (to err is human) states the Latin proverb. And indeed, even the great people make errors as can be seen from the examples in Problems 1.15 and 1.41. Here is another example.

At the beginning of the twentieth century airships and air balloons were filled with hydrogen. In the battles of the First World War they became easy targets, since if they were hit by bullets or projectiles, the hydrogen invariably exploded destroying the balloon and its crew. The losses were so great that soon the belligerents were forced to stop using the air balloons for military purposes.

However, a strange airship then appeared over London: she was hit repeatedly but with no catastrophic consequences. It turned out that the Germans started using helium-filled airships in 1918.

When this became known, one physicist said: "Helium is twice as heavy as hydrogen, hence the aerodynamic lift of balloons must be halved". In fact, the aerodynamic lift of a helium balloon is practically the same as that of a hydrogen balloon.

How can this be explained?

1.44. The Mystery of Garret Windows

This is what a reader of the Soviet magazine *Knowledge is Power* wrote about: "In autumn and winter the wind blows so strongly in our village that tiles fall off roofs.

"We were discussing how to save tiling, when an old man said: 'You need to put garret windows in the pediments.' We were astonished at this advice yet began to check it only to find that wherever there were garret windows the tiling was safe and sound, elsewhere they had lost tiles. What is the matter?"

Can you explain the "mystery" of the garret windows?

1.45. Why Do Velocities Differ?

We are not astonished if the velocities of ships going in the same direction are different—this can be explained by a difference in design and engine power.

But why can rafts which have no engines float down the river with different velocities, too? It has been even noticed that the heavier the raft the higher its velocity.

Why is this?

Chapter 2

Heat and Molecular Physics

2.1. Do Sunken Ships Reach the Bottom?

All bodies are known to contract under pressure: gases do so most, liquids less so and solids are the most resistant to compression.

Does not all this suggest that the ships which sink in deep waters may never reach the bottom since at great depths water is so compressed that its density exceeds that of the metal used to construct the ship?

Professor Aronax, whom we mentioned in Problem 1.40, stated that during his imprisonment aboard the submarine *Nautilus* he observed ghost ships suspended between the ocean's surface and the bottom.

Could the Professor's statement have been a reality?

2.2. What Is the Temperature at High Altitudes?

Even the first balloonists, who rose to comparatively low altitudes above the Earth's surface, noted a decrease in air temperature. At altitudes of a few kilometres, which is the height of modern passenger jets, the temperature is so low that passengers would be simply frozen to death were not the airplane cabins properly heated.

However, at higher altitudes the so-called inversion happens, i.e. the air temperature begins to rise up. And at a height of a few hundred kilometres the air molecules possess velocities corresponding to temperatures of several thousand degrees Centigrade!

Why then do artificial satellites, flying for long periods of time at such altitudes, neither melt nor burn away?

2.3. In Spite of the Thermal Laws...

There are three identical Dewar vessels *A*, *B*, and *C*. Two of them contain one litre of water each at temperatures 80 °C and 20°C, respectively. Vessel *D* with walls that conduct heat perfectly is empty. It is small enough to fit inside the three Dewar vessels.

Is it possible, by manipulating the four vessels, to heat the cold water using the hot one until its final temperature is higher than the final temperature of the hot water? The water in vessels *A* and *B* must not be mixed.

Generally the problem is thought to be unsolvable because heat transfer can only occur "by itself" from hot bodies to cooler ones and stops as soon as the temperatures of both bodies are the same. Nevertheless, the problem can be solved.

Try to find out the solution.

2.4. Why Doesn't Thermal Insulation Help?

A copper tube with an outer diameter of 1 cm serves as a vapor transfer line. To lower thermal losses it was blanketed with a 5 mm thick layer of thermal insulation. However, the losses not only decreased but, quite the reverse, increased.

Why is this the case?

2.5. Which Scale is More Advantageous?

In some countries temperature is measured using the scale proposed in 1730 by the French physicist René Antoine Réaumur (1683-1757). In this scale the melting point of ice, as in the Celsius scale, is fixed as 0°, but water is assumed to boil at 80° R[2] under normal pressure.

Let us calculate the amount of heat needed to heat 100 g of water to the boiling point from its melting point.

Using the International System of Units (SI) and the Celsius' scale, we get

$$Q_1 = 0.1 \text{ kg} \times 4.19 \text{ kJ/(kg}\cdot{}^\circ\text{C)} \times 100^\circ\text{C} = 41.9 \text{ kJ}.$$

The same calculations performed with the Réaumur scale yield

$$Q_2 = 0.1 \text{ kg} \times 4.19 \text{ kJ/(kg}\cdot{}^\circ\text{R)} \times 80^\circ\text{R} = 33.5 \text{ kJ},$$

i.e. in the latter case we appear to heat water by expending 8.4 kJ less heat.

Is this the case?

[2] In fact, the Swedish astronomer Anders Celsius (1701-1744) proposed a scale, for which the boiling point of water was denoted as 0°, while the ice melting point as 100° The Celsius scale was updated in 1745 by Celsius' compatriot Martin Stromer (1707-1770).

2.6. What Is the Source of the Work?

A system in order to perform work needs energy by the law of energy conservation. Thus, to lift a piston by expanding, the gas under a piston in a cylinder needs to be heated.

However, sometimes the same results can be achieved by acting oppositely. If we fill iron ball with water and seal it hermetically and then cool down the ball to below 0°C by taking away heat, the freezing water will break the ball, i.e. it will perform work.

Where is the energy source for destroying the ball?

2.7. Does a Compressed Gas Possess Potential Energy?

In many trolleybuses, buses, and trams the doors are opened and shut by compressed air. Very often it is also used to drive the brakes.

Any work can only be done at the expense of an energy source (from the formula $A = Fs$ we can, in reality, calculate the amount of energy transferred from one body to another) and sometimes one can hear that "compressed air performs work at the expense of the potential energy". However, this statement is profoundly fallacious.

In fact, around room temperature, moderately compressed air is like an ideal gas, whose internal energy is independent of volume, since the forces of interaction between its molecules are absent. Therefore under compression or expansion the internal energy of air does not practically change.

Then what is the source of the energy for brakes and doors?

2.8. Again Energy Vanishing...

Bending a steel strip we transfer energy to it. If we place a bent strip in a glass so that the walls do not allow the strip to straighten and pour concentrated sulphuric acid into the glass, the strip will gradually dissolve and the energy stored in it will vanish, too.

But can energy vanish in reality?

2.9. Where Does the Energy of the Fuel Burnt in a Rocket Disappear?

Let us imagine a vertically installed rocket. The thrust developed by its engines can vary widely. By regulating the fuel feed it is possible to produce a thrust identical to the rocket's weight. In such a case the rocket, like "Muhammad's coffin", which, according to Islamic belief, levitates without any support, will hang poised over the Earth's surface without falling or rising.

A seeming paradox arises: fuel is burnt in the engines to yield thrust, but the work produced according to the formula

$$A = Fs$$

is zero, since there is no displacement due to force.

Then where does the burnt fuel energy disappear?

2.10. Can a Body's Temperature Be Increased Without Heat Being Transferred to the Body?

The question in the heading might sound absurd, just like "Can we heat a body without heating it?". For all its seemingly absurd nature, the answer is yes.

Try to produce examples of a body whose temperature rose without heat exchange with environment!

2.11. From What Metal Should a Soldering Iron Be Made?

By looking up in a handbook of physical constants we can check that the heat capacity of iron is about 20% larger than copper's. Consequently, for an equal mass and heating temperature the internal energy store in a soldering iron made of iron is 20% larger than one made of copper.

Then why are soldering irons made of more expensive copper instead of cheap iron?

2.12. A Negative Length

The linear dimensions of bodies change with temperature, viz.

$$l_t = l_0 (1 + \alpha t).$$

Suppose that the temperature falls to $t = -\frac{1}{\alpha}$.

By substituting this temperature into the first expression we arrive at

$$l_t = l_0 \left(1 - \alpha \frac{1}{\alpha}\right) = 0!$$

Then let us lower the temperature still further. Could a body's dimensions become negative?

2.13. Is the Law of the Energy Conservation Valid in All Cases?

Figure 2.1 depicts two tubes differing in the positions of their bulges. If we pump air from the tubes, immerse their open ends in a cup filled

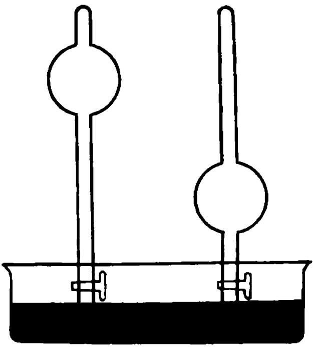

Fig. 2.1

with mercury, and open taps, then atmospheric pressure will drive mercury into the tubes. The work performed is, as is known from a physics textbook,

$$A = pV,$$

where p is atmospheric pressure, and V is the volume filled with mercury. If the internal volumes of the tubes and their bulges are the same, the work done to lift the mercury up will be the same.

However, the level of mercury in the left-hand tube will be higher than that in the right-hand one, whence it follows that for the same work performed the change in the potential energy of the tubes is different. This statement seems in evident contradiction with the law of the energy conservation.

What is the fault in the above analysis?

2.14. The Mystery of Capillary Phenomena

By immersing the end of a fine (capillary) glass tube in water you will observe the water column in the tube to rise. The height to which the water rises is inversely proportional to tube's diameter and in very fine capillaries it may be metres high. As water rises there are no visible changes either in the tube or the water.

What energy source makes capillary phenomena feasible?

2.15. "Clever" Matches

Pour pure water into a neatly washed plate (if there is no distilled water, thoroughly boiled water will do) and throw a few matches onto the water surface.

If you touch the water between the matches with a sugar lump, the matches, as if they have a sweet tooth, will approach the sugar. Yet if you touch water with soap, the matches scatter in every direction.

How can the "intelligent" behaviour of these inanimate objects be explained?

2.16. How Is a Wire Drawn?

Figure 2.2 is a sketch of how a thin wire is drawn from a thick one. As can be seen from the figure, the cross section of wire rod is decreased by passing it through a die hole. Naturally we might ask: why notwithstanding the huge stresses needed to draw the wire, does the thin wire that has passed through the die not break, while the thick one is deformed?

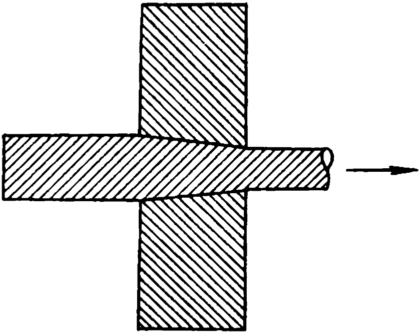

Fig. 2.2

2.17. Boiling Water Cools Down Ice

What happens when hot water is poured onto ice? It seems obvious that the ice will melt either totally or partially. However, if the amount of hot water is not large and the ice's temperature is well below 0°C, the ice will not be melted but only somewhat heated. Yet how can you explain the unexpected result obtained below?

Suppose a litre of boiling water having the temperature of 100°C is poured into a jug containing 1.3 kg of ice at 0°C. The jug has a mass of 1 kg and is made from a material with a specific heat capacity of 0.838 kJ/(kg·°C). What temperature will be set in the jug after the water and ice have mixed?

We set up a thermal balance equation: the heat given up by hot water is 1 kg × 4.19 kJ/ (kg·°C) × (100°C — t), the heat obtained by the jug is 1 kg × 0.838 kJ/(kg·°C) t, the heat needed to melt the ice is 1.3 kg × 335 kJ/kg, the heat needed to heat the water produced from the ice is 1.3 kg × 4.19 kJ/(kg·°C) t.

From the law of the conservation of energy we get:

$$1\text{ kg} \times 4.19\text{ kJ/(kg}\cdot{}^\circ\text{C)}\,(100^\circ\text{C} - t) = 1\text{ kg} \times 0.838\text{ kJ/(kg}\cdot{}^\circ\text{C)}\,t + 1.3\text{ kg} \times 335\text{ kJ/kg} + 1.3\text{ kg} \times 4.19\text{ kJ/(kg}\cdot{}^\circ\text{C)}\,t.$$

By solving this equation we find that $t = -1.6^\circ\text{C}$, i.e. the boiling water has cooled the ice. How can this strange result be explained?

2.18. Why Does Water Evaporate?

Heat transfer from one body to another occurs only when there is a temperature difference between them. Therefore it seems incomprehensible why water in a plate or a glass having the temperature of the surrounding air is gradually evaporated. You know that in order to evaporate a liquid heat has to be transferred to it but the water cannot obtain it from the environment because their temperatures are the same.

Then why does the water evaporate?

2.19. An Italian Question

An Italian student was asked during an exam: "As you know, the boiling point of olive oil is higher than the melting point of tin. Explain why it is possible to fry food in olive oil in a pan." (The best Italian saucepans are made from tinned copper.)

What is the answer?

2.20. How Can Water Be Boiled More Efficiently?

It is known that the boiling point of water falls with pressure. Why then do we not pump the air out of saucepans?

This would allow economizing on fuel, wouldn't it?

2.21. Is It Possible to Be Burnt by Ice or to Melt Tin in Hot Water?

Paradoxically enough both these phenomena are possible.

Could you explain under what conditions?

2.22. How Much Fuel Will Be Spared?

An inventor has learnt about three inventions: the application of the first one would save 30% fuel, the second one would spare 25%, while the third one would spare 45%. Our man decided to construct a machine to combine all three inventions and he aimed to spare 30% + 25% + 45% = 100%.

To what degree are the hopes of the inventor justified?

2.23. How Many Heat Capacities Has Iron?

Two iron balls with the same diameter are heated from 20°C to 100°C. One is placed on a horizontal plane, while the other one is suspended by a

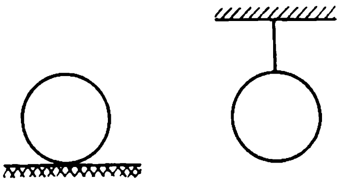

Fig. 2.3

virtually inextensible thread (Fig. 2.3). It would seem that the formula

$$\Delta Q = mc\,\Delta t$$

implies that the heat transferred to the balls must be identical. In fact, the quantity of heat transferred to the first ball is somewhat larger. This means that the heat capacity of the iron used to manufacture the first ball is also larger, although the balls were cut from a single ingot.

Why might this be? What is the heat capacity given for iron in tables?

2.24. Why Heat Stoves?

Suppose that in late autumn you visit your dacha in the country. You find the temperature in the rooms to be about 0°C and decide to warm the room using the stove. Let us calculate the changes in the internal energy of the air in the room if the thermometer's mercury column climbs to the level of 20°C.

As we mentioned in Problem 2.7, under near-to-normal conditions (i.e. at temperatures close to 0°C and under pressures of about 10^5 Pa) air behaves like an ideal gas, whose internal energy is proportional to the mass m and absolute temperature T of the gas

$$U = \alpha\, mT,$$

where α is a proportionality factor dependent on the gas and it can be shown that it is equal to the quantity of heat needed to heat a unit mass of the gas by one degree Centigrade at a constant volume; therefore in thermodynamics it is called the specific heat capacity at a constant volume and denoted as C_V.

When the rooms of dacha are heated the air warms and expands, so some of it passes under the closed doors, and through cracks in the walls and windows. The mass of air inside a room is changed, but the volume of the rooms and the air pressure remain constant. Therefore the change in the air's internal energy can be written as

$$\Delta U = U_2 - U_1 = \alpha m_2 T_2 - \alpha m_1 T_1$$
$$= \alpha\,(m_2 T_2 - m_1 T_1).$$

To solve the latter equation, we use Clapeyron's equation:

$$pV = \frac{m}{\mu} RT$$

Since, as we noted above, the pressure in the room always remains the same as the external one and the volume of the room is constant (only the mass of air is changed), we obtain

$$m_2 T_2 = m_1 T_1 = \frac{pV\mu}{R} = \text{const.}$$

Thus, the firewood is burnt but the internal energy of the air in the room remains unaffected.

Nevertheless, we do heat living rooms. Where does the energy which is released by burning the firewood go?

2.25. Why Is Such a Machine Not Constructed?

It is clear why no one has been able to construct a machine which operates without any energy source. The impossibility of a perpetual motion machine results from the law of the energy conservation whose validity is supported by the century-old experience of the whole mankind.

However, why has nobody succeeded in designing a machine working by the cooling sea water? The prospect seems tempting!

The volume of water in the oceans of the world is 1.37×10^9 km^3 (about 1/800th of the total volume of our planet). By assuming, for simplicity, that the density of sea water is the same as that of fresh water, we find the mass of water is about 1.4×10^{21} kg. Since the heat capacity of water is about 4.2 kJ/(kg·K), on cooling all the water in the oceans by 1 K

$$1.4 \times 10^{21}\text{kg} \times 4.2 \times 10^{3} \quad \text{J/(kg·K)} \times 1\text{K}$$
$$\approx 6 \times 10^{24}\text{J}$$

of heat would be released.

Since all the electric power stations in the world generate "only" about 2×10^{19} J per year this means that cooling the oceans would release so much energy that, at present consumption levels, it would be enough to last mankind for hundreds of millennia. Practically this sort of device would be a type of perpetual motion machine. In scientific terms it is called a perpetual motion machine of the second kind.

In passing note that such a machine would not violate the law of the conservation of energy.

2.26. When Is Car's Efficiency Higher?

The maximum efficiency of a heat engine with a heater and cooler can be calculated from the following formula

$$\text{Eff} = \frac{T_1 - T_2}{T_1},$$

where T_1 and T_2 are the absolute temperatures of the heater and cooler, respectively.

This expression shows that, the temperature of the heater remaining the same, the heat engine's efficiency increases as the cooler's temperature falls.

Why then does a car (which is also a heat engine) consume much more petrol in winter than in summer? The temperature of atmospheric air, which is car's cooler, is noticeably lower in winter, while the temperature of the gases produced by the combustion of petrol is the same both in winter and in summer.

2.27. Is Maxwell's Demon Feasible?

The last two sophisms show that for a heat engine to work there must be two bodies with different temperatures, i.e. a heater and a cooler. If there is no temperature difference, the heat engine will not operate, which is formally a result of the formula for efficiency given in the last problem.

However, is it impossible to design a device in which a temperature difference can arise inside it as a result of the engine operation itself? The English physicist James Clark Maxwell (1831-1879) proposed one in the mid-1800's.

Let us imagine, Maxwell wrote, two chambers separated by a trap door (Fig. 2.4). The trap

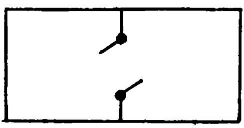

Fig. 2.4

door has a control mechanism (Maxwell called it a demon) that can distinguish between fast and slow molecules. If the demon opens and shuts the trap door so that fast molecules are allowed to go from right to left, while slow molecules are allowed to go in the opposite direction, then after a while all the fast molecules will be in the left-hand chamber, while all the slow ones will be in the right-hand chamber. As a result, the temperatures in the two chambers will be different, since a gas's temperature depends on the velocities of its molecules.

However, a temperature difference would induce an operation of heat engine. After the temperatures in both chambers were equalized by the operation of the engine, the process of sorting the molecules could be repeated and so on until the machine wears out.

Thus we would have a perpetual motion machine, would we not?

Chapter 3

Electricity and Magnetism

3.1. Is Coulomb's Law Valid?

The force F attracting the plates of a parallel plate air capacitor can be calculated by multiplying the charge Q of one plate by the electric strength E of the field produced by the charge on the second plate, i.e.

$$F = QE.$$

For a parallel plate capacitor the field between the plates is uniform and the field strength is independent of the distance between the plates, viz.

$$E = \frac{Q}{2\varepsilon_0 S}$$

where ε_0 is the dielectric constant and S is the plate area (this and the following formulas are written for SI units). Therefore the attraction force between the plates must be unaffected by changes in the plate separation.

Doesn't this contradict Coulomb's law, i.e.

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$$

according to which, the force of interaction between charges is inversely proportional to the square of distance R between them?

3.2. Should a Current Flow Through a Conductor Which Shorts Battery Poles?

Let us consider the two electric circuits in Fig. 3.1. If the current does not flow along conductor *A1B*, it will also be absent in conductor *A2B*,

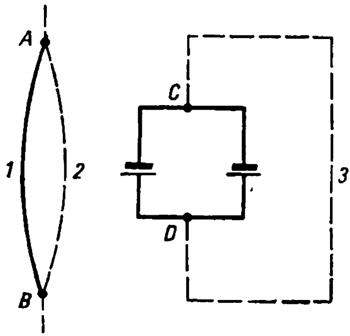

Fig. 3.1

since they are both connected to the same points, *A* and *B*.

If two identical galvanic cells are connected in parallel there will be no current in either cell. Thus, there is no current between points *C* and *D*, as we saw occurs between *A* and *B* in the first case. However, by reasoning in an analogous fashion, we would conclude that there should also be no current in the conductor *C3D*, which is connected to the battery, i.e. points *C* and *D*.

Does this not contradict experience?

3.3. Is the Current in a Branch Equal to That in the Unbranched Part of the Circuit?

Two electrical bulbs are connected as shown in Fig. 3.2. By denoting the current in *Bulb 1* as

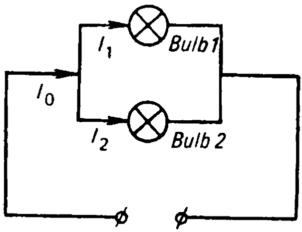

Fig. 3.2

I_1 and that in *Bulb 2* as I_2 we can write

$$I_1 + I_2 = I_0,$$

where I_0 is the current in the unbranched part of the circuit. By multiplying both sides of this equation by $(I_0 - I_1)$, we obtain

$$I_1 I_0 - I_1^2 + I_0 I_2 - I_1 I_2 = I_0^2 - I_0 I_1.$$

Let us transpose the third term from the left-hand side to the right-hand side:

$$I_1 I_0 - I_1^2 - I_1 I_2 = I_0^2 - I_0 I_1 - I_0 I_2.$$

Now we factor I_1 out from the left-hand side and I_0 from the right-hand side to get

$$I_1 (I_0 - I_1 - I_2) = I_0 (I_0 - I_1 - I_2).$$

By dividing both sides of this equation by the bracketed term we have

$$I_1 = I_0!$$

Had we multiplied the initial equation by $(I_0 - I_2)$ instead of $(I_0 - I_1)$ we would have $I_2 = I_0$!

What is the matter?

3.4. What Current Can an Accumulator Battery Generate?

An accumulator with lead-acid cells has an internal resistance of 0.1 Ohm and bears the label "Emf 4 V, maximum discharge current 4 A".

Meanwhile, by shorting the accumulator poles with a conductor whose resistance is also 0.1 Ohm we would have

$$\frac{4\ \mathrm{V}}{0.1\ \mathrm{Ohm} + 0.1\ \mathrm{Ohm}} = 20\ \mathrm{A},$$

i.e. a current 5 times the value advertised.

What is the reason for the discrepancy?

3.5. How Can Galvanometer Readings Be Decreased?

During a physics practical lesson the students decided to measure temperatures using a thermocouple and a galvanometer. However, at the end of the experiment the temperature had risen so much that the galvanometer pointer was going off scale. To decrease the galvanometer's sensitivity, one student proposed connecting in parallel a resistance box with the same resistance as galvanometer. He believed that the galvanometer would then pass only half the total current and the pointer would not sweep off-scale. The proposal was accepted, but the young physicists were surprised to find that the resistance box did not affect the galvanometer readings. After thinking for a while, they understood what had happened.

How did they explain their results?

3.6. Why Did the Current Fall?

To increase the current flowing in a circuit, an additional cell was connected in series with the galvanic cell used as the energy source. The current not only did not increase, but, quite the opposite, fell significantly.

In what case should this be possible?

3.7. What Is the Resistance of an Electric Bulb?

By measuring the resistance of a 100 W electric bulb with ohmmeter, a schoolboy found it to be 35 Ohm. To check the result, he decided to calculate the resistance from power and the rated voltage (220 V) indicated on the bulb.

Using the formula $R = U^2/W$, the schoolboy was surprised to get 484 Ohm, i.e. about 14 times higher than he had measured.

How can you explain this difference in the results?

3.8. What Will a Voltmeter Indicate?

The potential difference between any two points of an electric circuit can be measured by a voltmeter connected to these points. On the other hand, the voltage can be found from Ohm's law by multiplying the resistance of the section of the circuit between these points by the current running through it.

Consider a circuit composed of two identical galvanic cells connected as shown in Fig. 3.3.

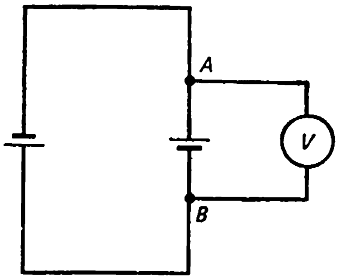

Fig. 3.3

By denoting the electromotive force of the cells as E and their internal resistance as R, the current in the circuit will be

$$I = \frac{2E}{2R} = \frac{E}{R}.$$

It would seem that if a voltmeter is connected to A and B it should indicate the potential difference

$$\varphi_A - \varphi_B = IR = \frac{E}{R} R = E,$$

since the current running in the circuit is E/R and the resistance of the section of the circuit connected in parallel with the voltmeter is R.

In fact, the voltmeter will read zero. A paradoxical situation that seems improbable at first sight has arisen: the circuit is passing current, yet the potential difference between its ends is zero.

Why is this possible?

3.9. What Value Must the Resistance Have?

Consider the circuit in Fig. 3.4. The resistance of the load is R and the resistance of the source is

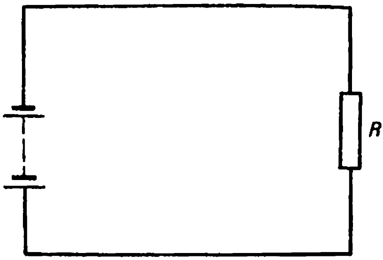

Fig. 3.4

r. After rearrangement we obtain the following expression for efficiency of electrical power:

$$K = \frac{W_{\text{used}}}{W_{\text{total}}} = \frac{I^2 R}{I^2 (R+r)} = \frac{R}{R+r}.$$

This formula can be rewritten as

$$K = \frac{1}{1+\frac{r}{R}}.$$

The last expression shows that the more R exceeds r, the higher the electrical power efficiency, in other words, the more efficient the whole installation will be.

Why then are the load and the source resistances matched to be as equal as possible, even though the efficiency will thus only be 50%?

3.10. How Much Current Does the Device Consume?

A device consuming 50 W is connected through another resistor of 40 Ohm to a power supply of 120 V.

Let us calculate the current through the device from these data.

To solve the problem, note that the voltage across the device and the voltage across the other resistor must be equal to the sum of the network voltage, i.e.

$$U_{dev} + U_{res} = U_{net}.$$

By expressing the first term on the left-hand side in terms of the power consumed by the device divided by the current running through it and the second term, as the product of the other resistance and the same current, we obtain the following equation:

$$\frac{W}{I} + IR = U_{net}.$$

All the quantities here except for the current are known. By substituting in the numerical values we get

$$\frac{50}{I} + 40I = 120,$$

or

$$40I^2 - 120I + 50 = 0.$$

If we solve this quadratic equation we obtain two values for the current, viz. $I_1 = 0.5$ A and $I_2 = 2.5$ A.

Which current will flow through the device?

3.11. The Mystery of Electrolysis

The following currents flow through any cross section drawn between electrodes perpendicular to the direction in which the ions move in an electrolylic bath

$$I_+ = q_+ n_+ v_+ \text{ and } I_- = q_- n_- v_-,$$

where q_+ and q_- are the charges on the positive and negative ions, respectively, n_+ and n_- are their concentrations, and v_+ and v_- are their velocities. Thus, the total current is

$$I = I_+ + I_- = q_+ n_+ v_+ + q_- n_- v_-.$$

At the same time the deposition of substance during electrolysis occurs due to the ions of a *single* polarity neutralized at the electrodes, for example, positive ions will be deposited at the cathode. Therefore, it would seem that the mass of the substance deposited at the electrode has to be calculated from the current I_+. Why, in fact, is the total current, i.e. the sum $I_+ + I_-$, used in the calculation?

3.12. How to Improve the Efficiency of an Electrolytic Bath

If the electrolytic current is I, then the mass of the substance (m) deposited on the electrodes during time t can be calculated from Faraday's law, viz.

$$m = kIt,$$

where k is the substance's electrochemical equivalent.

Suppose n identical baths are connected in series. Since the current in series circuit is everywhere the same, n times more compound is deposited overall than in one bath.

Doesn't this mean that the efficiency of the new installation is n times higher than that of the first one?

3.13. Once More about the Conservation of Energy

Let the capacity of each of the capacitors *C1* and *C2* shown in Fig. 3.5 be 20 μF and switch

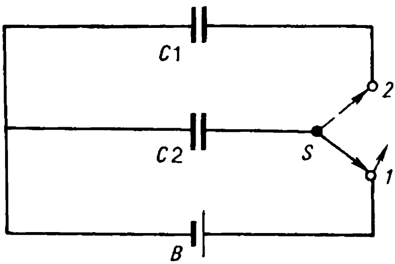

Fig. 3.5

S be set initially in position *1*. Capacitor *C2* is thus connected to the battery *B*. If its electromotive force is 100 V, then *C2* will store an energy of

$$W_1 = \frac{CU^2}{2} = \frac{20 \times 10^{-6}\ \text{F}\,(100\ \text{V})^2}{2} = 0.1\ \text{J}.$$

If then the switch is set in position *2*, the capacitors will be connected in parallel forming a battery with the capacity of $2 \times 20\ \mu F = 40\ \mu F$. The potential difference across the terminals will be 50 V, i.e. half the voltage developed across capacitor *C2*, because the initial charge is now divided into two.

By using these data let us calculate from the above formula the energy stored in the battery, i.e.

$$W_2 = \frac{40 \times 10^{-6}\ F\ (50\ V)^2}{2} = 0.05\ J.$$

This is only half the energy initially possessed by capacitor *C2*.

Where has the other half of the energy disappeared?

3.14. Why Does the Energy in a Capacitor Rise?

A plane capacitor with a capacitance C_1 1 μF is made using a thin glass plate, with a relative dielectric permittivity of $\varepsilon_{rel} = 5$, as the dielectric, and is charged to a potential difference $U_1 = 100$ V.

Using the formula given in the last problem we obtain for the energy stored in the capacitor

$$W_1 = \frac{1 \times 10^{-6}\ F \times 10^4\ V^2}{2} = 0.005\ J.$$

When the glass is removed, the capacitance is decreased by ε times to $C_2 = C_1/\varepsilon = 0.2\ \mu F$. Since the capacitor charge remains intact, the potential difference between the capacitor plates has been increased as much as the capacitance has been decreased ($q = CU$), i.e. to $U_2 = 500$ V

Whence, after removing the dielectric, we have for the capacitor energy

$$W_2 = \frac{0.2 \times 10^{-6}\ \text{F} \times 25 \times 10^4\ \text{V}^2}{2} = 0.025\ \text{J}.$$

What is the source of the extra energy? The capacitor has not been connected to a current source, has it?

3.15. A Single-Pole Magnet

It is commonly believed that each magnet must have two poles, N and S. However, the argument below seems to disprove this.

Let us take a steel ball and cut it through from the surface to the centre into pyramidal polyhed-

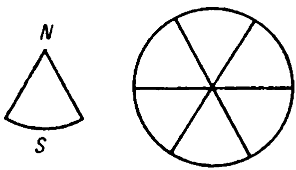

Fig. 3.6

ra. Then we magnetize these so that their vertices are like poles and then reassemble the ball, as shown in Fig. 3.6.

As a result, there will only be one pole on its surface. Hence, is it possible to produce a single-pole magnet?

3.16. Where Is the Energy Source of a Magnet?

Let us bring a magnet from above onto an iron object. If the weight of the object and the distance between it and the magnet are not too large, the iron object will be attracted to the magnet. If the object's weight is P and the vertical distance to the magnet is h, then the work of the magnet against gravity is $A = Ph$.

The magnitude of the work in any particular case is not large, yet the experiment can be repeated as many times as desired without visible changes with the magnet and its "magnetic force" does not weaken.

Does this not violate the law of the conservation of energy?

3.17. Are the Resistances of All Conductors Identical?

A metallic ring (Fig. 3.7) is placed in an alternating magnetic field. As a result, an induction cur-

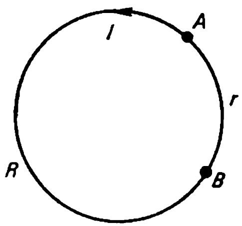

Fig. 3.7

rent arises in the ring, which at a certain moment is I.

We choose two arbitrary points on the ring A and B and use R to denote the resistance of the larger arc of the ring separated by these points, and r for the resistance of the smaller arc.

By Ohm's law, the potential difference across ArB in terms of the current and the resistance of the smaller arc is

$$\varphi_A - \varphi_B = Ir. \tag{1}$$

The same argument yields the potential difference for larger arc BRA, viz.

$$\varphi_B - \varphi_A = IR. \tag{2}$$

Since both parts have the same end points the left-hand sides of both equations must be the same, because each point can only have one potential value. Whence we conclude that the right-hand sides of both equations must be the same, too, i.e.

$$Ir = IR.$$

By cancelling out the current I, we arrive at the absurdity that

$$r = R.$$

Note. It would certainly be more logical to equate both right-hand sides of eqs. (1) and (2) but with opposite signs. However, the final result would then be even more absurd, viz.

$$r = -R.$$

3.18. Does Transformation Ratio Change for a Variable Transformer Load?

When a larger load is connected to a transformer power consumed from the network is increased. Hence, the current in the primary coil increases, too. The higher current must magnetize transformer core more strongly and while before the maximum magnetic flux was Φ_1, after the increase in the load it will be $\Phi_2 > \Phi_1$.

The electromotive force induced in the secondary coil is determined by the number of turns and the rate of change of magnetic flux in time:

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t}.$$

With the initial load the magnetic flux has changed from 0 to Φ_1 during a quarter of a period, while for the larger load it changed from 0 to Φ_2. Since $\Phi_2 > \Phi_1$, the rate of change of magnetic flux is larger for the larger load. Therefore the electromotive force induced in the second coil must also increase.

In practice the transformation ratio is independent of the load which means that our reasoning was faulty.

Where exactly did we go wrong?

3.19. At What Voltage Does a Neon Lamp Ignite?

To determine the ignition potential of a neon lamp a setup was assembled whose scheme is given in Fig. 3.8. If the setup is plugged into an AC mains and the potentiometer slide arm made to move upwards, thus increasing the voltage applied across the lamp, the latter will flash when the voltmeter (an electromagnetic one)

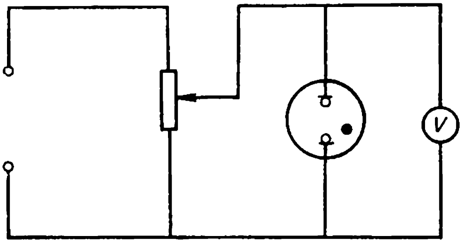

Fig. 3.8

reads 50 V. If we repeat the experiment by plugging the apparatus into a DC electricity source, the lamp will flash when the voltmeter pointer approaches 70 V.

What is the actual ignition potential of a neon lamp?

3.20. Which Ammeter Readings Are Correct?

The circuit in Fig. 3.9 was assembled with amme-

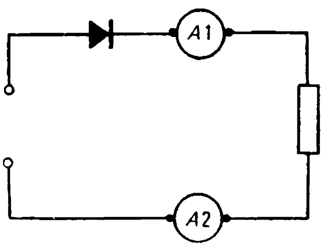

Fig. 3.9

ters, one of a moving-coil type and another of an electrodynamic type. Both devices were tested and there is no doubt that they are in good repair.

However, when the circuit was connected to an AC mains the second ammeter gave readings more than one and a half much those on the second ammeter.

What is the reason for this discrepancy?

3.21. Why in a Series Circuit Is the Current Different?

Two ammeters *A1* and *A2* are connected in the circuit in Fig. 3.10. They are both accurate but their readings differ when switch *S* is closed.

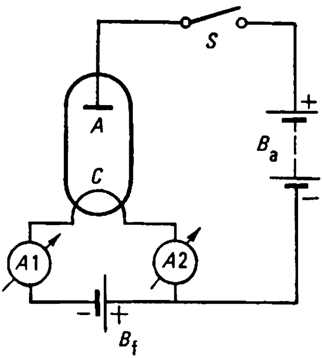

Fig. 3.10

Why? Don't they record the same current running in the filament of the electron tube shown?

3.22. How Can the Decrease in Temperature Be Explained?

When the switch S in the circuit in Fig. 3.10 is closed, filament's temperature falls. Meanwhile, this seems not to be the case.

In fact, when the switch is open, only a filament current I_f runs through the filament, releasing the heat per unit time of

$$Q_1 = RI_f^2,$$

where R is the filament resistance.

When the switch is closed, the current through the left-hand half of the filament increases by 0.5 of anode current, while that through the right-hand half decreases by the same value (see the solution of the preceding problem).

Hence, we do not err much to assume these currents to be the same:

$$I_1 = I_f + 0.5I_a \text{ and } I_2 \quad I_f - 0.5I_a.$$

Thus, the heat released by both currents per unit time is

$$Q_2 = \frac{R}{2} I_1^2 + \frac{R}{2} I_2^2 = RI_f^2 + 0.25RI_a^2,$$

i.e. more than with the switch open.

Then why does the temperature fall?

3.23. Why Is the Magnetic Field Unchanged?

A laboratory was investigating the behaviour of semiconductors in AC magnetic field. In order to produce the magnetic field a coil was wound around a cardboard spool and the AC mains current passed through it. Then an alternating magnetic field is produced inside the coil in which a specimen can be placed.

Since it was desirable for the experiment that the magnetic field be as intense as possible, the technical assistant wound *three identical* coils, one on top of the other, expecting to produce, when connected in parallel, a field three times more intense.

However, it was demonstrated that the magnetic field of the three coils was about the same as that of a single one. The laboratory head whom the technical assistant asked for help explained what had happened although he noted that the triple-coil circuit was not senseless.

Why did the field remain unchanged and why, nevertheless, were three coils better than one?

3.24. How to Check Fuses?

In apartment No. 19, where I lived, the light once went out suddenly. Using a control lamp it was cleared up that the electricity to the apartment's fuse box had ceased. Looking for the cause of the damage I went to main's staircase fuse box with the control lamp to check the fuses. (A four-wire supply circuit is used in our apartment block as shown in Fig. 3.11.)

To check the fuses, I reasoned it would be sufficient to plug the control lamp between points *A* and *D*. However, I couldn't do this, because the connections to the "zero" wire were thoroughly insulated, which I did not want to remove.

Well, I decided, then the control lamp must be plugged between points *A* and *B* or *A* and *C*.

It will only flash if both fuses are in good repair, those for wires *3* and *2* or *3* and *1*, respectively.

To my great surprise, the lamp flashed in both cases. It also flashed when I plugged it between

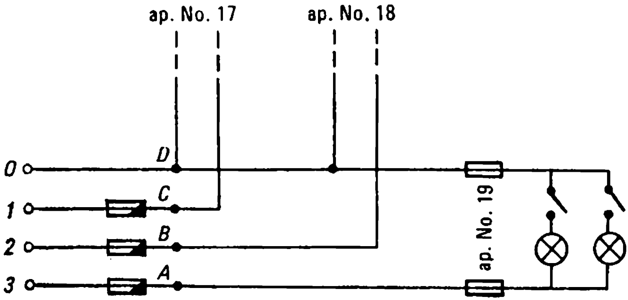

Fig. 3.11

points *B* and *C*, although that test was certainly unnecessary.

Everything is clear, I thought, there is a break in the wiring between the fuse box and my apartment! However, as I was returning I rechecked my deductions and

Meanwhile, you think it over, too.

3.25. Why Did the Lamps Flash?

Practical lessons are being held in two adjacent rooms. The bell rang and the pupils took their seats. One quickly assembled a circuit and after showing to the teacher, he plugged it into the mains. However, the circuit did not work. Whilst the first pupil was checking the connections, other pupils assembled and plugged their circuits into mains, yet none of these operated. They soon found out that electricity had stopped. Suddenly a current appeared, but the voltage was somewhat higher than normal. Attempts to detect the cause of the distortion led to an unexpected "discovery": it turned out that the mains voltage appeared whenever an electric stove in the room next door was plugged into the mains. Once it was switched off, the current disappeared. The pupil who was operating the stove noted that it was not heated as much as it should.

Can you help the pupils find the cause of these phenomena?

3.26. Why Are the Voltmeter Readings Different?

A voltmeter of electromagnetic type was plugged directly into AC mains, and gave a reading of

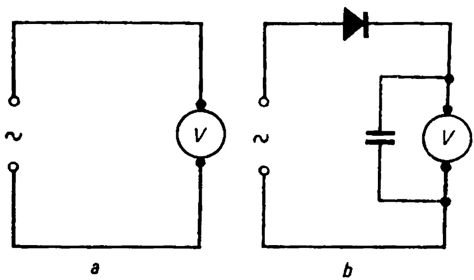

Fig. 3.12

125 V Then the same voltmeter was plugged into the mains through a rectifier (for example, a selenium column or germanium diode) as shown in Fig. 3.12.

Since a rectifier only passes current in one direction, i.e. 50% of total time, the voltmeter readings should, as it would seem, be half as much. In fact, the voltmeter gave a reading of 175 V! How can this be explained?

3.27. Six Hectowatts "Are Equal to" Sixty Kilowatts!

We know that 2 hW = 200 W and 3 hW = 300 W.

By multiplying both equations we obtain

6 hW = 60,000 W or 6 hW = 60 kW!?

3.28. The Certificate of an Electric Motor

The rating plate on an AC motor contains the specifications: $V = 220$ V, $I = 5$ A, and $W = 0.9$ kW.

However, if we multiply the two first numbers we get 1.1 kW.

Why then does the plate indicate a power of 0.9 kW?

3.29. Will the Capacitor Be Charged?

Attempts to construct perpetual motion machines still continue in our days, too. Officials at the Soviet Government's Committee for Inventions and Discoveries report on an average of eight patent applications per month for such machines. Some of these projects are very interesting.

This is one example. It is known that in the absence of an electric field electrons moving in any conductor are in a state of perpetual motion. The total randomness of the motion might lead to a situation in which the upper part of a conductor (see Fig. 3.13) contains more electrons than the lower one. Here the question is electron density fluctuations. These fluctuations will result in a potential difference across the conductor ends

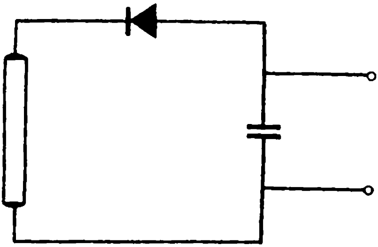

Fig. 3.13

which can be used to charge a capacitor. A detector will prevent the capacitor from discharging when the potential difference across the conductor ends changes sign. The charged capacitor can then be used as a source of "gratuitous" energy. The power will of course be low, but it is the principle that is important.

3.30. A Strange Case of Iron Magnetization

In 1827 a French scientist Félix Savart (1791-1841) discovered that after a Leyden jar was discharged through a wire wound around an iron spoke, the spoke was often magnetized. The most interesting thing about this was that the same spoke end could acquire either an N-pole or an S-pole, even if the discharge current was always in the same direction, the jar always being charged in the same way.

The first exhaustive explanation of the phenomenon was provided by the German physicist Heinrich Hertz (1857-1894).

How could he do it?

Chapter 4

Optics and Atomic Structure

4.1. A Simple Method of Travelling into the Past

In a science fiction story by the French astronomer Camille Flammarion (1842-1925) the following method of looking into the past was proposed.

Light rays transport visual images of the world though very quickly but not instantaneously, as was once believed. Suppose an observer moves away from the Earth. So long as his velocity is low, light waves will catch up with the experimentalist and he will see images of events that occurred on the Earth after his departure. However, once he starts moving fast enough he will begin to overtake the light waves. He will pick up image that overtook him and then images of events that occurred before he left. The events will thus evolve before him in reverse order, i.e. he will observe his departure from the Earth, will be "present" at his own birth, will be able to observe the events from the distant past, and will see great personalities long since dead.

It is not difficult to imagine what an invaluable aid this might be to scholars studying the distant past, historians, paleontologists, archeologists, i.e. the people who now study the past from books and relics of bygone days.

This project would, of course, require a powerful telescope and powerful engines to accelerate the rocket to the velocities needed.

Doesn't it seem to you that our century, when man controls atomic energy and subjugates outer space, is the right time to consider sending an expedition into the remote past?

It remains only to regret that this "time machine" would not, as Herbert Wells's one could, show us the future!

4.2. The Overalls of a Metallurgist

Steel workers labour under hard conditions, dealing with molten metal which with the hot breath could burn man to a cinder. It would seem that to protect a steel worker at blast furnaces, open-hearth furnaces his overalls should be made of a material with low thermal conductivity. However, these overalls are often covered with a thin layer of a metal which is an excellent heat conductor.

What is the reason for this?

4.3. Where to Place a Mirror?

The closer to window we stand, the more we see of the street. It would be natural to assume the same situation happens when using a mirror. Actually, this is not the case.

We look in a mirror vertically hanging on the wall and only see down to knees. All our attempts to see more by approaching the mirror or moving away from it fail.

What is the difference between these two cases?

4.4. An Uncommon "Mirror"

Everybody knows that a plane mirror reflects light rays back to the light source only if the light is incident normal to the mirror, i.e. at 0°. If the mirror is tilted slightly, the reflected light rays will miss the source. However, a device can be constructed to reflect light rays along the path they travelled whatever the angle of incidence. You would be able to see your own reflection in such a "mirror" irrespective of your position relative to the "mirror"!

Try to design an optical system that might possess such an interesting property.

4.5. Why Does a Rainbow Happen?

When explaining the origin of rainbows it is assumed that a light ray falling upon a raindrop is totally reflected from its back wall and then passes through the front one. Each transition from one medium to another is accompanied by refraction, which results in the origin of the rainbow.

However, it can easily be shown that after one total reflection a light ray can never leave the drop to return to the air.

Suppose a ray of light enters a drop and propagates along *AB* so that the angle of incidence *1* at the drop's back wall, i.e. the angle ray *AB* makes with radius *OB* (where *O* is the centre of the drop), exceeds the critical angle (see Fig. 4.1). Then at point *B* we would have total internal reflection, after which the light would follow *BC*.

Since *COB* is an isosceles triangle, we have $\angle 3 = \angle 2$ and, due to the second law of reflection, $\angle 2 = \angle 1$. Thus, if angle *1* exceeds the critical angle, the same will be true of angle *3*. In other

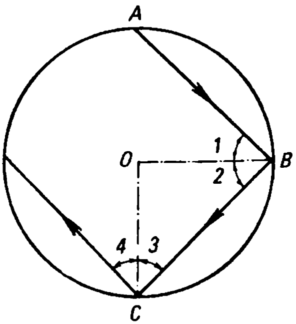

Fig. 4.1

words, we would also have total internal reflection at *C*. The argument would continue indefinitely.

How then do you explain the origin of a rainbow?

4.6. Is It Possible to Increase Illumination Using a Diverging Lens?

A converging lens and a screen are placed perpendicular across a parallel beam. A circular spot of differing diameters can be obtained on the screen by adjusting the lens. Naturally, when the spot's area is changed, the illumination within it does the same.

If the illumination produced by the beam on the lens surface is E_1, and the lens diameter is

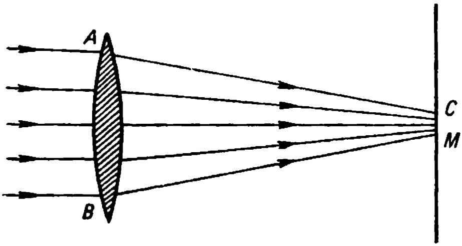

Fig. 4.2

AB (see Fig. 4.2), then the light flux passing through the lens will be

$$\Phi = E_1 \frac{\pi (AB)^2}{4}.$$

Since this light flux is distributed on the screen over the area of a circle of diameter CM, the illumination inside the spot will be

$$E_s = \Phi \quad \frac{\pi (CM)^2}{4} = E_1 \frac{(AB)^2}{(CM)^2}.$$

This expression demonstrates that, with the spot diameter less than that of the lens, the illumination on the screen will exceed the illumination produced directly by the light beam.

Is it possible to increase the illumination using a diverging lens?

4.7. The "Vice Versa" Lenses

We often use the expressions "a biconcave lens" and "a diverging lens" as synonyms. However, the biconcave lens is not always diverging, while a biconvex one is not always converging light.

Can you guess when these lenses might be "interchanged"?

4.8. When Do We Need a Longer Exposure Time in Photography?

A man was photographed upright, then a close-up was taken of his face. Although the illumination of the pavillion remained the same, the photographer decided to increase the exposure time.

Why did he do this?

4.9. A Wonderful Eye

When looking through air at a river bottom you can see it clearly through the transparent water. But, when you dive in the river with open eyes, the shapes of all the objects on the bottom are blurred and indistinct. This is because human eyes do not have enough refracting ability to see clearly in water.

By contrast, fish have an almost spherical crystalline lens with which they see well under water, but become short-sighted in air.

Is it possible to design an eye that would see distant objects in both air and water?

At first sight the task seems impracticable, yet under some conditions such an eye is feasible.

Could you indicate which conditions?

4.10. Why Do Wheels Rotate in the "Wrong" Direction?

We often see in the cinema an amusing thing: the wheels of a moving carriage rotate in the wrong direction.

How can this paradox be explained?

4.11. How Does a Refracting Telescope Work?

When constructing a refracting telescope, a long-focus lens is used as the objective, while a short-focus lens serves as the ocular. Since the objects observed in the telescope are very distant, their images are actually obtained in the objective's focal plane.

The image due to the objective lens is the object for the ocular lens, which is positioned so that its front focal point matches the objective's back focal point.

Since the object is in the ocular's focal plane, its image cannot be formed from the rays that leave the ocular in parallel (to be more exact, these rays form an image at infinity).

So how do astronomers make their observations?

4.12. Do Astronomers Need Telescopes?

A telescope magnification is the ratio of the angle subtended by the object in the telescope to the angle at which it is subtended at the naked eye.

In view of the remoteness of stars (recall that even light from the nearest star which travels at 300,000 km/s takes about four years to reach us) the angle subtended at the naked eye by a star is practically zero. Therefore even using the most powerful telescope you can only observe stars as luminous infinitesimal points (however many times zero is magnified, it finally remains zero).

Does this suggest that telescopes are only useful when observing comparatively near objects like the planets, while stars can be observed with the naked eye?

4.13. What Aperture Setting Should Be Used?

In order to increase the sharpness of an image on a photographic film the lens is stopped down, i.e. a diaphragm between the camera lens and the film is contracted. However, if the aperture is too small the image again begins to be blurred (this is why modern cameras have a minimum aperture setting of 1 22 instead of 1 36 as old cameras used to have).

Why, notwithstanding the production of very sensitive photographic emulsions which could be used with very small aperture settings, do we still not use apertures less than 1 : 22 in practice?

4.14. Is the Construction of Hyperboloid Realizable?

Peter Garin, the main character in Aleksei Tolstoy's novel *Engineer Garin's Hyperboloid*, was explaining his invention to Zoe Montrose. He said: "It is as plain as a pikestaff. The secret is hidden in the hyperbolic mirror *A* resembling in shape the mirror of a conventional searchlight, and the piece of chamonite *B*, which is made in the shape of a hyperbolic sphere. A hyperbolic mirror works as follows:

"Light incident on the internal surface of a hyperbolic mirror is collected at a single point called the focus of the hyperbola. This is a well-known fact. Now at the focus of the hyperbolic

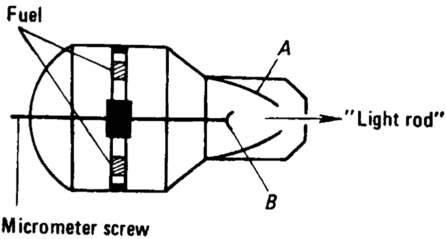

Fig. 4.3

mirror I place a second inside-out hyperbola, a hyperboloid of revolution. This second hyperboloid is made of a refractory mineral that can be polished to a mirror finish, i.e. chamonite deposits of which in Russia are practically inexhaustible. As to the rays?

"The rays collected at the focus of mirror *A* are then incident on the surface of hyperboloid *B* and are reflected in parallel. In other words, hyperboloid *B* concentrates all the light rays into a single "light rod" of arbitrary diameter. By shifting hyperboloid *B* using micrometer screw, I can increase or decrease the diameter of the light rod at will and adjust the rod (practically) down to a needle's diameter. Nature can do nothing to resist this light rod. Buildings, battleships, airships, cliffs, mountains, the earth's crust—everything can be pierced, destroyed, or cut by my beam."

The scheme of Garin's hyperboloid is shown in Fig. 4.3. Is such an apparatus possible? To be more precise, would it be as powerful as stated by Garin?

4.15. Instead of a Laser

The energy flux density in the beam of a modern laser (optical quantum generator) is so high that like Garin's hyperboloid of which we spoke in the previous problem, it can readily cut the metal sheets several centimeters thick and drill fine channels in the crystal of diamond, the hardest substance in nature. At present lasers are used to more prosaic ends, such as cutting out cloths in big factories under programmed control.

Lasers can be used in many ways as well. Yet their other applications are hindered by their rather high cost due to the complicated manufacturing process.

We shall try to design a simpler device which like a laser yields narrow light beam with a high energy density.

Suppose that the light rays from a powerful searchlight are incident from the left (see Fig. 4.4) into the funnelled opening of a conical tube whose internal surface is polished and silvered. After a succession of reflections the light rays will emerge through the right-hand opening. Since its cross-section can be made arbitrarily small, it would seem, we can provide a very high energy concentration in the light flux emerging from the cone.

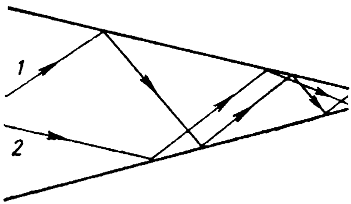

Fig. 4.4

Since no such device, for all its simplicity, is in use our reasoning seems faulty. Try to detect the fault.

4.16. Will the Colour Be Changed?

The wavelength λ is related to the velocity of light c in a medium and to the medium's refraction index n, i.e.

$$\frac{\lambda_1}{\lambda_2} = \frac{c_1}{c_2} = n_{1,2}.$$

It can be seen from this equation that in passing from one medium to another the wavelength of the light is changed. For example, the wavelength in air is 0.65 μm, whereas in water, whose refraction index relative to air is 1.33, the wavelength will be

$$\lambda_2 \quad \frac{\lambda_1}{n_{1,2}} \quad \frac{0.65\ \mu\text{m}}{1.33} = 0.49\ \mu\text{m}.$$

The wavelength 0.65 μm corresponds to red, while 0.49 μm, to blue colour, Does this mean that a diver underwater will see the light coming from a red lantern as blue?

4.17. What Is the True Colour?

The colour of a body coated with a layer of white zinc paint is perceived as white. If Prussian blue paint were used, the body would be blue. In both cases the body seems to be coloured definitely.

However, sometimes the colour of a body cannot be described so easily and this can be illustrated as follows.

If you look at cigarette smoke it seems to vary from bluish to reddish yellow, depending on where the observer is relative to the smoker, smoke, and light source.

Why does smoke's colour depend on the observer's "point of view"?

4.18. An Incident with R. Wood

The American optician Robert Wood (1868-1955) was a great jester and a lover of fast driving. He was once driving fast in the city and could not brake in time to stop when traffic lights turned red. He was arrested by a policeman and the conversation developed along the following lines:

"I am not guilty," said Wood defending himself. "I was a victim of the Doppler effect."

"What?" asked the astonished policeman.

"The Doppler effect," answered Wood and then added, "You must have noticed how an engine whistle or car horn coming towards you rises in pitch? This happens because your ear receives more sound waves per unit time and so the whistle sounds higher. An analogous phenomenon is observed for light, too. If a light source approaches you or you approach it, then its colour changes for you because its colour shifts to the blue end of the spectrum. I was driving rather fast and so the red traffic light appeared to be green!"

We do not know how the conversation ended (it is said that the policeman eventually fined Wood for driving too fast). However, we are interested in another aspect: could R. Wood legitimately refer to Doppler effect?

4.19. Negative Light Pressure

Both the electromagnetic theory of light developed in 1860-1870 by Maxwell and the quantum theory, whose foundations were laid in 1900 by Max Planck (1858-1947) and which was successfully applied in 1905 to light by A. Einstein (1879-1955), predict that light has a pressure. The phenomenon was first experimentally detected by Pyotr Lebedev (1866-1912) in solids and then in gases, thus having a universal character.

Light pressure is significant in nature. For example, it prevents the gravitational collapse of stars, plays the decisive role in forming comet tails, and reduces the lifetime of artificial Earth satellites.

Science fiction writers long ago dreamed about interplanetary ships driven by light pressure, hoping that advances in chemistry would provide a light strong plastic for the sails of a space yacht. Yet light pressure only acts "downstream" of a source. What is to be done, if the astronaut wants to return to the Sun? Tacking like yachtsmen on water do would be fruitless because interplanetary space is practically devoid of substance. However, it appears that a system can be designed that effected by light rays will be attracted to the source of the light. Try to imagine such a system.

4.20. Why Do Identically Heated Bodies Glow Differently?

How do you explain the following paradox: a piece of iron heated to 800°C glows very bright, while a piece of quartz (a less successful experiment can be performed with glass) heated to the same temperature barely glows?

4.21. The Paradox of Rulers

Place two rulers to make an acute angle α (see Fig. 4.5). Suppose one ruler begins to move rel-

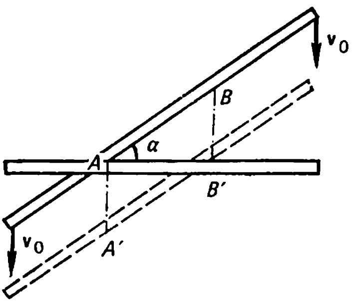

Fig. 4.5

ative to another translationally with a velocity v_0 in the direction indicated in the figure. Then the point where the rulers intersect will begin to move with a velocity **v** whose value can be determined in the following way.

During a time interval Δt the moving ruler covers a distance

$$AA' = BB' = v_0 \Delta t$$

and so the intersection point moves by

$$AB' = v \Delta t.$$

The triangle ABB' yields

$$AB' = BB'/\tan\alpha.$$

Substituting this into the relations for AB' and BB' and cancelling by Δt yields

$$v = v_0/\tan\alpha.$$

Let $v_0 = 14{,}000$ km/s and $\alpha = 10°$ ($\tan 10° = 0.035$), then for the velocity of the intersection point we have

$$v = \frac{14{,}000 \text{ km/s}}{0.035} = 400{,}000 \text{ km/s!}$$

How can this be reconciled with the postulate of the theory of relativity that the velocity of light is the maximum possible?

4.22. The Paradox of the Lever

Imagine that you have a lever whose shoulders have lengths l_1 and l_2 (see Fig. 4.6). Assume the lever is rotated one turn during a time T Then the distance covered by the left-hand end can be written in two ways, viz.

$$S_1 = v_1 T = 2\pi l_1.$$

Analogously, for the distance S_2 covered by the right-hand end we have

$$S_2 = v_2 T = 2\pi l_2.$$

By dividing these equations term-by-term we obtain:

$$v_1/v_2 = l_1/l_2,$$

whence

$$v_2 = v_1 \frac{l_2}{l_1}.$$

If the right-hand shoulder is 100 times longer than the left-hand one, which has a velocity of

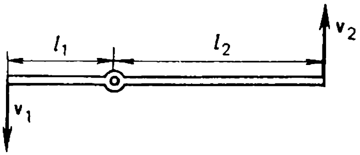

Fig. 4.6

4000 km/s, then the velocity of the lever's right-hand end will be

$$v_2 = 4000 \text{ km/s} \times 100 = 400{,}000 \text{ km/s}.$$

This exceeds the velocity of light!

Is this really possible?

4.23. How Much of Radium Did the Earth Contain When It Was Born?

Suppose that there is only 1 kg of radium presently on the Earth. This is a gross underestimate because laboratories and hospitals all over the world possess much more. However, for simplicity we shall use this quantity.

Radium's halflife is 1620 years. This means 1620 years ago there was twice as much radium as there is now, i.e. 2 kg; 3240 years ago there was 4 kg, etc. We can compile a table and obtain the mass of radium on Earth, taking its age to be 10^{10} years, this being in accordance with the latest geological and astronomical data.

Number of years to the present	Number of halflives	Mass of radium, kg
0	0	$1=2^0$
1620	1	$2=2^1$
3240	2	$4=2^2$
4860	3	$8=2^3$
6480	4	$16=2^4$
10^{10}	$10^{10}/1620$	$2^{10^{10}/1620}$

The tabulation indicates the amount of radium on Earth 10^{10} years ago was

$$m=2^{10^{10}/1620}\ \text{kg} = 2^{6.17\times 10^6}\ \text{kg}.$$

Taking logarithms of both sides of the equation yields

$$\log m = 6.17 \times 10^6 \times \log 2 = 1{,}857{,}000.$$

Whence the mass of radium on the Earth when it coalesced was

$$m=10^{1{,}857{,}000}\ \text{kg!}$$

How can this be reconciled with the fact that the mass of the whole Earth is now "only" about 6×10^{24} kg?!

4.24. How Do Cosmic Rays Originate?

At the beginning of this century the Austrian physicist Victor Hess (1883-1964) was one of many researchers to show that Earth's surface is continuously being irradiated by a flux of cosmic rays, i.e. fast protons and α-particles originating from outer space. Their energy is colossal (certainly, for particle physics), with a magnitude of 10^{19} eV The largest accelerators now can only generate charged particles to energies of about 1×10^{12} eV The Italian physicist Enrico Fermi (1901-1951) advanced the following hypothesis concerning the origin and genesis of cosmic rays which is at present accepted as the most probable.

Astrophysicists have observed moving clouds of interstellar gas and the accompanying magnetic fields due to the motion of charged particles inside these clouds. Fermi's hypothesis holds that cosmic particles encounter these randomly wandering magnetic fields and are accelerated by them.

However, we know that the force acting normal to the magnetic field on a moving charged particle (Lorentz's force) is directed perpendicular to the velocity of the particle. It can only change the direction of the velocity, not its magnitude.

How does Fermi's hypothesis explain the necessary acceleration?

4.25. Nuclear Reactions and the Law of Mass Conservation

The neutron was discovered in 1932 by Sir James Chadwick and is unstable. In due course it decays into a proton, an electron, and an antineutrino. The decay equation can be written down as

$$ {}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e + {}^{0}_{0}\tilde{\nu}, $$

when n, p, e and $\tilde{\nu}$ are the symbols for the neutron, proton, electron, and antineutrino, respectively. The superscript is the mass of the particle and the subscript is its charge (both in atomic units).

A process has been discovered in which a proton is transformed into a neutron, a positron, and a neutrino. The equation for the "nuclear reaction" is

$$ {}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{+1}e + {}^{0}_{0}\nu. $$

Thus, as a result of these two reactions following one another, a neutron is "reborn" but, in addition, four new particles, an electron, a positron, a neutrino, and an antineutrino, are produced.

How can this be reconciled with the law of energy conservation?

4.26. Are There Electrons Inside an Atomic Nucleus?

Soon after the experimental detection of neutron the Soviet physicist Dmitriy Ivanenko (b. 1927) and independently the German physicist Werner Heisenberg (b. 1901) proposed a theory of nuclear structure according to which each nucleus is composed of protons and neutrons generically called nucleons.

If the atomic number of an element and its mass number (the atomic mass rounded-off to an integer) are Z and A respectively then the number of protons in a nucleus is Z, while the number of neutrons is $A - Z$. There are no other particles in nucleus.

The validity of the proton-neutron model of the nucleus is not doubted.

On the other hand, one type of radioactive transformations, i.e. β-decay, is the decay of an atomic nucleus with the ejection of a β-particle, i.e. a common electron.

Where did the electron come from?

Solutions

Ch. 1. Mechanics

1.1. Subway trains strictly follow a time-table and arrive at stations at definite intervals of time. Let us use this to solve the problem graphically.

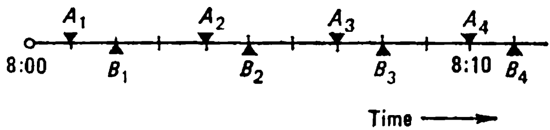

Fig. S.1.1

Figure S.1.1 shows a time axis whose origin is at 8 a.m. Along the axis the lower triangles label the arrivals of the trains going the passenger's way, while the upper triangles do the same for the trains going in the opposite direction. The trains arrive every three minutes in both directions.

Since we stated that the time when the passenger arrived at the platform is random, the event might occur in the intervals A_1B_1 or B_1A_2 (or A_2B_2 or B_2A_3, etc.). If the passenger arrives at the platform within the intervals A_1B_1, A_2B_2,. . ., the first train to arrive, after he reaches the platform, will be in his direction, while if he reaches the platform during B_1A_2, B_2A_3, . , the first train will be in the opposite direction. Since the latter intervals are twice as long as the former, the probability that the passenger will reach the platform in time to see a train going in the opposite direction will be twice the probability that the first train will be going his way. At another station these relationships would be different.

This problem effectively illustrates the utility of graphical methods of solving problems.

1.2. This problem is often solved in completely contradictory ways. Some people argue that the propeller-driven sledge will remain immobile, while others state it to move ahead.

Meanwhile the correct answer is that the problem as given has no solution at all.

Let us consider two extreme cases. Suppose that there is no friction between the conveyer belt and the sledge. Then the belt's motion will not tell on the sledge's velocity, since the sledge is driven by a propeller. The vehicle will sort of hover over road and the latter's motion will not influence the motion of the sledge just as the motion of a road cannot affect the velocity of an aeroplane flying above it.

At the other extreme case, if there is a great deal of friction between the sledge and the conveyer such that the thrust produced by the propeller cannot overcome the friction, the sledge can be treated as being fixed to the conveyer belt. Then, of course, the vehicle will move in the same direction and with the same velocity as the conveyer belt.

There are as many different sledge velocities as there are intermediate cases. A special case is when the sledge's position relative to environment remains unchanged, i.e. it will be motionless. This will occur when the propeller's thrust equals the friction (the air resistance being neglected). However, this state would be unstable, since even a slight push one way or the other, due to irregularities in the conveyer, will start the vehicle moving relative to the ground in the appropriate direction.

1.3. The resultant velocity of point A (Fig. S.1.2) is the one observed in reality, i.e. the boat's

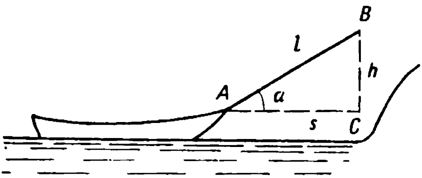

Fig. S.1.2

horizontal velocity v_b. Hence, the velocity at which the rope is hauled in is one of its components. But what is the direction of the second component?

This direction must be chosen so that the motion in that direction does not change the magnitude of the rope velocity, $|\mathbf{v}_r|$, and only changes direction. We can easily see that this will only be the case when the direction of the second component is at a right angle to the rope. Otherwise, we might always decompose the second component $\mathbf{v}_2$ as indicated on the left-hand side of Fig. S.1.3, so that one of the new components $\mathbf{v}_2'$ changes the value of $\mathbf{v}_r$.

This suggests that in our case the parallelogram of velocities must be a rectangle, inside which the resultant velocity is horizontal, while one

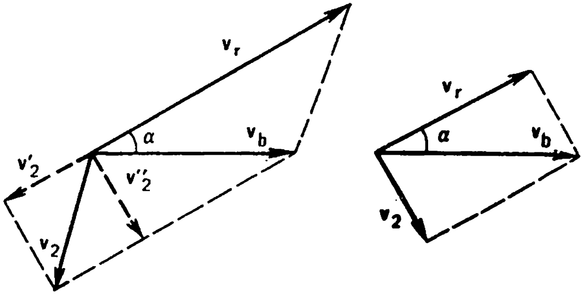

Fig. S.1.3

of the components coincides with the rope's direction. By making the corresponding drawing (the right-hand side of Fig. S.1.3), we find that

$$|\mathbf{v}_b| = \frac{|\mathbf{v}_r|}{\cos\alpha}$$

this being a correct solution of the problem.

Thus, although any vector can be decomposed in arbitrary directions, not every decomposition is meaningful. The decomposition shown in Fig. 1.1 is physically meaningless since the resultant is the motion along the horizontal rather than along the rope and it is this resultant that must be decomposed.

The problem can be solved most easily by differential calculus.

From triangle ABC (see Fig. S.1.2) we get

$$(AB)^2 = (BC)^2 + (AC)^2.$$

By differentiating this expression with respect to time and assuming $AB = l$, $BC = h = \text{const}$ and $AC = s$, we find that

$$2l\frac{dl}{dt} = 2s\frac{ds}{dt}.$$

Given that $s/l = \cos\alpha$, we get

$$\frac{dl}{dt} = \frac{ds}{dt}\cos\alpha.$$

However, $\frac{dl}{dt}$ is the magnitude of the rope's hauling-in velocity, $|\mathbf{v}_r|$, while $\frac{ds}{dt}$ is the magnitude of the boat's velocity, $|\mathbf{v}_b|$, therefore,

$$|\mathbf{v}_r| = |\mathbf{v}_b|\cos\alpha,$$

or

$$|\mathbf{v}_b| = \frac{|\mathbf{v}_r|}{\cos\alpha}.$$

1.4. The velocity at which the load is raised will not, naturally, be changed if both ropes are passed over a block and pulled with the same velocity as before. It is only necessary to put guides *KL* and *MN* to lift the load *P* along the same path as before (see Fig. S.1.4). The new forces (the friction between the load and the guides) should not disturb us since the problem is purely kinematic.

Its solution will not change either if both ropes are substituted by a single rope, since the second one becomes useless. Thus, this problem is analogous to the preceding one, which becomes evident if we rotate Fig. S.1.4 by 90° clockwise and compare the result with Fig. 1.1 or S.1.2.

The load acts as the boat and the velocity **u** is the resultant one. At the same time the rope velocity **v** is one of the components. For the second component **v′** to have no projection oriented along the rope, it must make right angle with the first component.

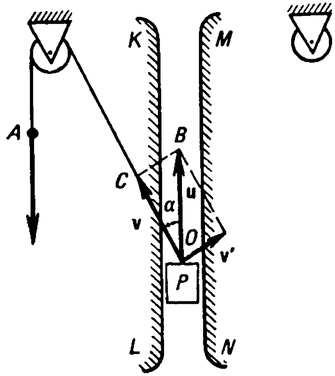

Fig. S.1.4

Then from the velocity triangle OBC it follows that

$$|\mathbf{u}| = \frac{|\mathbf{v}|}{\cos\alpha}.$$

1.5. The average velocity of the motor-cyclist is rigorously zero according to the formula

$$\mathbf{v}_{av} = \mathbf{s}/t,$$

since he finally returned back to point A (i.e. the displacement $\mathbf{s} = 0$).

However, in everyday life the average speed is the average of the magnitude of the velocity, which can be found by dividing the distance covered by the time taken. Let us consider the speed.

An intuitive and usual answer of 50 km/h is wrong. Let the distance between points A and B be l. Then the time spent by the motorcyclist travelling from A to B will be

$$t_1 = l/v_1.$$

The return run requires

$$t_2 = l/v_2.$$

The whole distance back and forth will require

$$t = t_1 + t_2 = \frac{l}{v_1} + \frac{l}{v_2} = \frac{l(v_1+v_2)}{v_1 v_2}.$$

The average speed is thus

$$v_{av} = \frac{2l}{t} = \frac{2l}{\dfrac{l(v_1+v_2)}{v_1 v_2}} = 2\,\frac{v_1 v_2}{v_1+v_2}.$$

Substituting the values for v_1 and v_2 yields $v_{av} = 48$ km/h.

The equation for the average speed can be transformed to

$$\frac{1}{v_{av}} = \frac{1}{2}\left(\frac{1}{v_1} + \frac{1}{v_2}\right).$$

This value of v_{av} is called *the harmonic mean* of v_1 and v_2. Hence, the harmonic average of two numbers is the reciprocal of the arithmetic mean of the reciprocals of the component numbers.

The harmonic mean of two numbers a and b can be drawn geometrically.

Figure S.1.5 presents the hyperbolic curve of the function $y = \frac{1}{x}$. Let us plot the segments

$OA_1 = a$ and $OA_2 = b$ on the x-axis. Then let us draw from points A_1 and A_2 normals with respect to the x-axis, to intersect with the hyperbola. So we find points B_1 and B_2. We then find

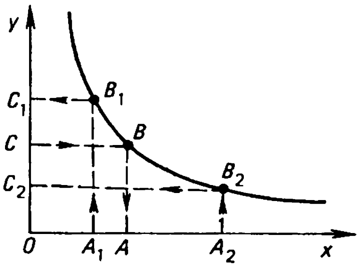

Fig. S.1.5

C_1 and C_2; they are the points where the horizontals, passing through B_1 and B_2, intersect the y-axis. We find the mid-point C between C_1 and C_2, and reverse the procedure to find B and A. The segment OA is then the harmonic mean of OA_1 and OA_2 and thus the numbers a and b. This follows from the system

$$A_1B_1 = \frac{1}{a}$$

$$A_2B_2 = \frac{1}{b}$$

$$AB = \frac{1}{h}$$

$$AB = \frac{1}{2}(A_1B_1 + A_2B_2).$$

One can show that the harmonic mean h of two numbers a and b, their geometric mean $g =$

$\sqrt{ab}$, and their arithmetic mean $m = \frac{a+b}{2}$ are related as

$$m \geqslant g \geqslant h$$

(the equality sign is valid when $a = b$).

The average velocity is only occasionally the arithmetic mean, for example, for motion with a constant acceleration. In this problem though the average velocity is the harmonic mean of v_1 and v_2.

1.6. Let us check the solutions by calculating the time needed to reach the height 6 m for the initial velocities 21.5 and 13.0 m/s, respectively.

The relation

$$t = \frac{-v_0 \pm \sqrt{v_0^2 + 2as}}{a}$$

yields two times for the initial velocity 21.5 m/s: $t_1 = 0.3$ s and $t_2 = 4$ s and two times for 13.0 m/s: $t_1 = 0.6$ s and $t_2 = 2$ s.

Thus, for any initial velocity satisfying the condition

$$v_0 > \sqrt{2 \times 10\ \text{m/s}^2 \times 6\ \text{m}} \approx 11\ \text{m/s}$$

the stone will be at a height of 6 m twice, once when moving upwards and once when failing. The higher the initial velocity, the longer the stone will take to rise to the culminating point of its trajectory and the later it will reach, when falling, the given height.

The times given in the text were specially selected to match the descent time.

All this is illustrated in Fig. S.1.6, where height versus time charts are given for both cases. The upper parabola is for the initial velocity of 21.5 m/s, and the lower one for 13.0 m/s.

1.7. When a car is braking, a passenger's body conserves its former velocity and so is thrown forwards. In order to resist the fall, the passenger

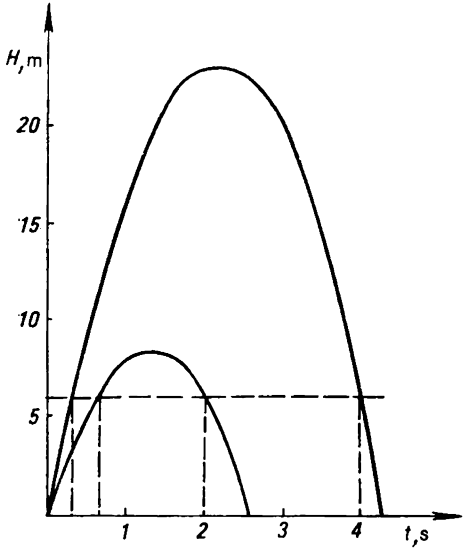

Fig. S.1.6

instinctively strains his leg muscles. When the car stops, the passenger could not immediately relax his muscles and these push him backwards. Carriage springs act in the same way as human muscles.

In case of braking the car suddenly the passenger's muscles are unable to adjust to the situation and his (her) body is thrown forwards, in accordance with the law of inertia.

1.8. It was once believed that a steam locomotive could not start a train moving if its weight exceeds that of the locomotive. Therefore the first designers provided their locomotives with legs to push against the ground (Brunton's steam engine, 1813) or proposed geared drive wheels and rails (Blackinson's steam engine, 1811).

The error of these inventors as well as that in the sophism in the text is that the friction coefficients between the carriage's wheels and the rails and between the locomotive's driving wheels and the rails were unjustly assumed to be equal.

The fact is that the points of contact between the locomotive's and the carriage's wheels and rails are stationary. This means that in both cases we are dealing with static rather than dynamic friction, the coefficient of static friction is not a constant but changes from zero to a threshold value after which the locomotive breaks away and motion is possible. Since the wheels rotate without skidding (i.e. the wheels are not blocked and can freely rotate), the friction coefficients for the wheels of the locomotive and the carriage are lower than the maximum, but not the same: the friction coefficient for the locomotive's driving wheels is higher. The weight (more exactly, the coupling weight) of a locomotive multiplied by the high friction coefficient observed under uniform motion is equal to the product of the weight of the train and the low friction coefficient. These coefficients, k_1 and k_2, may substantially differ and cannot be equated together, as is done in the sophism. Headley, who constructed in 1813 his steam engine "Puffing Billy", was the first engineer to demonstrate this fact experimentally. However, the first true steam engine was built somewhat later by George Stephenson (1781-1848).

1.9. Of course, the friction force does not decrease due to the measures suggested in the question. However, rotation is affected not so much by the force itself but its moment. It can be shown that the moment of braking friction force and hence the losses due to the work against the forces of friction decrease with the radius of the friction part.

1.10. At the right and left dead centres of the cylinder in the figure the piston of an operating engine stops for a short time to reverse the direction of motion. At these moments the lubricating oil is squeezed out from the space between the piston and cylinder walls and in the moment after this event the piston runs along dry surface until it comes in contact with a lubricated surface. Naturally, dry surfaces wear faster than lubricated ones.

1.11. The part of the problem related to the bar is correct. The second part would also be true if we were able to make a perfectly hard ball and a similar surface. However, all real bodies are somewhat deformed due to loading (including the ball's own weight) and as a result the ball and the plane contact over some finite surface rather than at a point. Within the limits of the surface the reaction at support can be slightly shifted and compensate for the moment of a couple of the force applied and the force friction. However, the deformations are usually not very pronounced and the support reaction cannot produce a noticeable moment. As a result, a ball can be set in motion more easily than a bar.

1.12. An experiment can be performed to sort out the faulty and the true arguments. To do this a model table should be placed on two dynamometers (it is convenient to use "clock-dial" type) so that table legs are at different heights. The readings on the dynamometers will be different—the higher reading for the lower legs. If the model is set up so that the perpendicular dropped from the centre of gravity onto horizontal plane passes through the table legs at point B (see Fig. 1.7), then the left-hand dynamometer readings will vanish at all.

The conclusion that the pressure should be different can be drawn from the following simple argument.

The table will rest on an inclined surface only if the sums of the forces acting on it are zero (no

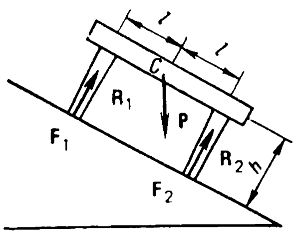

Fig. S.1.7

translation) and the moments of the forces vanish (no rotation). Since there is no rotation, for example, with respect to the axis passing through the centre of gravity of the table, then the sum of the moments of the force of gravity $\mathbf{P}$, the friction forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the table legs, and the support reaction forces $\mathbf{R}_1$ and $\mathbf{R}_2$ is zero (the forces are all labelled in Fig. S.1.7). By taking clockwise moments around the axis passing through the centre of gravity of the table C to be positive, and anticlockwise moments negative, we arrive at the following relation

$$|\mathbf{F}_1| h + |\mathbf{F}_2| h + |\mathbf{R}_1| l - |\mathbf{R}_2| l$$

$$+ |\mathbf{P}| 0 = 0.$$

Whence

$$|\mathbf{R}_2| l - |\mathbf{R}_1| l = |\mathbf{F}_1| h + |\mathbf{F}_2| h$$

or

$$|\mathbf{R}_2| - |\mathbf{R}_1| = (|\mathbf{F}_1| + |\mathbf{F}_2|) \frac{h}{l} > 0,$$

i.e.

$$|\mathbf{R}_2| > |\mathbf{R}_1|.$$

Note that if there is no friction at all, i.e.

$$|\mathbf{F}_1| + |\mathbf{F}_2| = 0,$$

then should the table slide along an inclined plane we would have

$$|\mathbf{R}_1| = |\mathbf{R}_2|,$$

i.e. the pressures of the table's legs at points A and B on the inclined plane would be the same.

1.13. Let us extend $\mathbf{R}_1$ to intersect with the extension of $\mathbf{F}_1$ and substitute it at the intersection point C for forces $\mathbf{F}_2$ and $\mathbf{F}_3$, as shown in Fig. S.1.8.

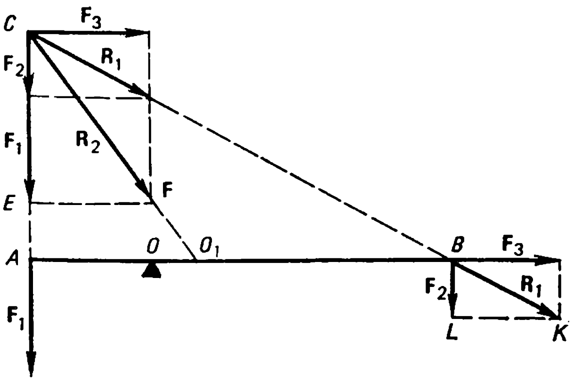

Fig. S.1.8

Triangle ABC is similar to BLK, hence we can write

$$\frac{AB}{AC}=\frac{LK}{LB}, \quad \text{or} \quad \frac{AB}{AC}=\frac{|\mathbf{F}_3|}{|\mathbf{F}_2|}$$

In the same way from the similarity of triangles AO_1C and EFC we have

$$\frac{AO_1}{AC}=\frac{EF}{EC}, \quad \text{or} \quad \frac{AO_1}{AC}=\frac{|\mathbf{F}_3|}{|\mathbf{F}_1|+|\mathbf{F}_2|}.$$

From a term-by-term division of these equations we get

$$\frac{AB}{AO_1}=\frac{|\mathbf{F}_1|+|\mathbf{F}_2|}{|\mathbf{F}_2|}, \quad \text{or} \quad \frac{AO_1+O_1B}{AO_1}=\frac{|\mathbf{F}_1|+|\mathbf{F}_2|}{|\mathbf{F}_2|}.$$

After reducing to a common denominator and some other transformations the last equation can be rewritten as:

$$\frac{|\mathbf{F}_2|}{|\mathbf{F}_1|} = \frac{AO_1}{O_1B}.$$

Thus, point O_1 divides AB inversely proportional to the moduli of forces $\mathbf{F}_1$ and $\mathbf{F}_2$, i.e. O_1 must coincide with point O. Hence, Figs. 1.8 and S.1.8 are not correctly drawn.

1.14. Since at each instant the line along which the reel is in contact with the floor is stationary, it can be regarded as an instantaneous rotation axis.

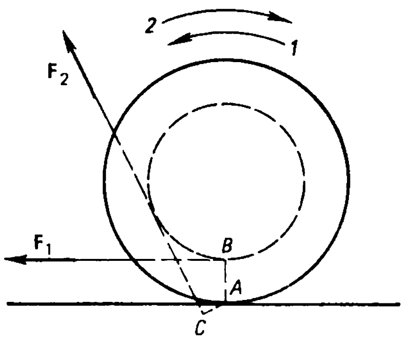

Fig. S.1.9

Figure S.1.9 shows that the horizontal force $\mathbf{F}_1$ has a moment with respect to this axis that tends to rotate the reel anticlockwise in direction *1*. This will make the reel move towards the experimenter.

When the thread is inclined at a sufficiently large angle, the moment of force $\mathbf{F}_2$ relative to the instantaneous axis tends to rotate the reel clockwise in direction *2* and the coil runs away from the experimenter.

1.15. Aristotle assumed that the upper stone only pushes the lower stone. In reality, the upper stone not only (or, to be more precise, not so much) sets the lower stone in motion as sets itself in motion.

In other words, the force acting to set the stones in motion (i.e. the force of gravity) is doubled, but so is the mass being acted upon, and hence acceleration remains the same in agreement with Newton's second law:

$$\mathbf{a} = \frac{\mathbf{F}}{m}.$$

1.16. The error in the deduction in the text is the baseless assumption about the force $\mathbf{F}$ being fully transferred via the bar of mass M_1 to the bar of mass M_2. This does not follow from the laws of mechanics. It is more reasonable to assume that the bar of mass M_2 is affected by some other force $\mathbf{F}^* \neq \mathbf{F}$. Then the bar of mass M_1 is exposed to $\mathbf{R} = \mathbf{F} - \mathbf{F}^*$ and by Newton's second law

$$\mathbf{a}_1 = \frac{\mathbf{F} - \mathbf{F}^*}{M_1} \quad \text{and} \quad \mathbf{a}_2 = \frac{\mathbf{F}^*}{M_2}.$$

Since the bars are constantly in contact, their accelerations must be equal to

$$\mathbf{a}_1 = \mathbf{a}_2 = \mathbf{a}, \quad \text{i.e.}$$

$$\frac{\mathbf{F} - \mathbf{F}^*}{M_1} = \frac{\mathbf{F}^*}{M_2}$$

whence it follows that

$$\mathbf{F}^* = \frac{M_2}{M_1+M_2}\mathbf{F}.$$

Thus, the bar of mass M_2 is not exposed to the whole force **F**, but only to $\frac{M_2}{M_1+M_2}$ of it. By substituting **F*** into any expression for acceleration (for simplicity, into the second one) we get

$$\mathbf{a} = \frac{\mathbf{F}}{M_1+M_2}.$$

The same result would be obtained, if the force applied to the bars is merely divided by their common mass.

1.17. Before proceeding with the solution of this problem, let us solve another one.

Physicist A. was born in 1870. When he was 37 he was elected to the Academy of Sciences and then he lived another 40 years until his death in 1960. How old was he when died?

After a brief contemplation we inevitably arrive at the conclusion that the data given cannot apply to a single person. Actually, depending on whether we use the birth and death dates or use the other biographical data, we obtain two different answers, viz. 90 or 77 years. Thus, the problem is inherently contradictory. A unique solution can be ensued if some of the data are corrected or ignored.

In just the same way the values given in our problem cannot be related to the motion of a single body: the problem contains redundant and contradictory data. For example, by excluding from the statement the information on the distance covered (or the time of motion), we obtain a correct, conventional, and readily solvable problem.

To conclude, we must note that the reservation concerning the direction of force and the absence of friction is not redundant, and we suggest the reader verify this independently.

1.18. In both cases the force acting to set the body in motion is two newtons. However in the first case the force of gravity acts on the hand-cart and the weight itself, while in the second case the force imparts an acceleration only to the hand-cart.

1.19. First, let us find the accelerations of all the balls immediately after cutting the threads. By taking both the forces and accelerations directed downwards as positive we get the following for the accelerations

$$a_1 = \frac{M_1 g + f_1}{M_1} = \frac{Mg + 2Mg}{M} = 3g,$$

$$a_2 = \frac{M_2 g - f_I + f_{II}}{M_2} = \frac{Mg - 2Mg + Mg}{M} = 0,$$

$$a_3 = \frac{M_3 g - f_{II}}{M_3} = \frac{Mg - Mg}{M} = 0.$$

Consequently, the acceleration of ball M_2 is indeed not g. Nevertheless, this in no way contradicts the assertion that under free fall the centre of gravity of a system must move with the acceleration due to the Earth's gravity. In fact, the system's centre of gravity only coincides with the centre of ball M_2 at rest, (this follows from the equality between the masses of the balls and the distances AB and BC).

However, the last equality is violated after the thread is cut. In fact, the elasticity of spring *I* is more than that of spring *II*, since they are stretched the same amount by different forces. Therefore after the thread is cut the first spring contracts faster than the second; the distances *AB* and *BC* will cease to be identical and the centre of gravity of the system will move downwards with respect to M_2. To find the position of the centre of gravity at some instant we use the conventional formula (it is derived below when solving Problem 1.30):

$$x_c = \frac{M_1x_1 + M_2x_2 + M_3x_3}{M_1 + M_2 + M_3} = \frac{M(x_1 + x_2 + x_3)}{3M} = \frac{x_1 + x_2 + x_3}{3}.$$

Now let us superpose the origin of coordinates to coincide with the centre of ball M_3 with the x-axis pointing downwards. Since the initial acceleration and velocity of ball M_3 are zero, we have $x_3 = 0$. After cutting the thread the coordinate x_2 of ball M_2 will be a constant value l (l is the length of spring *II*), since both the initial velocity and acceleration are zero, too. Meanwhile for ball M_1 we get

$$x_1 = -\left(2l - \frac{a_1t^2}{2}\right) = -2l + \frac{3gt^2}{2}.$$

By substituting these values into the expressions defining the centre of gravity, we get:

$$x_c = \frac{-2l + \frac{3gt^2}{2} - l + 0}{3} = -l + \frac{gt^2}{2}.$$

We thus have an equation which demonstrates that, as one might expect, the centre of gravity of the system moves downwards (the acceleration is positive, while initial velocity vanishes!) with an acceleration g. We can show (but it is difficult to do) that the same is valid at any arbitrary instant as well as immediately after cutting the thread.

1.20. Most people find the error in that an increase in velocity brings about an increase in resistance forces (friction and air drag), while the resultant force and acceleration decrease so that the velocity is limited.

However, this is not the main reason, since there can exist in principle conditions under which no velocity-dependent resistance forces are present. For example, a rocket moving in outer space, where, although there are some such forces, they can be taken as negligible.

To find the error, let us estimate how much power is developed by the cyclist at the end of twenty minutes. If the thrust force is 100 N and the velocity 600 m/s, we get from a well-known formula

$$P = 100 \text{ N} \times 600 \text{ m/s} = 60{,}000 \text{ J/s} = 60 \text{ kW},$$

i.e. again an absurd result, since a power like this can be developed by a man for an extremely short period of time, for example, when he jumps.

But it is here that the key to the paradox lies: since the power must remain within reasonable limits, the thrust force due to the cyclist must diminish with velocity.

A more detailed solution can be formulated as follows. At each given angular velocity of pedals, v, there exists a maximum force, F_{max}, with which the pedals are pressed. The F_{max} versus v graph is shown in Fig. S.1.10 by solid curve, which shows that a human foot cannot act on even a resting object ($v = 0$) with a force exceeding a certain value F_0. On the other hand,

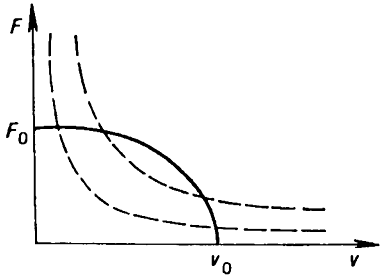

Fig. S.1.10

as the velocity of a pedal approaches a certain value v_0, the force F_{max} must tend to zero, since at high velocities the cyclist will be unable "to press on the pedals." In the same figure the dashed curves of equal power are presented (the hyperbolas $Fv = \text{const}$). The lower the power corresponding to a hyperbola, the closer the dashed line approaches the coordinate axes. It follows from the figure that at high velocities the net power that can be developed by the cyclist may even diminish with an increase in velocity and vanish at $v = v_0$!

A high power does not mean a higher thrust force. For example, the photon rockets designed have the thrust force of only a few tens or hundreds of newtons (compared with the engine of a modern jet which exerts a thrust force of hundreds of thousands of newtons), hence such a rocket could not take off independently because the available thrust force is much lower than the force of gravity. Therefore photon engines will only be started after the rocket is already in space having been given a sufficient velocity by another type of engine, for example, a conventional jet.

1.21. There is a big difference between Münchausen and a cyclist. According to the tale, Münchausen "succeeded" in lifting the centre of gravity of the whole system rider-horse by means of own efforts (we might say internal forces). This contradicts the laws of physics and therefore is impossible. The cyclist, however, when he pulls handle bars towards himself, and thus lifts them above the ground, simultaneously pulls himself towards the handle bars. The centre of gravity of the whole system remains unchanged.

It is essential to note that as long as the bicycle moves along the ground the bicycle-cyclist system is not closed and its centre of gravity can be raised by using the ground's reaction. For example, a circus trick involves a cyclist who starts riding on two wheels, then jerks the front wheel up and continues riding on the back. The height of the centre of mass of the system is thus increased by pushing against the ground (think over what movements the cyclist must do to accomplish this).

1.22. When written in the form

$$F = \gamma \frac{m_1 m_2}{R^2} \qquad (1)$$

the law of gravitation is indeed for point bodies, i.e. objects whose linear dimensions l are significantly smaller than the distances R between their centres of mass.

If R is not greater than l, then we must be careful and allow for the shape and size of the bodies involved. Generally, in order to find the gravity force between two bodies we must divide them into small parts such that each can be taken as a point, find the forces between each point in the first and second bodies and find their vector sum. This is how we must calculate the forces acting between a person and a chair or a stone and the earth, there will then be no infinity.

Generally, finding the vector of many forces is mathematically very complicated. However, if the interacting bodies are hollow spheres or balls (with a uniform mass distribution over the surface or volume!), then the gravity force can be calculated from formula (1) even when the distance between their centres is comparable with their radii, and this relationship is valid down to $R = R_1 + R_2$ (R_1 and R_2 are the radii of the spheres or balls).

For the bodies of arbitrary form at small R (R is assumed to be equal to the distance between the centres of mass) the interaction force cannot be calculated from formula (1). For example, the centre of mass of torus is outside the torus. Therefore a small ball can be placed inside the torus so that its centre of mass coincides with that of the torus: R will thus be zero, yet the gravity force between the ball and the torus will not tend to infinity. On the contrary, geometrically summing the forces acting between separate parts of the ball and the torus will yield a zero resultant.

To conclude, let us indicate the slip in the text of the problem: that two bodies are in contact does not mean the distance between their centres of mass is zero. Thus, as we have seen, for touching balls we have $R = R_1 + R_2$.

1.23. The magnitude of the high and low tides is not so much determined by the gravity force

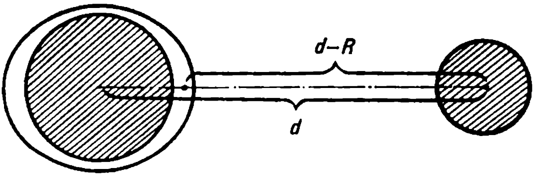

Fig. S.1.11

of the Sun or Moon, as by the difference between the gravity forces of the Sun, etc., and a body at the Earth's centre and at its surface. If they were equal, these forces would impart the same acceleration to the Earth and the ocean so that both would move as a single whole and there would be no tides.

However, the centre of the Earth is farther from the Moon (or the Sun) than the water in the ocean on the side facing the Moon (or the Sun). Therefore their accelerations will differ by

$$\Delta a = \frac{\gamma M}{(d-R)^2} - \frac{\gamma M}{d^2} = \gamma M \frac{d^2 - (d-R)^2}{d^2 (d-R)^2}$$

where M is the mass of the Moon (Sun), d is the distance between its centre and the centre of the Earth, R is the radius of the Earth, and γ is the gravitational constant (Fig. S.1.11).

Since in both cases $R \ll d$, we have

$$\Delta a \approx \frac{\gamma M \cdot 2Rd}{d^2 d^2} = 2R \frac{\gamma M}{d^3}.$$

This difference based on the "normal" gravitational acceleration g will be

$$\frac{\Delta a}{g} = 2R \frac{\gamma M}{d^3} \div \frac{\gamma M_E}{R^2} = 2 \frac{M}{M_E} \left(\frac{R}{d}\right)^3$$

where M_E is the Earth's mass.

For the Moon

$$\frac{M}{M_E} = \frac{1}{81} \text{ and } \frac{R}{d} = \frac{1}{60}.$$

Whence for the relative reduction in acceleration (and, consequently, for the relative reduction in the force of gravity on the side facing the Moon), we get

$$\frac{\Delta a}{g} = \frac{\Delta P}{P} = \frac{2}{81 \times 60^3} = \frac{1}{9 \times 10^6}.$$

For the Sun

$$\frac{M}{M_E} = 332{,}400 \text{ and } \frac{R}{d} = \frac{1}{23{,}500}.$$

Whence we get

$$\frac{\Delta a}{g} = \frac{\Delta P}{P} \approx \frac{1}{19 \times 10^6}.$$

Thus, solar tides must really be half as strong as lunar ones.

1.24. Formula (1) expresses the fact that, for the same distance covered, the work done will be larger, the larger the force performing the work. Similarly, expression (2) implies that for the same force, the work increases with the distance covered. (The work can only be doubled by doubling the force if the distance remains the same; or the work is proportional to the distance if the force remains unchanged.) Thus, the constant term in formula (1) is a variable in formula (2) and vice versa.

Hence, equations (4) and (5) derived from (1) and (2) by term-by-term multiplication and division must take both k_1 and k_2 as variables. Whence it follows that k_3 and k_4 are not constants. Indeed, from the meaning of k_1 and k_2 we can deduce that

$$k_3 = \sqrt{Fs} \quad \text{and} \quad k_4 \quad \frac{F}{s}.$$

By putting k_3 and k_4 into (4) and (5), respectively, we obtain the tautologous expressions

$$A = \sqrt{Fs} \cdot \sqrt{Fs} = Fs \quad \text{and} \quad F = \frac{F}{s} s = F$$

This sophism is not only related to the formula for work but, of course, to all formulas expressed analytically by monomials.

For example, in the case of uniform motion the distance covered is proportional to both velocity and time. By writing this as two formulas
$s = k_1 v$ and $s = k_2 t$
we get

$$s = k_3 \sqrt{vt},$$

where

$$k_3 = \sqrt{k_1 k_2} = \sqrt{vt} \quad \text{or} \quad v = k_4 t,$$

where

$$k_4 = \frac{k_2}{k_1} = \frac{v}{t}.$$

1.25. No violation of the law of energy conservation is possible. It is valid in all known processes. We needed only take into account that the collision between the projectile and the handcart is inelastic, i.e. part of the projectile's energy, namely, one half, is spent overcoming the force resisting its motion inside the handcart and finally it is transformed into heat, while the truck is set in motion with the remaining energy.

1.26. If coal is burnt on the third floor height, the potential energy of the combustion products (water, ashes, carbon dioxide, carbon monoxide, and unburnt coal particles) will increase exactly as much as the coal potential energy.

1.27. The fuel contained in the tank of a flying rocket possesses kinetic energy due to the speed resulting from burning fuel. Therefore the energy possessed by each kilogram of unburnt fuel will be produced from both the heat of combustion independent of the rocket's velocity and an ever increasing kinetic energy. At a velocity of about 3 km/s the kinetic energy of a kilogram of fuel is comparable to its heat of combustion, i.e. the store of chemical energy; by the time it reaches the orbital velocity the kinetic energy of the fuel is three times that of the heat of combustion. It is this fact that explains the paradox.

1.28. A gas balloon displaces a greater mass of air than the mass of the balloon, since the latter is filled with a gas whose density is lower than air's. At a height H the balloon's potential energy is increased by $VDgH$, where V is the volume of the balloon, D its average density and g is free fall acceleration.

At the same time the volume the balloon occupied now contains air that was pushed down and its potential energy is reduced by $VD'gH$, where D' is the air density.

As a result, the potential energy of the atmosphere-air balloon system is lowered by

$$VgH\,(D' - D) > 0.$$

It is this energy loss that lifts the balloon. Thus, the same reason that forces wood and gas bubbles to float in water is evident, i.e. the system tries to attain the state of minimum potential energy.

We assumed for simplicity the densities of the air and the gas in the balloon were constant. In reality the air's density decreases with height and so does that of gas filling the balloon since the balloon expands in order to keep the pressure inside the balloon and in the atmosphere the same. However, the gas's density cannot decrease infinitely because of the shell. Therefore at a certain height the air density becomes equal to the average density of the balloon and the further ascent is stopped.

1.29. When rolling down a hill, the hoop moves along the ground and it also rotates about its centre.

Therefore the right-hand side of the equation in the problem for the law of energy conservation must be supplemented by an additional term for the kinetic energy of rotation, W_k^{rot}:

$$mgH = W_k^{trans} + W_k^{rot}$$

From Fig. S.1.12 it can be seen that the hoop moves relative to its centre with the same velocity with which the centre itself moves relative to Earth's surface. Let us choose a point B on the hoop which at a given instant is in contact with ground and therefore stationary with respect to it. If the hoop's centre possesses a velocity

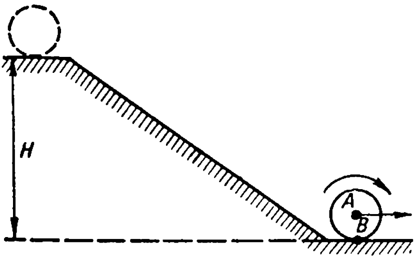

Fig. S.1.12

v relative to the Earth, then it will possess the same velocity relative to the point B. Yet if the centre A moves relative to the point B with a velocity v, then the point B must do so relative to the point A[3]. Every point on the hoop is equivalent and if one moves relative to hoop's centre with a velocity v, then the others have to do so too. We can conclude that the following equation is valid:

$$W_k^{\text{trans}} = W_k^{\text{rot}}$$

---

[3] We used the expression "the velocity of point A relative to point B" for brevity; of course, it will be more correct to speak about the velocity of a point relative to the reference frame related to the Earth's surface (or one related to the hoop's centre, when later we consider the velocity of the hoop relative to the hoop centre).

However,

$$W_k^{trans} = \frac{mv^2}{2},$$

therefore

$$mgH = mv^2.$$

Whence for the velocity gained by the hoop we have

$$v = \sqrt{gH}.$$

By substituting $H = 4.9$ m into the last expression we get:

$$v = \sqrt{9.8\ \mathrm{m/s^2} \times 4.9\ \mathrm{m}} \approx 6.9\ \mathrm{m/s}.$$

The problems on calculating the velocity of a ball, disk, and other objects rolling down a hill have to be solved in a similar fashion. However, these cases are more difficult to handle, since the velocities of the points at various distances from the centres of these objects differ and this substantially complicates the calculations. To solve these problems, it is useful to introduce the notion of the moment of inertia which plays the same role in the dynamics of rotation as does mass in the dynamics of translational motion. If a body slips down an inclined plane without rotating, then $W_k^{rot} = 0$ and the velocity can be calculated from the formula given in the sophism.

The validity of this argument can be easily checked experimentally, i.e. by comparing the time taken for two identical bottles of the same mass, one filled with water and the other with a mixture of sand and sawdust to roll down an inclined plane.

The potential energy of the first moving bottle is almost completely transformed into the kinetic energy of translational motion, since water does not rotate, except for a very thin layer next to the walls of the bottle (to simplify matters, we neglect here bottle's own mass).

The sawdust-sand mixture rotates with the bottle and a significant fraction of the bottle's potential energy is transformed into rotational kinetic energy. Therefore the kinetic energy and, consequently, the velocity of translational motion turns out to be lower for the second bottle.

1.30. The centre of mass of a system consisting of two point bodies is the point that divides the distance between the bodies into the segments inversely proportional to the masses of the bodies. Thus, if S is the centre of masses of two bodies m_1 and m_2 lying on the x-axis at x_1 and x_2, respectively, then

$$\frac{x_S - x_1}{x_2 - x_S} = \frac{m_2}{m_1},$$

whence

$$x_S = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

If the system contains one more mass m_3 at x_3 on the x-axis, then the centre of mass of the whole system will be the centre of mass of $(m_1 + m_2)$ at x_S and of the mass m_3. The centre of mass of the system will be

$$x_0 = \frac{(m_1 + m_2)\, x_S + m_3 x_3}{(m_1 + m_2) + m_3} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}.$$

For n points along the x-axis the centre of mass will be at

$$x_0 = \frac{m_1x_1 + m_2x_2 + \ldots + m_nx_n}{m_1 + m_2 + \ldots + m_n}.$$

If the points are randomly scattered in space rather than lying on the x-axis, then two equations are needed to define the centre of mass in space, viz.

$$y_0 = \frac{m_1y_1 + m_2y_2 + \ldots + m_ny_n}{m_1 + m_2 + \ldots + m_n},$$

$$z_0 = \frac{m_1z_1 + m_2z_2 + \ldots + m_nz_n}{m_1 + m_2 + \ldots + m_n}.$$

These formulas, called the Torricelli formulas, or, to be more exact, the first one can be used to solve our problem.

In order that the upper brick does not fall over the lower one, the centre of mass of the top brick must lie within the contour of the support, i.e.,

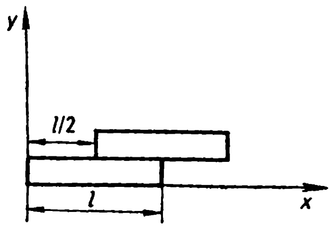

Fig. S.1.13

in one dimension the centre of mass of the upper brick x must not be larger than l (see Fig. S.1.13). Thus, the increment Δx_1 by which a brick in a wall can overhang the lower one must satisfy the condition

$$\Delta x_1 \leqslant \frac{l}{2}.$$

Now let us consider a system of three bricks and the admissible overhang of the middle brick in Fig. S.1.14. The desired increment Δx_2 can

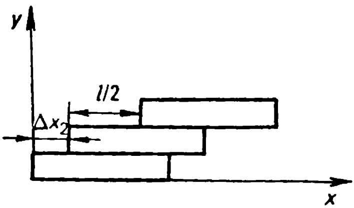

Fig. S.1.14

be found by taking into account that the centre of mass of the two upper bricks must lie within the contour of the bottom brick, i.e., an inequality must be satisfied:

$$l \geqslant x_0,$$

where x_0 is the centre of mass of the top two bricks

$$l \geqslant \frac{m(\Delta x_2 + l/2) + m(\Delta x_2 + l/2 + l/2)}{2m},$$

whence

$$\Delta x_2 \leqslant \frac{l}{4}.$$

For a system of four bricks (see Fig. S.1.15) we have

$$l \geqslant \frac{m(\Delta x_3 + l/2) + m(\Delta x_3 + l/4 + l/2) + m(\Delta x_3 + l/4 + l/2 + l/2)}{3m}$$

and

$$\Delta x_3 \leqslant \frac{l}{6}.$$

In a similar fashion we obtain

$$\Delta x_4 \leqslant \frac{l}{8}; \quad \Delta x_5 \leqslant \frac{l}{10}, \qquad \Delta x_n \leqslant \frac{l}{2n}.$$

The admissible total overhang of the upper brick is the sum

$$\Delta x_1 + \Delta x_2 + \Delta x_3 + \ldots + \Delta x_n = \frac{l}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right)$$

It is known that the sequence in the brackets[4] is divergent, i.e. the sum of the sequence grows

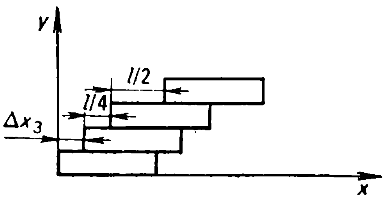

Fig. S.1.15

without limit as more terms are added. This means that as the number of bricks tends to

[4] It is called a harmonic sequence.

infinity the upper brick may overhang the lower as far as is desired.

1.31. Both statements are valid if we assume the third term in each formula to be constant.

Consider two points on a rotating disk at different distances from the axis of rotation. Then, for the same angular velocity ω, a point farther from the axis will be imparted a greater centripetal acceleration. This can be easily demonstrated experimentally by putting two objects on a record player: it is not difficult to select two positions, such that the object farther from the centre falls over, while the nearer remains standing.

By contrast, for the same linear velocity v two rotating points will have centripetal accelerations that are inversely proportional to their rotation radii. For example, the acceleration of the points at the periphery of two pulleys of different diameters linked by a belt transmission or two gear wheels with different numbers of teeth will be different.

Analogously, there are two formulas for calculating the power consumed in an electrical subcircuit:

$$P = I^2R \quad \text{and} \quad P = \frac{U^2}{R}.$$

If the currents are identical (this is the case for loads connected in series), the power dissipated in each subcircuit will be proportional to their resistances. When subcircuits are connected in parallel (e.g., household appliances connected to the mains) the voltages across the appliances will be the same, while the power consumed by each subcircuit will be inversely proportional to their resistances.

1.32. Needless to say, the engine is impossible. However, this is not because, as is sometimes stated, the sum of the centrifugal and centripetal forces vanishes. This is wrong, since it is senseless to add forces applied to different bodies.

The direction of the motion of a liquid not only changes along the arc *ACB*, it also changes near points *A* and *B*. We can assume that near these points the liquid moves along arcs of very small radius. Therefore the centrifugal forces acting onto the tube walls exist at *A* and *B* as well. Their direction and magnitude are such that their vector sum with the force **R** vanishes.

It is interesting to note that "the use of centrifugal force to ascend beyond the limits of the atmosphere and into the heavens" was suggested by Konstantin Tsiolkovsky, the founder of the theory of space flight, when young. His machine "consisted of a closed chamber or box, in which two inverted elastic pendulums with balls at the upper ends oscillate. These balls described arcs and the centrifugal force of the balls should lift the cabin and carry it into the heavens." (K. E. Tsiolkovsky, *My Life*). However, Tsiolkovsky soon realized that "the machine will only swing. Its weight will not decrease one bit". In the same fashion the weight of an oscillating pendulum suspended from a rectangular support will not decrease.

The simplest way of proving the impracticability of all such engines is to show the impossibility of setting a closed system in motion only using internal forces.

1.33. There is no contradiction, neither is the case when a walking man, after stumbling, falls forward, even though his feet have been subjected to a braking force in the opposite direction to his motion.

In both cases the explanation comes from the first law of mechanics, i.e. the law of inertia.

1.34. Usually the error in the "deduction" is thought to be the substitution of an arc by a chord. For small deflection angles the substitution is in fact rightful and admissible. The error lies in that when calculating the time of motion of the pendulum along chord AB (see Fig. 1.13), we assumed the acceleration in the direction of motion was constant and that $a = g \cos \alpha$, where α is the angle corresponding to the maximum deviation of the pendulum from its equilibrium position.

Actually the pendulum's acceleration in the direction of the trajectory is a variable reaching a maximum when farthest from its equilibrium position and vanishing when the pendulum passes through the equilibrium position. In other words, the error is the unlawful use of formulas for uniform motion, while in harmonic oscillatory motion, velocity, time, distance and acceleration are related in a much more complicated fashion.

1.35. The equation

$$m\omega^2 l \sin \alpha = m \mid \mathbf{g} \mid \tan \alpha$$

can be rewritten as

$$m\omega^2 l \cos \alpha \sin \alpha = m \mid \mathbf{g} \mid \sin \alpha.$$

Since the ball's mass is nonzero, both sides of the equation can be divided by m:

$$\omega^2 l \cos \alpha \cdot \sin \alpha = \mid \mathbf{g} \mid \cdot \sin \alpha.$$

However, you cannot infer that the sine of the angle sought for must remain nonzero. In other words, by eliminating sin α we eliminate a possible solution (viz., $\alpha = 0$), which corresponds to a vertical thread. Meanwhile, it can be easily shown that at low angular velocities this position is indeed possible.

Suppose the thread with the ball was pushed by chance through a small angle α from vertical. Then in the reference frame related to the disk there will be an extra force, acting on the ball, other than gravity, $\mathbf{P} = mg$, namely, the centrifugal force of inertia, viz. $|\mathbf{F}_{\text{in}}| = m\omega^2 R = m\omega^2 l \cdot \sin\alpha$. The thread will return to the initial position provided the moment of the force of gravity about A (see Fig. 1.14), which tends to move the ball towards the vertical, exceeds the moment of the centrifugal force relative to the same axis, which tends to move the ball away from the vertical, i.e.

$$|\mathbf{P}|\, R > |\mathbf{F}_{\text{in}}|\, l \cos\alpha$$

or

$$m\,|\mathbf{g}|\, l \sin\alpha > m\omega^2\, l \sin\alpha\, l \cos\alpha.$$

By dividing both sides of the inequality by m, l, and sin α (the latter is admissible since we have assumed that the angle α, though small, is nonzero), and transforming the resultant inequality we obtain:

$$\omega < \sqrt{\frac{|\mathbf{g}|}{l\cos\alpha}}.$$

Since the angle α is small its cosine can be approximated to unity and then we get

$$\omega < \sqrt{\frac{|g|}{l}}.$$

But if the angular velocity of the machine is such that $\omega > \sqrt{\frac{|g|}{l}}$, then the vertical position of the thread is unstable, i.e. a deviation due to a random cause will grow up to a certain angle α that can be found by the method described in the problem, since now $\sin \alpha$ will be nonzero and dividing both sides of the equation by it is admissible.

Eventually, the problem has two solutions depending on the angular velocity

(1) $\alpha = 0$ for $\omega < \sqrt{\frac{|g|}{l}}$,

(2) $\alpha = \arccos \frac{|g|}{\omega^2 l}$ r $\omega > \sqrt{\frac{|g|}{l}}$.

To summarize, it is of interest to note that for

$$\omega = \sqrt{\frac{|g|}{l}}$$

the period of the cone pendulum

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{l}{|g|}}$$

coincides with that of a mathematical pendulum of the same length.

This means that when the disk rotates slowly random displacements of the thread away from the vertical will induce harmonic oscillations of the ball in the vertical plane, while when the disk rotates rapidly oscillations are transformed into the rotation of the ball in a horizontal circle.

1.36. Often, when discussing waves, we imply the propagation of elastic oscillations which in liquids at low frequencies can indeed only be longitudinal. However, oscillations are not always due to elastic forces. In the given case the transversal waves on the pond surface are due to the force of gravity: the water pushed downward by the stone is then pushed upward by the weight of adjoining water layers. Once induced, the oscillation persists until the energy imparted by the stone is dissipated both to overcome liquid friction and to set in motion larger areas of liquid surface. Transverse oscillations maintained by the force of gravity can thus only exist at an interface between liquid and gas or between two liquids of different density. The source for the waves in a liquid bordering on a gas can be capillary forces which induce waves of a very short wavelength, i.e. ripples.

It should be noted that at high frequencies liquids show elasticity of shear, i.e. elastic transversal waves are thus possible.

1.37. A sound is softer when both prongs of the tuning fork are aligned along a single straight line to the ear. This can only be explained by the difference in the distances travelled by the sound waves if they emerge from the sound sources in phase or, more exactly, with a phase difference that is multiple of 2π. In fact the prongs oscillate out of phase: when one prong is transmitting a compression wave, the other is sending a rarefaction wave. As a result, when the prongs are aligned with ear, the sound waves are already out of phase. Therefore, as the fork is brought close to ear, the waves dampen one another and we discern a fall in the sound. The few centimetres that separate the prongs are insignificant, since the path difference produced is substantially smaller than a wavelength.

When the prongs are in a plane parallel to ear, we hear a louder sound, since the prongs act as a single acoustic source: when the prongs move together air is pushed out of the space between them and a compression wave comes to ear. When the prongs move apart a rarefaction wave begins. The alternating compressions and rarefactions are perceived as sound.

1.38. The sound produced by a tuning fork is gradually weakening, as the energy of its oscillations is released into the environment. The energy is dissipated faster if the tuning fork is fastened to a resonator or is simply in contact with a table, since the energy is released both from the tuning fork itself and from the surface of the resonator or the table. To summarize, though in the latter case the sound is louder it lasts for a shorter period, while the energy released will be the same in both cases.

1.39. The impossibility of realizing perpetual motion machine is due to the law of the conservation of energy. This sophism intentionally avoids the fact that the force due to pressure, like pressure itself, is normal to the surface. Therefore along any direction in which motion is feasible, i.e. horizontally, only the horizontal component and not the whole force of pressure acts.

Let the left-hand wall area be S and the average pressure acting on the wall be p. Then the modulus of the pressure force on the left wall can be expressed as

$$| \mathbf{F}_l | \quad pS.$$

As can be seen from Fig. 1.15, the area of the right-hand wall is $1/\sin \alpha$ more than that of the left-hand wall, since it is by this ratio that the lengths of the walls in the drawing differ.

It is for this reason that given the same average pressure p the modulus of the pressure force on the right-hand wall is different than that on the left-hand wall, namely

$$| \mathbf{F}_r | = p \frac{S}{\sin \alpha}$$

However, as has been stated above, only the horizontal force component $\mathbf{F}_r$ acts along the direction of feasible motion whose modulus is

$$| \mathbf{F} | \quad | \mathbf{F}_r | \sin \alpha = pS.$$

Thus, the forces acting from right to left and vice versa are equal.

This can be proved graphically without recourse to trigonometry. For example, the similarity of triangles can be employed (let the reader do the required construction independently). It is even simpler to take the angle $\alpha = 30°$ in order to use then the relationship between the triangle's leg and hypotenuse.

1.40. It turns out that the only thing to blame is... Archimedes' law. This states that a body immersed in a liquid and in equilibrium displaces its own mass of liquid.

Using this principle, let us consider the construction of a comfortable submarine. According to Jules Verne, the *Nautilus* had a volume of 1500 m^3, i.e. she displaced approximately 1500 tonnes (t) of water (approximately, since the density of sea water is somewhat higher than that of fresh water). Consequently, the ship's mass must be 1500 t as well, of which 150 t (and 150 m^3) is due to water ballast. Thus, the remaining volume of 1350 m^3 must accommodate 1350 t which must include the ship's body, machinery, instrumentation, crew, furniture, air for breathing, and food.

If everything is manufactured of metal, there is no special problem, since 1350 t of iron take as little a volume as 115 m^3 leaving lots of spare room. But machines and equipment are far from monolithic. Consider, for example, the 40D engine manufactured in the USSR. It has a mass of 10 t and requires 14 m^3 of space. Thus, in order to "sink" it needs extra force equivalent to the weight of about four tonnes! The situation looks somewhat better with lead-acid batteries which have a mass of about three tonnes. These batteries not only do not need any extra loading, they even compensate for the underweight of other objects. But you cannot fill a whole submarine with accumulators or even simply lead! We'd simply lack space for anything else!

This is why in order to balance weight and buoyancy the designers of submarines have to cut the volume of submarine rooms to a minimum: the cabins are very small, office space cramped, and compartments constrained at most.

1.41. A buoyancy force appears whenever we immerse body in a liquid, even in the water-in-water case, i.e. an imaginary volume of liquid is acted upon by two mutually balanced forces. However, it must be kept in mind that, according to Newton's third law, forces always appear in pairs. Therefore if we have Archimede's force directed upwards and acting on the volume of liquid, then this volume acts upon the rest of the liquid volume with a force equal to the weight of the "displaced liquid", i.e. its own weight. This latter force is directed downwards. Thus, although "water in water weighs nothing", nevertheless upper layers press down on lower layers and on the bottom of the container with a force equal to its own weight.

The same reasoning obviously applies to air too.

1.42. The liquid filling a vessel exerts a pressure on the side walls as well as on the bottom. The pressure p is always directed normal to the surface it acts upon. Therefore the pressure forces

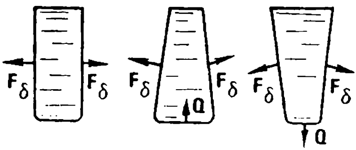

Fig. S.1.16

$\mathbf{F}_\delta$ on the side walls of a cylindrical vessel balance out, whereas the forces on the sides of a conical vessel have a resultant $\mathbf{Q}$ directed either up or down depending on vessel's shape (see Fig. S.1.16).

If the vector sums $\mathbf{R}_1 + \mathbf{Q}$ and $\mathbf{R}_2 + \mathbf{Q}$ (see the statement of the problem) are calculated and compared, these will be, as might be expected, the same.

1.43. The lift force $\mathbf{F}_g$ of a certain gas volume V_g is equal to the difference between the weight $\mathbf{P}_a$ of air displaced by the gas and the gas's own weight $\mathbf{P}_g$:

$$\mathbf{F}_g = \mathbf{P}_a - \mathbf{P}_g.$$

Since the modulus of a gas's weight can be expressed

$$|\mathbf{P}| = D_a\,|\mathbf{g}|\,V_g,$$

where D is the gas's density and g is the acceleration due to gravity, we can write

$$|\mathbf{F}_g| = (D_a - D_g)\,|\mathbf{g}|\,V$$

For helium

$$|\mathbf{F}_{He}| = (D_a - D_{He})\,|\mathbf{g}|\,V$$

Analogously, for the same volume of hydrogen

$$|\mathbf{F}_{H_2}| = (D_a - D_{H_2})\,|\mathbf{g}|\,V$$

Let us consider the ratio of the lift forces

$$\frac{|\mathbf{F}_{He}|}{|\mathbf{F}_{H_2}|} = \frac{D_a - D_{He}}{D_a - D_{H_2}} \qquad (1)$$

By substituting in the densities, we get

$$\frac{|\mathbf{F}_{He}|}{|\mathbf{F}_{H_2}|} = \frac{(1.29-0.178)\ \mathrm{kg/m^3}}{(1.29-0.089)\ \mathrm{kg/m^3}} = 0.92.$$

Thus, the lift force remains practically unchanged.

For an arbitrary gas Eq. (1) can be rewritten as

$$|F_g| = \frac{|F_{H_2}| D_a}{D_a - D_{H_2}} - \frac{|F_{H_2}|}{D_a - D_{H_2}} D_g = A - B \cdot D_g.$$

Hence, we readily see that a balloon's lift force linearly diminishes with an increase in the density of the gas filling the balloon. The force vanishes when the density of the gas equals the density of the surrounding air. On the other hand, if the gas density were zero, we see that even then the balloon's lift would be only 1.06 times higher than that of the hydrogen-filled one[5)].

1.44. The pressure in a wind flow blowing over a roof is lower than that in stationary air. Therefore, if a garret has no windows, a lift force emerges that tends to pull off the roof or its tiles. Yet if there are windows, the air under the roof will be set in motion, the difference between the pressures over the roof and under it decreases and becomes too small to damage the building.

The paradox can be formulated in another way: why during hurricanes are roofs not crushed by the wind pressure, but are instead thrown upwards? Or why do blast waves flow over continuous fences but leave thin poles intact? Remember also that one has to open his (her) mouth during gunshots in order to ensure the same pressure on both sides of ear-drum (from the side of the helix and from the side of Eustachian tube).

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5) Vacuum has zero density. It is of interest that as early as 1670 the Italian priest Francesco Lana (1631-1687) proposed employing thin-walled evacuated spheres for lifting in air.

1.45. Braking against banks, the bottom, and the air adjoining the surface results in that water in a river moves fastest in the middle of the river, just under its surface. The velocity distribution in depth is approximately as shown in Fig. S.1.17.

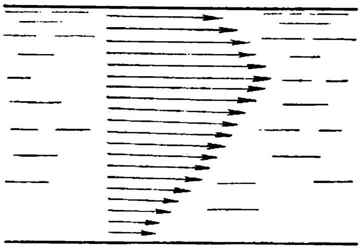

Fig. S.1.17

Therefore, as the loading on the raft increases, so does its draught, and thus the bottom of the raft is submerged down into faster layers and so the raft starts to move faster.

By the by, this was one of the reasons for the accident which nearly killed Henry Watzinger, a crew member on the Kon-Tiki[6].

Ch. 2. Heat and Molecular Physics

2.1. The compressibility of liquids is negligible: for water the decrease in volume amounts to about 0.00005 of the initial value per atmosphere of pressure applied.

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[6] T. Heyerdahl, *Kon-Tiki, Across the Pacific by Raft.*

We can easily calculate that water has the same density as steel at a pressure of 50,000 atm (and this will only be true if water's compressibility remains constant with pressure). Such a pressure would exist at a depth of 500 km. If iron's compressibility is taken into account, the required depth would increase even more. Meanwhile, the deepest point in the ocean only corresponds to about 11 km.

Note, by the way, that water is appreciably compressed by its own weight. Were it relieved from compression forces, the water level in the oceans would rise up by 35 m and a vast area (about 5,000,000 km^2) of the lowland regions of the Earth would be submerged.

2.2. High above the Earth's atmosphere the air is very rarefied and the number of molecules per unit volume is small. Therefore, although each molecule possesses a noticeable kinetic energy, the number of particles is too small to transfer much energy in collisions with a satellite. By contrast, when the satellite is not illuminated by the Sun, it gives up via radiation much more energy into outer space than it receives from the striking molecules and it could be cooled very severely if not suitably protected.

The heating of the satellite in the denser layers of the atmosphere when landing is entirely due to different reasons. It is due to the friction of the satellite's surface against the air.

Moreover, the notion of temperature is inapplicable to a single molecule. Temperature is a statistical quantity and is only applicable when there is a large enough number of particles. This was why the problem was formulated thus "...air molecules have velocities corresponding to temperatures of several thousands of degrees Centigrade".

2.3. Let us pour off half the cold water into vessel D and insert it into the vessel filled with hot water. It can be easily estimated that in the absence of thermal losses the temperature in vessels A and D will settle at 60°C. Then let us pour the warmed up water from vessel D into the empty vessel C and repeat the procedure with the remaining cold water. When put in contact using vessels A and D, the water temperature in both will settle at about 47°C. If we now pour the water from D into C, the temperature of the mixture in the latter will be about 53°C. Thus the problem is solved, since the cold water has been heated to 53°C by cooling the hot water to 47°C, without mixing the water.

If the water is not divided into two each time but into a larger number of portions, the difference in the final temperatures would be even more sizeable; at the limit of infinitesimal portions of water the final temperatures in A and C will be:

$$T_A = T_2 + \frac{T_1 - T_2}{e} \qquad T_C = T_1 - \frac{T_1 - T_2}{e}$$

where T_1 and T_2 are the initial temperatures of the hot and cold water, respectively, and e is the base of natural logarithm.

It is possible to construct a setup in which the temperatures are practically exchanged. This is done in industrial heat exchangers, where the hot and cold counterflows are passed through coaxial tubes. In long tubes the temperature exchange is practically complete, as is shown in Fig. S.2.1. If the two flows were passed in the same direction, the temperatures would at best only equalize.

We want to stress that no violation of the second law of thermodynamics (see Problems 2.25-2.27) occurs here since at every stage the temperature of the flow of liquid, to which heat is transferred, is lower than that of the flow donating the heat. This effect can be clearly seen

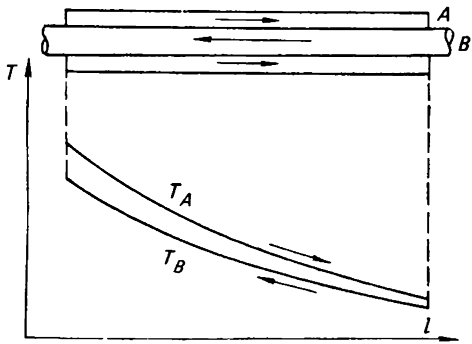

Fig. S.2.1

from Fig. S.2.1, where the lower curve represents the temperature of the receiving liquid.

2.4. Adding a layer of thermal insulation doubled the area of the heat radiating surface. Thus if the thermal conductivity of the insulation material is not too small, the losses may indeed increase.

2.5. To measure temperature any scale can be used. The laws of physics do not change from one scale to another, so the calculation of the amount of heat needed to heat the body can be made using either formula. However, the numerical values of the quantities whose definition contains the notion of temperature will be quite different. For example, the numerical value of the heat capacity of water in the Celsius scale will be 1.25 times larger than that in the Réaumur scale, i.e. it will change from 4.19 kJ/(kg·°C) to 5.23 kJ/(kg·°R). However, since according to Réaumur's scale water boils at 80°R, the heat needed to heat 0.1 kg of water from the melting point of ice to the boiling point of water will be the same and equal to 41.9 kJ.

2.6. This sophism only contains a seeming contradiction since in both cases the work done against external forces must come from some energy source. In the first case the energy is delivered to the system from without, i.e. from a heater, and in the second case the work is produced due to a loss of the internal energy of the body.

The internal energy depends on the velocities and arrangement of the molecules which correspond to the molecular kinetic and molecular potential components of internal energy. The solidification of a liquid is accompanied by a substantial change in the pattern of motion and arrangement of the molecules, while their velocities remain practically unchanged. The molecules in the crystal lattice of a solid are strictly ordered to match to a minimum molecular potential component. The excess energy is dissipated. This is called the heat of solidification (or melting). It is this heat that breaks a hermetically sealed iron shell when water inside it is frozen.

Let us consider another example. When steam from a boiler is supplied into the cylinder of a steam engine, the work done to displace the piston is performed due to the energy fed with the steam. However, when the cylinder is isolated from the boiler, the steam displaces the piston due to a loss of its internal energy, namely, its molecular kinetic component. The steam is thus cooled. This mode of operation was first employed in 1776 by James Watt (1736-1819) and enables using the steam's energy more effectively.

2.7. Since the forces of interaction between air molecules are practically vanishing, the energy of compressed gas is the kinetic energy of its molecules. The work done to compress the gas increases the kinetic energy of its molecules which manifests itself in a rise in its temperature. When the air expands it performs work due to a loss of its internal energy, i.e. due to a fall in the kinetic energy of the molecules. If external energy is not supplied from without, a gas's temperature when expanding falls. However, in practice it is quickly raised due to the heat exchange with the environment. Thus, compressed air performs work due to the internal energy of the environment. Though, it is to be noted that the same amount of energy is given up to the environment by the air heated during the compression stage.

2.8. Like the sophism concerning the disappearance of the potential energy of coal (see Problem 1.26), this problem does not take much trouble: energy cannot disappear, it is only transformed from one kind to another. In fact, precise measurements would demonstrate that dissolving the bent strip results in a higher temperature of the acid than dissolving the unbent strip. Though, the temperature elevation is so small that it cannot be detected by simple means and a conventional thermometer would be inadequate.

Note though that an unbent strip dissolves more quickly.

2.9. Work is a process resulting in an energy transfer from one body to another. The amount of energy transferred can be defined as force multiplied by distance (if their directions coincide, otherwise a third multiplier, i.e. the cosine of the angle between the directions of the displacement and the force, is required).

Work is a possible yet not the only method of energy transfer. In everyday life we no less often encounter a second method of energy transfer between bodies, that of heat transfer.

A rocket "hovers" immobile even though its engines are running. Since there is no displacement, there is no work. This does not at all mean that the energy of burnt fuel disappears, the gases ejected from the rocket nozzles are at high temperature and take away the energy previously contained in the fuel tanks.

2.10. As was indicated in the solution of the previous problem, there are only two radically differing forms of energy exchange, viz. work (the ordered transfer of energy from one body to another, i.e. energy transfer at the macroscopic level) and heat transfer (a disordered form, i.e. energy transfer at the microscopic level). Notwithstanding their qualitative difference, these processes can lead to the same result. Here lies "the equivalence between heat and work" Let us consider a specific example.

Suppose air is contained under a piston in a cylinder. The air can be heated in several ways. For example, heat can be supplied by a heater with the piston fixed in one position. This is called an isochoric process. Under isochoric heating the heat capacity of air is 0.73 kJ/(kg·K) and the amount of heat needed to heat 1 kg of air to 1 K is then

$$\Delta Q = mc_V \, \Delta T = 1 \text{ kg} \times 0.73 \text{ kJ/(kg}\cdot\text{K)}\cdot 1 \text{ K} = 0.73 \text{ kJ},$$

where c_V is the isochoric specific heat capacity.

Since the piston is fixed, the gas does not do any work against external forces and all the heat supplied, i.e. 0.73 kJ, is spent to increase the air's internal energy, this resulting in a temperature rise of 1 K.

However, the same temperature rise and the same increase in the internal energy can be reached by another method, namely, by supplying energy not via heat transfer, but via gas compression.

A process occurring without any heat exchange with the surroundings is called adiabatic. For the process to be adiabatic the air must be contained in a non-heat-conducting shell. A good approximation to this ideal is a system in which the gas is contained inside a glass vessel with double silvered walls and a vacuum in the space between the walls (a Dewar flask).

Very fast processes are also practically adiabatic. For example, when the gas is rapidly compressed heat exchange with the surroundings is impossible and the increase in the gas internal energy will be equal to the work done to compress it. This increase is accompanied by a rise in temperature cyclists know from pumping up tyres.

It is known from thermodynamics that in an adiabatic process gas's volume and temperature are related

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-}$$

where T is the gas temperature in kelvins and γ is a constant (the ratio between isobaric and isochoric heat capacities), which is 1.4 for air.

By assuming the initial volume occupied by 1 kg of air is 1 m³ and the initial temperature is 273 K, we get the volume to which the gas must be adiabatically compressed to rise its temperature to 274 K, i.e.,

$$V_2 = V_1 \sqrt[\gamma-1]{\frac{T_1}{T_2}} = 1 \text{ m}^3 \sqrt[0.4]{\frac{273 \text{ K}}{274 \text{ K}}} = 0.99 \text{ m}^3.$$

To sum up, it is worth noting that the equivalence between heat transfer and work (heating bodies without supplying heat to them) was unconsciously used even in ancient times when they produced fire by friction. It was only in 1795 that Sir Humphrey Davy (1778-1829) and Benjamin Thompson (Count Rumford) (1753-1814) in more detail in 1798 embarked on experimental research of this phenomenon the physics of which was fully understood after works by Julius Mayer (1814-1878), Ludwig Colding (1815-1888), James Joule (1818-1889) and Hermann Helmholtz (1821-1894).

2.11. When using a soldering iron it is not the internal energy of the iron that is important (especially, if the soldering iron is electric), but rather the rate at which it is released. Since the heat conductivity of copper is six times that of iron, heat is released much faster if the soldering iron is made of copper. Of vital importance in choosing material for a soldering iron is the better, as compared to iron, chemical resistance of copper.

2.12. Even if we take a substance with a high coefficient of thermal expansion, the value of $-1/\alpha$ is much less than $-273°C$, i.e. absolute zero. Thus, for lead $\alpha = 3 \times 10^{-5}\ °C^{-1}$ and $-1/\alpha = -3 \times 10^{6}\ °C$. Such temperatures are beyond reach (to state it better, they are nonexistent).

In analyzing the sophism it is important to keep in mind the dependence of the coefficient of thermal expansion on temperature, too.

2.13. We must always distinguish between a force to perform a work and a force against which a work is performed. Sometimes the magnitudes of these forces do not match; therefore, with the same displacements, the works performed by these forces may not only differ in sign (this is always so), but in magnitude, too.

For example, when lifting a balance weight of 1 kg at a height of 1 m, we perform work $A = mgH = 1\ \text{kg} \times 9.8\ \text{m/s}^2 \times 1\ \text{m} = 9.8\ \text{J}$ only when the force applied to the weight is 9.8 N. The energy expended is then transformed to the weight's potential energy.

Yet it is possible, when lifting a balance weight, to apply a force exceeding 9.8 N. In this case the excess force will result in an accelerated motion and in an increase in the weight's kinetic energy. The sum of the potential and kinetic energies at the height of 1 m will be equal to the work performed by the man's arm during this displacement.

The problem also deals with the force performing work, that of atmospheric pressure, the force against which work is performed, that of gravity on the mercury column in the tube.

The work of the first force is the same for both tubes and can be calculated from the formula

$$A_{at} = p_{at} \cdot V,$$

where p_{at} is the atmospheric pressure and V is the internal volume of each tube, the bulge included.

As to the second force, the work it performs can in principle be calculated from the same formula. However, in practice this is not easy, since the pressure in the mercury column is constantly changing as the tube is filled. Immediately after the taps are opened the pressure is zero, as there is no mercury in the tubes. Then the pressure forces grow as the column rises. The growth is slower in the right-hand tube, where the bulge is lower, and until it is filled with mercury the pressure only changes slightly. Therefore the mean pressure turns out to be higher in the left-hand tube, since here the mercury column rises fast. Hence, the work done against pressure forces will be larger.

Thus, the work done by the atmospheric pressure forces is the same in both cases, yet it is performed not only to increase the potential energy of the mercury, but to accelerate it and to overcome friction. When mercury stops rising, its kinetic energy transforms to heat. The same heating accompanies the friction of mercury against tube walls.

In both cases the sum of the energies spent heating the mercury and increasing its potential energy equals the work of the atmospheric pressure forces, but the second term is larger for the left-hand tube.

2.14. A liquid rises in a capillary tube when the force of attraction between the molecules of the liquid and those of tube exceed the cohesive forces between the molecules of the liquid. Thus, the work done to raise the liquid is performed at the expense of the fall in potential energy of the liquid-tube system configuration. (Analogously, a permanent magnet attracts an iron piece due to the accompanying decrease in the potential energy of the magnet-iron piece system.)

Yet if the cohesive forces between the molecules of the liquid exceed the forces of attraction between these molecules and those of the tube material, the potential energy of the system decreases with the liquid falling in the tube.

2.15. A sugar solution in water has a larger coefficient of surface tension (larger specific surface energy) than does water. This is why the surface occupied by the sugar solution tends to shrink and carries with it the matches to the lump of sugar.

When soap is dissolved the water tension decreases (by the way, this is the cause of the washing action of water), the surface occupied by the soap solution extends and the matches follow the interface with pure water retreating to the rim of the plate.

2.16. The answer will be assisted by an unsophisticated experiment. Take some steel or copper wire 1-2 mm in diameter and heat it in a flame. When it has cooled slowly the wire is flexible, it can be easily bent into circles. However, when bent from side to side a few times, the wire gradually becomes more and more rigid. This hardening due to loading is called cold-work hardening. It can be explained by the mutual compensation of the different defects in crystal lattice. The theory is rather involved and cannot be presented here in a rigorous way [7].

Drawing results in a cold-hardening which hardens the wire passing through dye hole (drawing eye). With repeated drawing it is preceded every time with annealing of the wire to facilitate the process.

2.17. The error in the solution in the text is the wrong assumption that after the addition of boiling water all the ice will melt. Actually, there is too little hot water and some ice will survive, while the temperature will not rise above 0°C. Therefore, to draw up the heat balance equation we must account for the amount of heat supplied by the boiling water

$$1 \text{ kg} \times 4.19 \text{ kJ/(kg} \times {}^\circ\text{C)} \times 100^\circ\text{C}$$

and that expended in melting the ice

$$m \times 335 \text{ kJ/kg},$$

where m is the mass of melted ice.

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[7] Anyone interested in it should turn to a course on the theory of solids.

The jar does not enter the heat balance, since its temperature is left unchanged. By equating the above expressions, we find $m = 1.25$ kg.

Hence, the jar will contain 2.25 kg of water and 0.05 kg of ice and their mixture will have a temperature of 0°C.

To sum up, the heat balance equation, when a substance goes over from one state of aggregation into another (for example, from solid to liquid or from liquid to gas) has to be carefully compiled with due account for the possibility of an incomplete transition [8].

2.18. At any temperature a liquid contains both fast and slow molecules. Evaporation proceeds due to the escape from the liquid of fast molecules which possess energies sufficient to overcome the forces of cohesion with the remaining liquid. After the escape of the fast molecules, the average velocity of the remaining ones decreases. The same holds for the average liquid temperature governed by the average velocity of the molecules. Following this, the temperatures cease to be equal and heat exchange becomes feasible.

2.19. Without being embarrassed (one of the examiners was the young but already well-known Enrico Fermi) the student nevertheless understood the question and answered correctly: "When food is fried, it is not the oil that boils but the water in food."

In fact the temperature of the food will not rise above 100°C until all the water boils away.

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[8] Generally, here we should speak not of the transition from one state of aggregation into another, but the phase transition of the first kind.

This is why water can be boiled in a paper bag.

2.20. As the pressure is lowered, the water boils at a lower temperature. Therefore in mountains, where the atmospheric pressure is lower than at sea level, water can boil at 80°C or lower, since the drop in the boiling point of water amounts to 3°C per kilometer of altitude above sea level. This phenomenon is well known to mountaineers who use it sometimes to check the height they have reached.

However, it is not the fact of the water boiling itself that is important but the effect of the boiling water. Who needs "boiling water" with a temperature of, say, 60°C? It cannot soften meat or fish, nor sterilize medical equipment, nor even satisfy a tea lover.

A.G. Dralkin, the leader of the Fourth Soviet Antarctic expedition, reported that during their journey by caterpillar vehicles from Mirny to the South pole and farther to the *Vostok* station, they were forced to ascend to above 3500 m. At these altitudes it was impractical to boil food because the atmospheric pressure was too low and water boiled at between 55 and 60°C. It could take as much as 6 or 7 hours to cook meat. Later they installed pressure cookers in the vehicles' galleys. [9)]

2.21. Here as in the previous problem, we need know that the boiling point of water is pressure-dependent. For instance, at 40 atm the boiling

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[9)] Bear in mind that atmospheric pressure in the Antarctic is in any case lower than elsewhere. Such regions are called in meteorology atmospheric depressions.

point is 249.3°C. At the same time the melting point of tin at this pressure is practically 232°C, i.e. the temperature tin melts under normal atmospheric pressure. Thus, at 40 atm water can indeed melt tin.

A lump of ice can burn. Gustav Tammann (1861-1938) and Percy Williams Bridgman (1882-1961) demonstrated in a series of experiments that the structure of ice is changed at elevated pressures and it turns into new crystal modifications. Tammann discovered three isotopes of ice which are known as ice II, III and IV (the usual isotope is ice I). Bridgman discovered two more modifications, i.e. ice V and ice VI. At 20,000 atm ice VI remains solid even at +75°C. At even higher pressures ice VI can exist at even higher temperatures, and this ice would be able to burn somebody's hand.

2.22. The inventor's calculations were wrong, since a 100% saving in fuel would mean a perpetual motion machine would be possible.

The use of all three improvements together does ensure more savings than when each invention is used separately but they still will be less than 100%.

To simplify the calculation we assume that before the inventions are used the setup consumed 100 kg of fuel per hour. The first invention reduces the fuel consumption to 70 kg/h. The second invention provides a further 25% but now the base figure is 70 kg. Thus, putting into practice the two inventions the fuel rate will be 52.5 kg/h. Finally, the third invention saves a further 45% and lowers the consumption to 28.9 kg/h. The final value is independent of the sequence of calculations. In each case the saving will total to about 71.1%.

2.23. The paradox is resolved by the first law of thermodynamics. This can be stated as: the amount of heat transferred to a system equals the increase in its internal energy plus the work performed by the system against external forces:

$$Q = \Delta U + A.$$

Since the initial and final temperatures of both balls are equal, the changes in their internal energies are the same. However, thermal expansion raises the centre of gravity of the first ball and lowers that of the second ball. Thus, to heat the first ball needs additional energy (heat) for the positive work against gravity (i.e. to increase the potential energy of the ball). The centre of gravity of the second ball falls and the work against gravity is negative. To sum up, for the same rise in temperature we have $Q_1 > Q_2$. Hence, the heat capacity of matter is not a constant. Depending on the heating conditions it may take on different values from $-\infty$ to $+\infty$! However, for solids we need generally know only one heat capacity, namely, that under a constant pressure which is cited in handbooks of physical constants. For gases we often need the heat capacity at a constant volume, too (the ratio between these two heat capacities appears, for example, in the equation for expansion in an adiabatic process (see the solution of Problem 2.10).

This first person to draw attention to the fact that both solids and gases possess different heat capacities was the French physicist Jean Baptiste Biot (1774-1862).

2.24. On this problem the German astrophysicist Robert Emden (1862-1940) wrote in 1938 to *Nature*: An amateur would answer the question why we heat rooms in winter thus: to warm the rooms; someone who is good at thermodynamics would say: to supply missing energy. Such being the case, the layman will be right rather than the scientist[10].

In fact we demonstrated that the increase in the internal energy of the air in the room due to the rise in temperature is equal to the energy loss because the air mass falls. Thus all the energy supplied to the air in the room by heating it escapes through the cracks in the walls. We might say, we heat the environment rather than cottage.

Then why do we burn firewood in a stove? The answer is that although we do not care what the total internal energy of the air in the room is, our bodies are very sensitive to the temperature, which is a function of the energy per molecule.

We heat a chamber for the same reason that life on Earth is only possible with a permanent inflow of solar energy. Again it is not because the incident energy will almost all be reemitted, rather like someone who does not gain weight even though he continues to eat, it is because we need physiologically a certain temperature in order to survive.

2.25. Together with the law of energy conser-

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[10] Cited from A. Sommerfeld *Thermodynamik und Statistik*, Wiesbaden: Dieterich, 1952.

vation, there is another thermodynamic law. It is expressed as follows: nature is so arranged that each heat engine has both a "heater", and a "cooler". For example, in a steam engine the heater is the furnace and the cooler is the atmosphere.

The ocean can indeed be considered to be a giant heater but for it to be the heat source for an engine there must be an equivalently large cooler but this we cannot provide.

Heat engines of any kind need a temperature difference to operate. This is required by the second law of thermodynamics.

This law is of prime importance in maintaining life. To illustrate it, Emden wrote: "in the giant factory of Nature the principle of entropy (the second law of thermodynamics) is like the manager who determines how and which transitions can be handled. The law of energy conservation is like the accountant who puts into balance debit and credit."[11]

The construction of a machine using the heat of the ocean does not contradict the first principle of thermodynamics, but it contradicts the second one. Note that machines which are based on the temperature difference between surface and the depths of the ocean are quite realizable, but their efficiency, which is calculated by

$$\mathrm{Eff} = \frac{T_1 - T_2}{T_1}$$

(see the next problem) turns out to be rather low, the smaller the difference between the temperatures of the heater and cooler the lower the

[11] *Ibid.*

efficiency. If the temperature at the depth is 2°C and the temperature at the surface is 27°C, then the efficiency is 8.3%. The temperature difference could easily be utilized in a thermoelectric cell. Possibly in the future, when their cheaper production is found, the powerful ocean-based thermoelectric stations might contribute to the energy balance of our planet.

2.26. The ratio

$$\frac{T_1 - T_2}{T_1}$$

is the fraction of burnt fuel energy convertible into work by a heat engine when there are no losses of useful energy (it is the efficiency of an ideal heat engine). The ratio indeed increases in winter. However, during winter the oil in the motor, gear-box, the differential, and bearings becomes so thick that even if the winter oils which have lower viscosity are used, the friction losses are larger and the overall efficiency of the car is lower than it is in summer. Besides, in winter we spend petrol to warm up a cold engine when starting it. This is why the petrol consumption in winter is larger than it is in summer.

2.27. Maxwell's paradox has only been solved recently. We tacitly assumed that the device for sorting molecules by velocity could operate either without spending energy or by only consuming it in small amounts. In fact, a demon needs energy to operate. In particular, the demon must "see" the molecules, i.e. they must be illuminated, which needs an energy.

Exact calculations by the French physicist Louis Marcel Brillouin (1854-1948) demonstrated that even when there were no losses, say, due to friction, velocity after being sorted by the device would be just sufficient to allow it to operate once more. Thus, at best the demon could only make itself go while to use the whole device as an engine is out of question.

Ch. 3. Electricity and Magnetism

3.1. Coulomb's law describes the interaction of pointlike charges. This means that in practice the dimensions of the charged bodies must be much smaller than the distances between them. If the charged bodies are not too far from each other, it is unclear how to measure their separation. In order to calculate the force of close bodies they must be divided into very small ("pointlike") parts and the resultant of all the forces acting upon all the points of the first body by all the points of the second body must be found. This resultant ensures that an electric force applied to the first body is dependent on its shape and charge distribution over the surface. Thus, calculating the forces of interaction is a complicated problem which can sometimes (but not always) be solved by integral calculus.

Capacitor plates are not pointlike bodies and the force of interaction between them cannot be calculated from the formula

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2},$$

where R is the distance between the plates.

It is also necessary to note that the field between the plates of a parallel-plate capacitor is not, actually, uniform and the deviations from uniformity are larger the farther apart the capacitor plates. The field strength and the force with which the plates are attracted together are inversely proportional to the distance between the plates. When the plates are far apart the charges on them can be taken to be pointlike and the force of interaction can be calculated according to Coulomb's law.

To conclude, note that Coulomb's law correctly describes the force of interaction between two spheres which have the charge uniformly distributed over their surfaces, or balls whose charge is uniformly distributed over their volumes. The parameter in the Coulomb equation is then the distance between the centres of the spheres or balls.

3.2. For a current to be in an electric circuit it must be closed and there must exist some force to set the current carriers, i.e. electrons or other charged particles, in ordered motion. The most frequent cause is the presence of a potential difference between two points in the circuit[12].

The absence of any current in subcircuit *A1B* suggests that the potentials of *A* and *B* are equal. The situation is not changed by connecting the points to conductor *A2B*, as no current will flow for the same reason.

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[12] Other forces can cause the charges to move. This will be considered in more detail in the solution of Problem 3.17.

The second case is different. There is a potential difference between C and D equal to the electromotive force of each cell, since two cells connected in parallel can be considered to be one cell with electrodes double in area. However, until a conductor $C3D$ is connected to the cell electrodes, the circuit is open and therefore no current can flow between C and D. We might say that without the conductor $C3D$ there is only the internal circuit section, i.e. the battery, while the external section is missing.

3.3. This sophism is mathematical rather than physical.

$$I_0 = I_1 + I_2.$$

Therefore

$$I_0 - I_1 - I_2 = 0,$$

but it is well-known cancelling by zero is prohibited. In the algebra involved in a physical problem we cannot therefore ignore mathematical rules.

3.4. The accumulator battery is marked with the maximum current permissible under standard operational conditions. By overloading it much higher currents can be obtained; however overloading damages the cell plates and the battery may be put out of operation. Conventional lead cells are especially sensitive to overloading. The batteries in cars are specially designed to allow, in starting the engine, to draw for a short time currents of some hundreds of amperes without damaging the electrode plates. Alkaline cells are less susceptible to overloading since they have a substantial internal resistance which effectively limits the current.

3.5. The current through the galvanometer can be calculated from Ohm's law for a complete circuit:

$$I_1 = \frac{E}{r+R},$$

where E is the electromotive force of the thermocouple, r its resistance, and R the galvanometer's resistance.

After connecting a shunt with resistance R, the current through the galvanometer will be

$$I_2 = \frac{1}{2}\,\frac{E}{r+R/2}.$$

Since the resistance of the thermocouple is substantially lower than that of the galvanometer, the first term in the denominators of both fractions can be neglected and we then get

$$I_2 = \frac{1}{2}\,\frac{E}{R/2} = \frac{E}{R} = I_1.$$

Thus, the current flowing through the galvanometer and, hence its reading will not really change.

If the resistance of the galvanometer substantially exceeds that in the remaining circuit, as is the case here, then the galvanometer operates as a voltmeter. To bring down its reading a resistance is to be connected in series. This is what student physicists would have had to do.

3.6. Let the resistance of the external subcircuit be R, the electromotive forces of the cells be E_1 and E_2, while their internal resistances, r_1 and r_2, respectively. Since the current in the first case is higher than in the second, i.e.

$$\frac{E_1}{R+r_1} > \frac{E_1+E_2}{R+r_1+r_2},$$

the situation described can really take place if

$$E_1 > E_2 \frac{R+r_1}{r_2}.$$

3.7. The current running through the lamp during the measurement of its resistance is too low to change appreciably the temperature of its filament and we can assume that the resistance of the cold filament is being measured.

The resistance obtained by calculation, the power in the formula corresponds to a current which heats the filament white hot. Yet the filament's resistance rises with temperature according to the law

$$R_t \quad R_0 (1 + \alpha t).$$

By putting into this expression the resistances of the cold and hot filaments as well as the temperature coefficient of resistance for tungsten, $\alpha = 0.0046°C^{-1}$, we can find the temperature of the heated filament, i.e.

$$t = \frac{484 \text{ Ohm} - 35 \text{ Ohm}}{35 \text{ Ohm} \times 0.0046° \text{ C}^{-1}} = 2800 \text{ °C},$$

i.e. the value very close to the true one.

The change in temperature can be judged from the change in a conductor's resistance and this fact forms the basis for resistance thermometers, i.e. thermoresistors.

Note that the electrical conductivity of semiconductors is especially temperature sensitive and their resistance quickly drops with an increase in temperature. Therefore, the most sensitive thermistors, barretters, which can sense the heat from a lighted match a few kilometres distant are made of semiconductor materials. Resistance is more weakly dependent on temperature in metals and even weaker in metal alloys. The resistance of constantan composed of copper, nickel, and manganese does not practically change with temperature and this is essential when constructing very precise electrical devices.

3.8. The potential difference across a subcircuit is only equal to the product of the current flowing in the subcircuit and its resistance when it does not contain a current source. Otherwise the potential difference must be calculated from the formula

$$\varphi_A - \varphi_B = IR - E, \qquad (1)$$

in which φ_A and φ_B are the potentials of the initial and final points of the subcircuit, respectively, R is its resistance, I is the current flowing through it, and E the e.m.f. present in the subcircuit. To use this formula correctly the current I must be taken to be positive when it is flowing from A to B, and negative in the opposite case. In turn, the e.m.f. E must be positive if it forces positive charges to move from A to B, and negative, if the opposite is true. (In other words, the extra e.m.f. is positive if it "assists" the current to flow from A to point B, and vice versa.)

In our case the extra e.m.f. must be taken with the "plus" sign, since the right-hand cell sends current in the same direction as the left-hand one. Thus we have

$$\varphi_A - \varphi_B = IR - (+E) = \frac{2E}{2R} R - E = E - E = 0.$$

It should be remembered that formula (1) can be derived from Ohm's law for a complete circuit (see Fig. S.3.1):

$$I = \frac{E}{R+r},$$

where E is the e.m.f., R is the resistance of the external section, r is the internal resistance of the cell, and I the current flowing in the circuit.

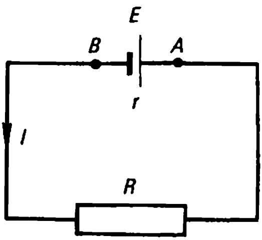

Fig. S.3.1

Let us rewrite the last equation as

$$IR = E - Ir.$$

In this expression all the quantities and their products are positive. Therefore, on the basis of Ohm's law for the subcircuit BRA, the right-hand side of the equation has to be equated to the potential difference $\varphi_B - \varphi_A$ (and not

$\varphi_A - \varphi_B$), since the potential at B is higher than that at A. Thus, we have

$$\varphi_B - \varphi_A = E - Ir,$$

whence

$$\varphi_A - \varphi_B = Ir - E,$$

which is Ohm's law applied to the ArB subcircuit containing the e.m.f. equal to E. Note that the subcircuit involving an e.m.f. used to be called irregular.

3.9. Indeed, the larger the resistance, the higher the efficiency of electric power utilization. It reaches unity when the load resistance is infinite but this, of course, is unrealizable in practice, although it can be approached.

However, making the load resistance too large is impracticable. True, the voltage across the load increases, but it cannot exceed the current source's e.m.f., while at the limit of infinite R the current decreases and tends to zero. Thus, in the formula for the power,

$$P = I \cdot U$$

the first multiplier will tend to zero, while the second does not exceed a certain limit (the e.m.f.). It can thus be seen that the power consumed by the external section from the source tends to zero.

The load resistance must not be too low either, since in the above formula for power the first multiplier cannot be higher than E/r (this is the short-circuit current, when the load resistance vanishes), while the voltage across the load tends to zero as the load resistance infinitely decreases.

It can be shown that the maximum power is consumed by the external subcircuit when its resistance is equal to that of current source. Let us write the conventional expression for this power

$$P=\frac{E^2}{(R+r)^2}R,$$

where notations are as before.

Let us multiply both the numerator and denominator by $4r$, then we get

$$P=\frac{E^2 4Rr}{4r(R+r)^2}.$$

By using the identity

$$4Rr=(R+r)^2-(R-r)^2,$$

and transforming the equation we get

$$P=\frac{E^2}{4r}\left[1-\frac{(R-r)^2}{(R+r)^2}\right].$$

Whence it can be seen that $P=0$ at $R=0$ and $R=\infty$, while at $R=r$ the power reaches a maximum (since both the numerator and denominator of the fraction in the brackets are positive, and the fraction as a whole is minimum at zero, when $R=r$).

The proof is even simpler using a numerical example.

Suppose we have a current source with an e.m.f. of 4 V and an internal resistance of 1 Ohm. Then for different load resistances we obtain the following values of the power consumed by the load:

Load resistance, Ohm	Consumed power, W
0.7	3.87
0.8	3.95
0.9	3.98
1.0	4.00
1.1	3.99
1.2	3.96
1.3	3.92

In relation to the solution of this problem it is useful to dwell on the output transformers in valve radios. The output tubes putting in operation a loudspeaker have resistances of tens and hundreds of kOhm. Meanwhile the loudspeaker's coil has a resistance of only 5-10 Ohm since high-resistance coils are technically difficult to make. Connecting a low-resistance loudspeaker directly to the anode circuit of an electronic valve only yields a low acoustic power. A matching output transformer with a high-resistance primary coil and a low-resistance secondary coil should therefore be connected between the valve and the loudspeaker. Besides, its incorporation is also useful because in this case the sole AC component of anode current will pass through the loudspeaker.

3.10. Both answers are correct (cf. the solution of Problem 1.17). This does not however mean that two different currents run through the same device simultaneously. It turns out that two dissimilar electric circuits can be assembled such that each satisfies the requirements given in the problem.

Given the two currents and the resistance of the series resistor we can find two voltages:

$$U'_{res} = 0.5\ \text{A} \times 40\ \text{Ohm} = 20\ \text{V} \text{ and}$$
$$U''_{res} = 2.5\ \text{A} \times 40\ \text{Ohm} = 100\ \text{V}$$

Analogously, one can obtain the two voltages across the device:

$$U'_{dev} = \frac{50\ \text{W}}{0.5\ \text{A}} = 100\ \text{V} \text{ and}$$
$$U''_{dev} = \frac{50\ \text{W}}{2.5\ \text{A}} = 20\ \text{V}$$

and so two resistances of the device:

$$r' = \frac{120\ \text{V}}{0.5\ \text{A}} - 40\ \text{Ohm} = 200\ \text{Ohm} \text{ and}$$
$$r'' = \frac{120\ \text{V}}{2.5\ \text{A}} - 40\ \text{Ohm} = 80\ \text{Ohm}.$$

The problem contains no data by which we can prefer the first solution to the second one or vice versa. True, if we calculate the power consumed by the series resistor, then in both cases we get

$$P' = (0.5\ \text{A})^2 \times 40\ \text{Ohm} = 10\ \text{W} \text{ and}$$
$$P'' = (2.5\ \text{A})^2 \times 40\ \text{Ohm} = 250\ \text{W}$$

and compare it with the power of the device itself (50 W), then the second solution is highly improbable, however we cannot be fully certain that the solution is totally useless.

Hence, we must believe both solutions are correct. If we are going to have the unique solution those composing problems should have added extra data, for example, the power consumed by the series resistor.

3.11. The current intensity can be expressed via the amount of electricity Q passing through a cross section of the conductor during time t, namely,

$$I = \frac{Q}{t}.$$

From this relationship we can see that in the formula

$$I = I_+ + I_- \qquad q_+n_+v_+ + q_-n_-v_-$$

the product n_-v_- stands for the number of negative ions escaping from cathode per unit time. As a result, as many positive ions remain near the cathode as positive ions reach its surface. Besides, the cathode is approached by n_+v_+ positive ions per second. Thus, the total number of the positive ions neutralized near the cathode is governed by the total current. A similar situation is observed near the anode for negative ions.

3.12. The statement composed in italics in the problem contains no errors. Actually, since the current in all the baths is the same, then n-fold more substance must be deposited in the n baths than in one bath. However, this does not imply that the amount of substance deposited in the second installation will be n times larger than that in the first one for the same period of time. The reason is that if the power source is the same in the new installation, then the current intensity must drop for two reasons.

The first one is only too evident. Connecting the electrolytic baths in series increases the length of the "liquid conductor", which results in an increase in the resistance of the circuit, i.e. a decrease in the current.

The second reason is less obvious. The passage of a current through an electrolyte induces a number of physical and chemical changes at electrodes due to which the electrolytic bath starts to operate as a galvanic cell with e.m.f. opposed to the applied e.m.f. This phenomenon is called galvanic polarization, was discovered in 1802 by N.N. Gautreau (1753-1803). Antoine César Becquerel (1788-1878)[13] was the first who described it in 1824. It was studied in detail in 1842-1845 by the Russian physicist A.G. Savelyev (1820-1860) who concluded that "polarization cells" could be made[14]. The total polarization e.m.f. increases with the number of baths, which also decreases the current.

Thus, although an increase in the number of baths should increase the total mass of substance deposited on the electrodes, in fact it will at best remain unchanged (in practice the total mass always decreases), since the mass of substance deposited in each bath is smaller due to the fall in the current.

3.13. The very attempt to discover a process that contradicts the law of energy conservation

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[13] A.C. Becquerel was a French physicist and the grandfather of Antoine Henry Becquerel (1852-1908) who discovered natural radioactivity in 1896.

[14] This prediction was implemented in 1859 by the inventor of acid accumulator, the French scientist Gaston Plante (1834-1889).

is doomed to failure, as all the previous ones were.

When charging capacitor $C1$ the energy of the electric current is partly expended for heating the conductors (Joule heating) and partly radiated into surroundings as electromagnetic waves. We need only here that at $C1 = C2$ the energy "losses", independent of the resistance of connecting conductors, always amount to 50%. If $C1 \ll C2$, there is practically no energy dissipation, while at $C1 \gg C2$ it reaches 100%.

3.14. On the sides of the glass sheet facing the capacitor plates charges are accumulated due to dielectric polarization. These charges have signs opposite to those of the charges on the capacitor plates. When the glass sheet is removed the work has to be done against the forces of Coulomb attraction between unlike charges. This work increases the energy of the capacitor.

3.15. The system will not possess magnetic properties, since its complete symmetry implies that each point in the reassembled ball and the space around it will be passed through by equal numbers of magnetic lines of force of opposite sense. To put it another way, the "magnet" will instantaneously demagnetize itself. This does not mean that a ball-shaped magnet is impossible, only that unlike poles, even though in unequal numbers, must be found on its surface. For example, a ball can be magnetized so that there will be two north poles and one south pole on its surface. While discussing the subject of magnetizing a ball (some people assume it to be impossible!), recall that the Earth which is a ball is a giant magnet.

In conclusion let us note that the English physicist Paul Dirac (1902-1984) pointed out that Maxwell's equations which are the basis of electrodynamics, in principle, permit the existence of isolated magnetic poles (i.e. particles having one pole, either south or north). However, attempts over a long period of time to detect "Dirac monopoles" have failed and most contemporary physicists think there are none.

3.16. As the magnet and the piece of iron are brought together, the potential energy of the permanent magnet-iron object system is decreased by the amount of the work performed against gravity. To restore the potential energy needs the iron to be removed from the magnet. It is clear that to do this requires as much work as was performed by the magnet in lifting the iron.

Thus, the magnet can be compared to a spring that lifts a weight: to do work the spring must first be stretched, thus spending energy.

3.17. This sophism is related to the one in Problem 3.8. As before, the absurd conclusion is due to the incorrect application of Ohm's law.

The potential difference across a subcircuit is only equal to the product of the resistance of the subcircuit and the current flowing through it when the subcircuit has no e.m.f. source. Such subcircuits are called uniform.

In this case each section of the ring is nonuniform, since the induced e.m.f. is uniformly distributed around the perimeter of the ring.

According to Ohm's law for a subcircuit involving an e.m.f. (a nonuniform section) we can write an equation for ARB by taking a point A as the origin of the subcircuit and B as its end, i.e.

$$\varphi_A - \varphi_B = IR - E_{ARB},$$

where E_{ARB} is the e.m.f. induced in ARB.

If the total e.m.f. present in the ring is E, then the current flowing through the ring is

$$I = \frac{E}{R+r}.$$

On the other hand, the e.m.f. concentrated in a ring section should be directly proportional to the length of the section or, if the conductor is uniform, to its resistance. Thus we get

$$E_{ARB} = E\frac{R}{R+r}.$$

By substituting the last two expressions into the first we find

$$\varphi_A - \varphi_B = \frac{E}{R+r}R - E\frac{R}{R+r} = 0.$$

The same conclusion can be arrived at by applying Ohm's law to section BrA:

$$\varphi_B - \varphi_A = Ir - E_{BrA}.$$

The current, as before, is equal to

$$I = \frac{E}{R+r},$$

while from the above considerations for E_{BrA} we can write

$$E_{BrA} = E\frac{r}{R+r},$$

so we get

$$\varphi_B - \varphi_A = \frac{E}{R+r} r - E \frac{r}{R+r} = 0.$$

Thus, a current can also flow in an electric circuit when the potential difference across two arbitrary points of the circuit is zero. This is not really surprising if we recall the definition of electric current, i.e. the motion of charged particles in one direction. Yet this motion need not be due to electrical forces. An electric current, for example, can be produced by a flux of charged sand particles falling through Earth's gravitational field or by a cloud of charged smoke particles being driven by the wind.

Moreover, an electric current can even run from a high potential to a lower one, as happens in a galvanic cell or other current source.

The following examples illustrate this.

Water moves in the river due to a potential difference, while in a cup of tea slowly stirred by a spoon water moves too, even though there is no potential difference. When pumped from a well, water particles move against the potential difference, i.e. the force of gravity of water.

3.18. The magnetic flux in a transformer core is not only produced by the current in primary coil, but by that in the secondary coil too. According to Lenz's law the directions of the appropriate magnetic fluxes are opposite (shifted in phase by almost 180°), hence in the ideal case the resultant magnetic flux in the core must vanish. An increase in the load on the transformer is accompanied by an increase in the primary coil current and in the magnetic flux produced by it.

Simultaneously the secondary coil current and the "secondary" magnetic flux increase. The resultant magnetic flux changes within these limits and, as a result, the AC e.m.f. induced in the secondary coil does not change.

3.19. The absolute value of the voltage across any subcircuit of an AC circuit varies 100 times per second from zero to a maximum called the peak voltage. A moving-iron voltmeter connected in parallel to this section will read an intermediate value which is called the root-mean-square (r.m.s.) or effective voltage and is $\sqrt{2} = 1.41$ times lower than the peak value. If the moving-iron voltmeter reads 50 V, then at some instant of time the voltage is 50×1.41 V, i.e. about 70 V, when the DC lamp flashes. Therefore, the experiments do not involve any contradiction.

This simple and easy experiment is a useful school demonstration as it assists students to understand the relationship between the root-mean-square and peak voltage in an AC circuit.

3.20. To answer the question we must know how the ammeters of each system are designed.

In moving-coil devices a frame coil through which the current being measured is passed is placed between the poles of a permanent magnet. The displacement of the frame coil is directly proportional to the current. If the latter rapidly changes, the pointer will indicate the average value of the current flowing through the device.

In electrodynamic devices the current being measured passes through an immobile coil and then through a moving coil placed inside the first one. Thus, the pointer moves proportionally to the currents passing in both coils, i.e. to the square of the current since in both coils it is the same. In this case if the current changes quickly the pointer moves in proportion to the mean square of the measured current.

Therefore, if both ammeters read the same value in a DC circuit, then their readings will differ in an AC circuit. It can be proved using higher mathematics that these readings will differ by $\pi/2$.

In fact, the mean value of a pulsed current can be calculated as

$$\langle I \rangle = \frac{\int_0^{T/2} I_0 \sin \omega t \, dt}{T} = \frac{2I_0}{\omega T} = \frac{2I_0}{2\pi} = \frac{I_0}{\pi},$$

where I_0 is the peak value of the pulsed current, ω is the angular (cyclic) frequency of industrial AC, and T is the oscillation period. This expression shows that the readings of a moving-coil ammeter in an AC circuit with a peak AC value of I_0 amperes will be π times smaller than if the ammeter were connected to a DC circuit with a current of I_0 amperes passing through it.

Analogously, the mean square of a pulsed current with the same peak value is

$$\langle I^2 \rangle = \frac{\int_0^{T/2} I_0^2 \sin^2 \omega t \, dt}{T} = \frac{I_0^2 \frac{T}{4}}{T} = \frac{I_0^2}{4}.$$

In both cases the upper integration limit is $T/2$, since the current only passes through an ammeter for half a period.

This expression shows that the pointer of the electrodynamic device will indicate a value which is smaller by a factor of

$$\frac{I_0^2}{\langle I^2\rangle} = 4$$

than the indicated value the ammeter would have if it were connected to a DC circuit with a current of I_0 amperes passing through it. However, because the ammeter scale is quadratic (i.e. the pointer deflection is proportional to the square of the current), the reading is only a factor of $\sqrt{4}$ 2 smaller than it would be in a DC circuit.

Thus, the readings of a moving-coil and an electrodynamic ammeters which detect the same current intensity in a DC circuit will differ

$$\frac{\pi}{2} = 1.57 \text{ times.}$$

But which ammeter reading is true, which ammeter is trustworthy? Obviously the answer will depend on the application of the ammeter and the circuit it is connected to.

For example, if it is necessary to control the current passing through an electroplating bath, with the unsmoothed current from a rectifier, then the ammeter should be a moving-coil device, since the mass of the substance deposited during the electrolysis, like the reading of the ammeter, depends on the mean value of the current passing through the bath (since it is the first power of the current intensity in Faraday's laws).

If one is interested in the thermal effect of a pulsed current, the circuit should incorporate an electrodynamic ammeter, since both its readings and the amount of the heat released by a current passing through a wire depend on the mean square current (the Joule-Lenz law involves the square of the current).

3.21. As a component, the loop $B_fA1CA2B_f$ is part of complicated circuit, in which the cathode filament C is heated by current I_f and an anode current I_a passes from battery B_a. We can see that the fraction of I_a passes through ammeter *A1* in the same direction as I_f, while in ammeter *A2* the anode current opposes I_f.

If both ammeters pass the same fractions of the anode current, then

$$I_1 = I_f + 0.5I_a \quad \text{and} \quad I_2 = I_f - 0.5I_a,$$

whence it can be seen that $I_1 \neq I_2$.

3.22. The cathode filament's temperature is controlled both by the heat released by the current flowing through it and by the way it is cooled.

While the switch is off, the filament is surrounded by a space charge cloud and the number of electrons emitted from the metal to vacuum is equal to the number of electrons returning from vacuum to the filament. When the switch is on, all the electrons (only a fraction at a low anode voltage) are "sucked out" from the filament by the "cathode-anode" field. There remains only a unidirectional flux of electrons from the metal to the vacuum and the loss of heat by the filament via the heat capacity of its terminals and via radiation is added by the loss of the energy carried away by fast electrons and partly expended in their detachment from metal. It is this effect, which leads to the temperature drop after closing the switch.

3.23. Three coils in parallel wound onto a single coil can be considered to be a coil wound by a wire with a cross section three times thicker than a single coil. When plugged into DC mains, this system will pass three times the current, since the resistance of the coils in parallel will be a third of a single coil. Hence in the case of a DC source the magnetic field would be trebled, but not if connected to an AC source.

The impedance of a coil plugged into an AC mains is much higher than its resistance in a DC circuit. For example, the DC resistance of a coil in a knock-down school transformer is about 3.3 Ohm, but when plugged into the 50 Hz AC mains the current passing through it corresponds to a resistance of 20 Ohm.

The AC impedance of a coil (the sum of its resistance and reactance) depends both on the resistance calculated by

$$R = \rho \frac{l}{S}$$

and on the number of turns in the coil, its size, and the AC frequency. If the number of turns in the coil is large, then even for low resistances (the coil is made using very thick copper wire) its impedance can be quite high.

By connecting two more coils in parallel, the technical assistant had in fact slightly reduced the resistance, while the system's inductance did not practically change, since the number of turns and the coil shape have been left as before.

The small change in the resistance has almost no effect on the impedance. Therefore the current consumed by the mains and the magnetic field inside the coils do not substantially change either.

The extra coils are useful because only a third of the current flows through each coil, thus diminishing the heating of each coil.

3.24. The fuse blew in the group fuse box in phase *3* to our apartment.

Then why did the control lamp light?

The answer is that when the power was cut, the switches and all the lamps inside the apartment were left on (so that this error would not be striking, the switches in Fig. 1.31 are shown in the "off" position). Therefore, a control lamp connected between *A* and *B* received a voltage from phase *2* and the neutral wire, with which the lamp was connected via the apartment wiring.

A more attentive and experienced observer would, of course, have noticed that using the checking technique and a blown fuse in phase *3*, when the control lamp is connected between *A* and *B* or *A* and *C*, it will light dimly since the lamps in parallel inside apartment No. 19 will provide some resistance. Had the fuse been whole the lamps would have been too bright since then the control lamp would receive a voltage 1.73 times larger than the standard one, i.e. 380 V instead of 220 V.

3.25. In big buildings the AC power is supplied using the four-wire system. Therefore we can assume that the power was supplied to lecture-halls Nos. 1 and 2 as shown in Fig. S.3.2.

The circuit shows that, if fuse *A* in the "neutral wire" has blown, lamps *L1* and *L2* in the first lecture-hall will not light, unless another device, for example, an electric heater, in the second lecture-hall is switched on. Then the heater and the lamps will receive the voltage from phases Ph_1 and Ph_2 via operating fuses *B* and *C*.

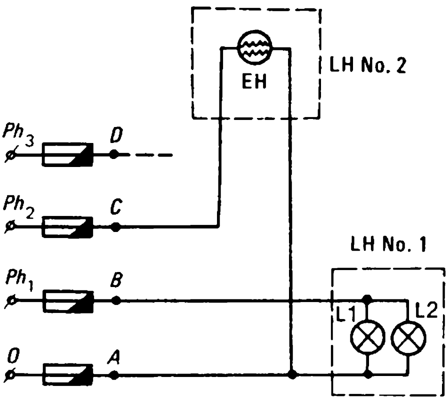

Fig. S.3.2

It is also understandable why the heater did not heat up fully, while lamps were heated above normal. For example, if the voltage between the "neutral wire" and one phase is 127 V, then the voltage between phases Ph_1 and Ph_2 will be $\sqrt{3}$ times higher, i.e. 220 V. According to Ohm's law, this voltage is distributed between the first and second lecture-halls in proportion to the resistances of the devices switched on inside. The heater has a comparatively low resistance, while the seemingly low-power lamps

L1 and *L2*, even though in parallel, have a sizeable resistance. As a result, the heater voltage will be low, while the lamps will have larger than normal voltage.

3.26. When plugged into the AC mains, a moving-iron voltmeter indicates the effective voltage. However, as was stated in the solution of Problem 3.19, the maximum of a sinusoidal voltage is $\sqrt{2} \approx 1.41$ times the r.m.s. voltage. A capacitor in parallel with the voltmeter will be charged by a potential difference of 125 V × 1.41 = 175 V It is this value that the voltmeter reads. The rectifier in effect prevents the capacitor from discharging when the current reverses its direction.

This phenomenon is well known to electronics enthusiasts who are aware of the fact that the voltage is raised by the introduction of a filter consisting of a choke and a capacitor into a rectifier.

The sophism could have been formulated as an experimental problem. It is essential therefore to take a capacitor with a high capacitance (some μE) and a voltmeter with a high resistance. If the resistance and capacitance are low, the system will have, as the professionals say, a small time constant, i.e. the capacitor will discharge quickly through the voltmeter resulting in a potential difference lower than 175 V across them. In very unfavourable cases the voltmeter reading may even be less than the effective voltage in the mains, i.e.

$$\frac{E_0}{2} = \frac{E_{eff}\sqrt{2}}{2} = \frac{125\ \text{V} \times \sqrt{2}}{2} \approx 88\ \text{V},$$

where E_0 is the peak voltage value in the AC mains (cf. the solution to Problem 3.20).

3.27. Notwithstanding the naivety of the sophism, we believe it is useful so as to stress the need to perform the same operations on unit dimensions, as on the numbers to which they relate.

3.28. The current and voltage in a device plugged into an AC mains can be varied so that their maxima and minima will be reached simultaneously. We then say that the current and voltage are in phase. This occurs if the device only possesses resistance. The power consumed by the device can be calculated from the formula

$$P = IU, \tag{1}$$

where I and U are the effective current and voltage, respectively.

Examples of devices with resistance only are incandescent lamps and electric heaters.

However, if a coil or capacitor are plugged into an AC mains, the maxima and minima of the current and voltage will occur at different times. In this case the current and voltage are said to be out of phase and the formula for the power has a third multiplier k called the power factor:

$$P = kIU. \tag{2}$$

The power factor k is represented as $\cos \varphi$, where φ is the phase shift angle between the current and voltage.

The windings of an electric motor have both resistance and reactance (which together make up its impedance). As a result the voltage and current are out of phase after passing through the windings of the motor. The phase shift cosine for the normal operation is

$$k = \frac{P}{IU} = \frac{900\ \text{W}}{220\ \text{V} \times 5\text{A}} = 0.82.$$

The value of cos φ corresponding to optimal operation is indicated in the motor's specification.

3.29. First note that the design does not contradict the law of the conservation of energy, since the capacitor would be charged using the thermal energy of the electrons. However, the device shown in Fig. 1.33 would contradict, if operated, the second law of thermodynamics (cf. Problem 2.25), since the process requires the spontaneous initiation of "condensations" and "rarefactions" of the electronic cloud. This is as impossible as the spontaneous separation of electrons or molecules into fast and slow ones (the latter corresponding to the spontaneous initiation of a temperature difference).

Nevertheless, the second law of thermodynamics, as opposite to the first law, is not absolute in that it does not fully forbids the spontaneous separation of molecules into fast and slow species or the initiation of density fluctuations (local condensations and rarefactions). It only states that the events have low probabilities and that this is lower the larger the fluctuation desired.

Due to the random nature of the motion of electrons, a potential difference may really arise across the ends of a conductor. However, as has been noted, the appearance of a substantial voltage has a low probability, while at low voltages the capacitor will be discharged through the detector, since at low voltages the current-voltage characteristics of all detectors are linear (cf. Fig. S.3.3) and there is no rectification effect.

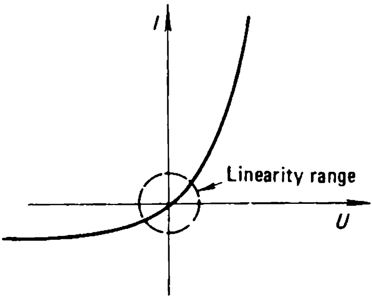

Fig. S.3.

The nonlinearity of rectification explains why it only occurs at rather high voltages, the probability that it will initiate due to density fluctuations in electron distribution is practically nil.

The proposed method of charging is as impossible as a situation (in principle possible) in which the chaotic motion of the air molecules in a sealed can can lift it above the Earth's surface. It has been calculated that can will jump spontaneously due to fluctuations in the motion of molecules about once every billion billion years.

Ariel, the hero in the scientific fiction novel by A.R. Belyaev, could govern the way the molecules in his body moved by transforming the motion from chaotic to directed. This permitted him to fly in any direction. Unfortunately, the effect may only occur in a scientific fiction novel, since it contradicts the second law of thermodynamics.

To be more exact, Ariel's flying not only violates the second law of thermodynamics, it also defies the law governing the motion of the system's centre of mass. In fact, internal forces cannot change both the value and direction of a centre of mass's velocity, since in the Earth's gravitational field it can only move (without intervention from without) with an acceleration **g** directed downward.

3.30. The discharge of a Leyden jar is oscillatory since the helix wound around the spoke and the jar together form an oscillatory circuit. Since the wire is resistive and the energy is released into the environment as electromagnetic waves, the oscillations gradually damp. The spoke maintains the magnetic field direction corresponding to the last oscillation swing that was strong enough to break down the spark gap through which the Leyden jar was discharged.

Ch. 4. Optics and Atomic Structure

4.1. However tempting Flammarion's technique for looking in the past of our planet, we have to accept that it will never be implemented. The special theory of relativity states that no material body can move relative to another one faster than the velocity of light in a vacuum, i.e. about 300,000 km/s.

Moreover, even this velocity is beyond reach for bodies with nonzero rest masses (e.g. electrons, protons, neutrons, atoms, molecules, as well as pieces of metals, stones, etc). It turns out that as its velocity increases the mass of an accelerated body increases, i.e.

$$m=\frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}},$$

where m_0 is the mass of the body at rest (its rest mass), v is the velocity to which the body is accelerated, m is its mass at this velocity, and c is the velocity of light in a vacuum. This relation shows that as v approaches c, the body's mass tends to infinity. Thus acceleration becomes more and more difficult and absolutely impossible at velocities close to c. The dependence of mass on velocity is well known to physicists who study the properties of high energy particles. For example, protons accelerated in the Soviet synchrophasotron accelerator at the Joint Institute for Nuclear Research in Dubna have masses over 100 times larger than their rest mass.

Nevertheless, there are particles which move at the velocity of light, for example, the quanta of electromagnetic radiation or photons. Yet their rest mass is zero. This means that photons cannot be stopped. They are doomed to be permanently in motion. Any attempt to stop a photon results in its destruction. It is of interest that the velocity of a photon in a vacuum is 300,000 km/s in any coordinate system independent of the relative motion of the observer and the photon's source, i.e. the source of light. The uncommon kinematic (and dynamic) properties of photons show that they are not "genuine" particles, they should really be called quasi-particles.

4.2. Heat is mainly transferred from an incandescent metal to a man by radiation. Most of the radiated energy from a heated metal's surface is in the infrared rays which, like electromagnetic waves in general are reflected by metals. This is why the clothes of foundry workers is metallised.

4.3. As can be seen from Fig. S.4.1, the man can see the same part of his body, i.e. no lower

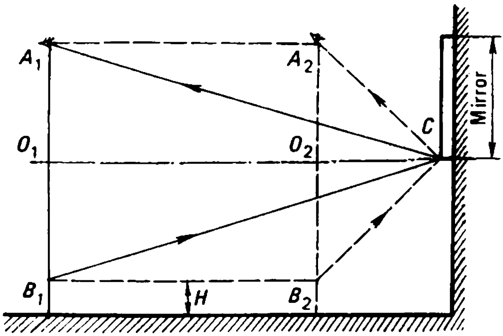

Fig. S.4.1

than a distance H above the floor, irrespective of the distance between the man and the wall on which the mirror is hanging.

4.4. This property will, for example, be possessed by a system of three flat mirrors at right angles to each other (like three faces of a cube with a common vertex). First, let us consider the two-dimensional case in Fig. S.4.2.

The vector of the velocity of light $\mathbf{c}_1$ can be decomposed into two components $\mathbf{v}_1$ and $\mathbf{v}_2$ normal and tangential, respectively to the first mirror's surface. When reflected by the first mirror the $\mathbf{v}_2$ component does not change, while the normal component changes in sign to $\mathbf{v}'_1$.

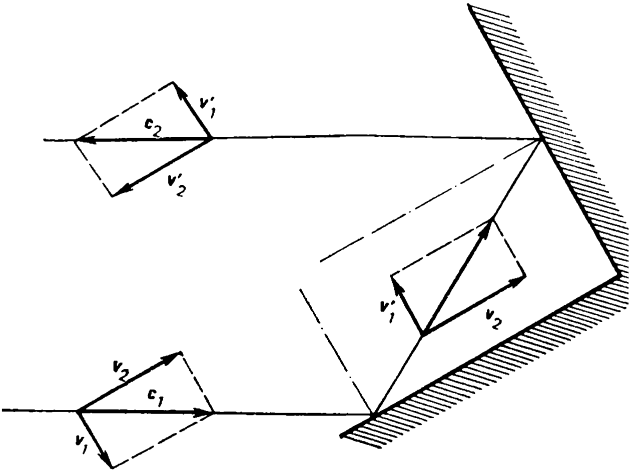

Fig. S.4.2

Since the mirrors are mutually at right angles, the $\mathbf{v}_2$ component which is tangential to the first mirror, is normal to the second one and therefore when reflected by the second mirror it too changes in sign. By contrast, $\mathbf{v}'_1$ retains its direction. Thus, after two reflections both components of the vector $\mathbf{c}_1$ had changed in sign, therefore the vector $\mathbf{c}_1$ is rotated by 180° while being shifted somewhat in the direction perpendicular to itself.

In three dimensions the vector c is decomposed into three components $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$, which are normal to the first, second, and third mirrors, respectively. Then when reflected from each mirror only one component reverses its direction at a time, so after the third reflection all the three components change their sign and the velocity vector is rotated to face the opposite direction.

Triangular prisms made by cutting a glass cube in half with a plane passing through the ends of three edges originating from a single vertex have similar properties. Here the role of mirrors is played by the side faces.

Both devices are called corner reflectors. French corner reflectors were installed on Soviet moon rovers. The time it took light from an Earth-based laser to travel to the reflector and back was used to measure the distance between the Earth and the Moon with very great accuracy.

Corner reflectors find some more mundane applications. For example, the triangular road signs with red glass placed near corners are made by die-stamping triangular pyramidal projections on the backs of the signs to form arrays of corner reflectors. When a passing car's headlights illuminate the sign in the dark, the reflectors flash attracting the driver's attention and warning him. Cyclists must be interested to know that the rear reflector in the metal casing on rear wheel splash guard is a corner reflector system.

To sum up, note that a corner reflector can be made of three flat mirrors. Experimenting with them affords much pleasure, it is of interest to observe that, independent of our position, we always see our reflected image (but only "upside down").

4.5. We assumed that rain drops are spherical in shape. In fact, if only molecular forces were involved droplet might be spherical. In this case the energy of surface tension is at a minimum (see the solution of Problem 2.15): among all the bodies of the same volume a ball possesses the least possible volume. Therefore, a droplet in a vacuum would be spherical. However, air drag distorts the sphericity and the droplet becomes stream-lined like a pear. The reflection conditions in the different points of the droplets cease to be identical. There may occur total reflection at one point and emission of rays at another.

It must be noted that total reflection never occurs. In any case, a fraction of radiation energy is "leaked" from the droplet into air.

4.6. Let us remove the collecting lens and place instead a diverging one. The ray's trajectory can then be displayed as shown in Fig. S.4.3. This figure demonstrates that within a zone limited by the circles $A'B'$ and CM the screen is illumined both by the rays scattered by the lens and those passing by the lens and therefore the illumination in the zone is higher than it would be without the lens.

This phenomenon can be observed in practice by placing glasses for the short-sighted on a sheet of paper in the sun light. A bright circle surrounded by a brighter fringe can be clearly seen and then the paper illumined by the sun. The illumination within the fringes will be at a maximum.

4.7. If a lens is placed in a medium whose refraction index exceeds that of the material making up the lens, then roles of the lens and

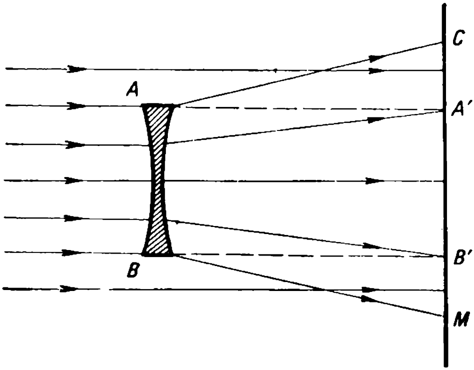

Fig. S.4.3

the medium will be exchanged. A biconvex lens will scatter rays (as does an air bubble in water), while a biconcave lens will collect them. This can be supported by a simple drawing (see Fig. S.4.4).

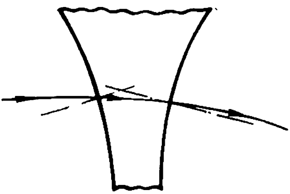

Fig. S.4.4

It can be seen that when entering and leaving a lens (only a part of the lens is shown) a ray of light is refracted towards the principal optical axis, parallel to which it was propagating.

4.8. The negatives of both photographs will be equal in density if the exposure is left unchanged. The exposure is the quantity characterizing the amount of light energy a photosensitive material receives during photography. The exposure is denoted by H and expressed via the product

$$H = Et, \tag{1}$$

where E is the illumination intensity on the film, and t is the exposure time.

The illumination intensity of an image is directly proportional to the light flux Φ passing through the objective lens from the object and inversely proportional to the image's area S_1:

$$E \sim \frac{\Phi}{S_1} \tag{2}$$

If the object (for example, a button on someone's jacket) reflects light uniformly in all directions, then the light flux passing through the objective lens is directly proportional to the solid angle subtended by the object at which the objective lens is seen:

$$\Phi \sim \Theta. \tag{3}$$

By assuming the area of objective lens is S_0, and the object to be a distance a_2 from the camera we get (Fig. S.4.5a) for the solid angle

$$\Theta = \frac{S_0}{a_2^2}. \tag{4}$$

By combining the three latter relations we have

$$E \sim \frac{S_0}{S_1 a_2^2}. \tag{5}$$

However, the areas of the object, S_2, and of its image, S_1, are related thus

$$\frac{S_1}{S_2} = \frac{a_1^2}{a_2^2} \tag{6}$$

where a_1 is the distance from the objective lens to the photographic film (see Fig. S.4.5b).

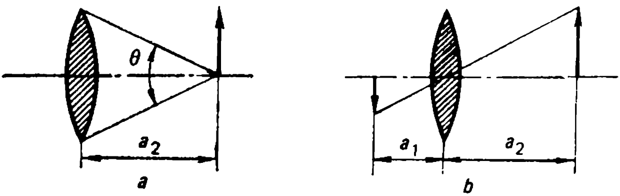

Fig. S.4.5

By substituting the value of a_2^2 from (6) into (5) we arrive at

$$E \sim \frac{S_0 S_1}{S_1 S_2 a_1^2} = \frac{S_0}{S_2 a_1^2}. \tag{7}$$

Since S_0 and S_2 are both constant,

$$E \sim \frac{1}{a_1^2} \tag{8}$$

Both the first and the last equations can be combined to yield

$$t \sim a_1^2. \tag{9}$$

However it follows from the lens formula, that the farther the object (i.e. the greater a_2), the shorter the distance a_1 must be from the camera's objective lens to the film.

Hence, the farther an object is the smaller the exposure must be and vice versa.

This is only true of a studio camera which has a large focal distance. For small cameras the situation is different. As a rule, the distance from the camera to the objects is many times longer, than its focal distances. Therefore in all cases a_1 practically coincides with the focal length (generally, 50 mm) and, hence, by relation (9), t turns out to be constant.

4.9. If the front (facing the object) surface of the cornea is flat, the focal length of the eye is the same in both air and water. As a result rays coming from distant objects (i.e. parallel rays) are not refracted by the cornea. Thus, eye can see distant objects equally well in air and water.

4.10. In each period of time each frame has the image of separate phases in the motion of the body. The human eye is such that if one frame is quickly changed to another the separate motion phases merge to produce an impression of a continuous motion.

The frames in a motion picture change 24 times a second. Suppose in a 1/24th of a second a carriage wheel, which is shown for convenience in Fig. S.4.6 as having only two spokes, OA_1 and OB_1, rotates so that the spokes move from A_1OB_1 to A_2OB_2. The eye will perceive the wheel to have rotated clockwise through an angle α.

If the vehicle moves faster between frame changes in the movie camera the wheel will rotate through a substantially larger angle. The spokes will move to $A'_2OB'_2$ as shown in the figure (bottom, right). Since the ends of both

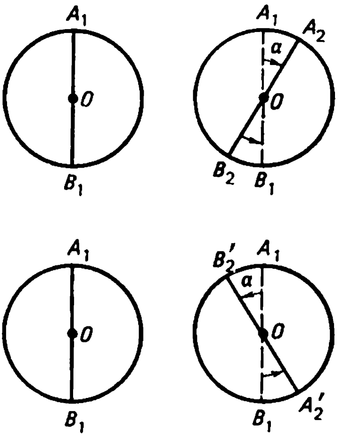

Fig. S.4.6

spokes are identical, the eye mistakenly perceives the rotation through angle $A_1OA'_2$ to have been through angle $A_1OB'_2$. As a result, the wheel rotation seems to be moving anticlockwise.

It can be seen that the paradox arises at frequencies such that during a frame change the spokes rotate through angles larger than half the angle between adjacent spokes. Therefore for a constant shooting frequency and constant carriage velocity wheels with different numbers of spokes will seem to be rotating in opposite directions, as can sometimes be seen in the cinema. Occasionally the rear wheels of a carriage have more spokes than the front wheels and appear to rotate in the opposite direction.

4.11. After passing through the eye's crystalline lens a parallel beam of rays leaving a telescope eye-piece converges to form the object's image on the eye's retina.

4.12. The argument was correct at all points except for the final conclusion. Telescopes, in fact, do not enlarge the image of stars, nevertheless they do assist astronomical observations. Progress in astronomy accelerated after 1609, when Galileo pointed a telescope at the sky.

The reason for using a telescope to observe stars is not to get an enlarged image, but to increase the light flux incident on the observer's eye from the object under observation. A telescope increases the light flux retina by as much as the area of the telescope's collecting lens is larger than the pupil.

A giant telescope was recently commissioned in the Soviet Union with a diameter of 6 m. The ratio of the areas of its mirror to an open pupil is about

$$\frac{S_t}{S_p} = \left(\frac{d_t}{d_p}\right)^2 = \left(\frac{600\ \text{cm}}{1\ \text{cm}}\right)^2 = 360{,}000.$$

Thus, the light flux on an observer's eye is 360,000 times as much collected by the naked eye. Therefore a telescope makes objects which are not observable with a naked eye available to observation.

4.13. At very small diameters of the objective the light is diffracted due to its wave nature. This phenomenon results in diffused image boundaries, thus deteriorating its quality.

4.14. Ways of constructing devices that generate energy beams have been thought up by many inventors, but none succeeded. This is not simply bad luck. The construction of the hyperboloid (A.N. Tolstoy should have called Garin's device a paraboloid since the description in the novel involved parabolic not hyperbolic mirrors) is blocked by a number of serious obstacles.

First, it is difficult to find an adequate refractory material to manufacture the mirrors because a mirror reflecting a powerful beam must itself be heated up to a high temperature.

Second, it is not clear what fuel is used to feed the hyperboloid, it must hold a colossal amount of energy in a small volume, otherwise the hyperboloid would be a harmless toy. Yet the gravest disadvantage is the difficulty of generating narrow beams of light that will travel long distances. This difficulty is due to the wave nature of light: the narrower the light beam, the more pronounced the diffraction which smears it. The phenomenon of diffraction prevents the generation of narrow beams of light using any technique dreamt of by Garin, i.e. mirrors, diaphragms, lenses or any other geometrical optical device.

Whilst this argument is valid for the device based on the principles of geometrical optics. However, two Soviet scientists Nikolai Basov (b. 1922) and Aleksandr Prokhorov (b. 1916) and the American physicist Charles Townes (b. 1915) independently invented devices in which pre-excited atoms radiate stored energy practically at once (this is the so-called coherent radiation). As a result, a very narrow, almost nondiverging beam of light is produced. The radiation energy density within the beam is so high that the beam can easily punch through wood a few centimeters thick several meters away from the generator. Now these lasers or quantum generators have been used to determine geographic and astronomic distances with breathtaking precision (cf. Problem 4.4), drill holes

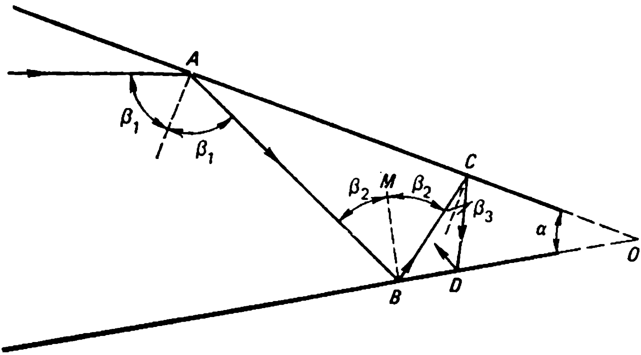

Fig. S.4.7

in hard jewel stones and metals, and in surgical operations, particularly, on human eyes. The applications of lasers are expanding incessantly.

4.15. Let us trace one of the rays leaving the source and incident at an angle β_1 on the inside surface of the light concentrator. To simplify, let us consider the two-dimensional (flat) case (see Fig. S.4.7).

From the triangle *AOB* we find

$$\beta_2 = 180° - (\angle BAO + \angle AOB + \angle OBM).$$

Since AN and BM are normal to the sides AO and BO, respectively, then

$$\angle BAO = 90° - \beta_1 \quad \text{and} \quad \angle MBO = 90°$$

The angle at the vertex of the cone $AOB = \alpha$. Therefore we have

$$\beta_2 = \beta_1 - \alpha.$$

Analogously, we can show that after the third reflection the angle β_3 will be

$$\beta_3 = \beta_2 - \alpha = \beta_1 - 2\alpha.$$

and after the nth reflection

$$\beta_n = \beta_1 - (n - 1)\,\alpha.$$

Thus, after each reflection the reflection angle is decreased by the same value; therefore, irrespective of the value of α at some stage the reflection angle will be zero and then become negative, i.e. reflected rays will turn back to the base of the cone.

By making $\beta_n = 0$, we can calculate how many reflections are needed before rays incident at an angle β_1 turn back:

$$n = \frac{\beta_1}{\alpha} + 1.$$

The smaller opening of the cone will be only reached by the negligible fraction of rays that is emitted by the source along the axis of the cone.

The argument can be supported by an accurate drawing. Our figure demonstrates that after the fourth reflection at D the incident ray turns to the broader opening of the cone.

Note that sometimes charged particles behave in magnetic fields like the rays of light trapped in a mirror cone.

Figure S.4.8 illustrates a simplified operation of the so-called "magnetic bottle". The magnetic field lines approach each other in a cone-shaped

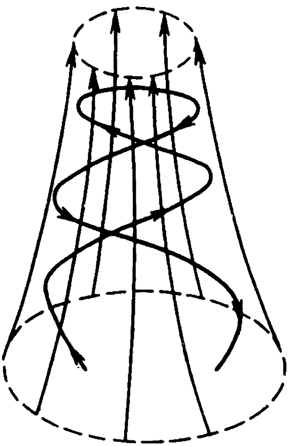

Fig. S.4.8

form and force the trapped particles to move along involved trajectories and to turn backwards. Regions limited by the lines of force of the magnetic field of the Earth are shaped like bent tubes narrower at the poles (Fig. S.4.9). The charged particles trapped by these tubes move from one pole to another to produce radiation belts around the Earth.

To summarize, we want to say that various schemes of controlled thermonuclear reaction involve plasma heated to millions of degrees Centigrade and isolated from the walls of the containing vessel by magnetic bottles, magnetic

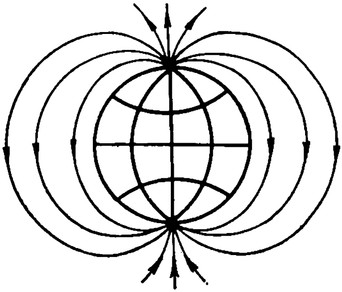

Fig. S.4.9

plugs or other setups based on the principles illustrated in Fig. S.4.8.

4.16. The perception of a colour by an observer's eye is not governed by the wavelength depending on the refraction index of medium, but by the frequency of the electromagnetic oscillations which affect the optic nerve endings. The frequency does not change on transition from one medium to another since it is determined by the source of light and not the medium.

Therefore a colour sensed in air as red is perceived as red in water, too.

4.17. The particles of tobacco smoke scatter the light incident on them depending on wavelength. Rays with short wavelength, i.e. violet, blue, and dark-blue are scattered most. Long wavelength rays from the opposite end of spectrum are scattered less, since the diffraction, viz. light bending round obstacles, is more characteristic of them. Therefore a beam of light passing through a smoke cloud is dominated by red hues. By contrast, when observing a smoke cloud from the side of the source or on the side we mainly see the short-wave rays and the smoke seems bluish.

The dependence of the absorption of light rays on their colour is always taken into account in practice: accident and hazard-warning lights have red filters (red traffic lights) and to ensure black-out (for example, during a war) room illumination is done by blue lamps.

As to the true colour of smoke, this must be the colour of the microscopic unburnt particles of carbon which make up the smoke, i.e. black.

4.18. As was mentioned in Problem 4.16, the wavelength corresponding to red is about 0.65 μm. Green has a wavelength of 0.55 μm. Thus, the wavelength shift due to the Doppler effect would be

$$\frac{0.55\ \mu\mathrm{m}}{0.65\ \mu\mathrm{m}} = 0.85.$$

This means that the frequency of electromagnetic oscillations falling on the driver's eye must have increased $1/0.85 = 1.18$ times, i.e. by about 20%. Such an increase in frequency is only possible at the velocities of about 20% that of the velocity of light, which of course is beyond the reach of any driver. The minimum velocity at which the Doppler effect starts to be detectable using very sensitive optical devices is 500 m/s, this too being far beyond the range of a normal car.

4.19. Consider a collecting lens with a source of light positioned at point A on its principal optical axis (Fig. S.4.10a). On passing through the lens, the velocity of the light quanta and,

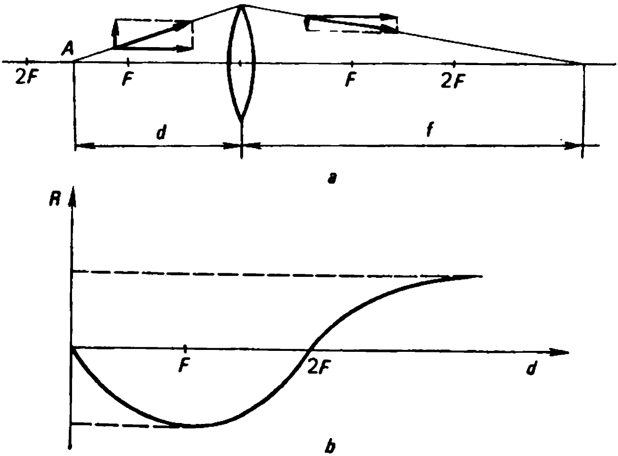

Fig. S.4.10

consequently, the magnitude of the momentum vector of the quanta remain the same as before; only the direction of the momentum vector changes. If the source of light is at a distance $0 < d < 2F$ from the lens then the projection of the photon momentum vector onto the optical axis before the light passes through the lens is smaller than that after (this can be easily proved using triangles and taking into account that $d < f$). Since the momentum component along the optical axis has been increased, the lens itself must have received a momentum in the opposite direction, i.e. towards the source of light!

For $d > 2F$ the lens will be exposed to the force R directed away from the source (Fig. S.4.10b). Of course, the lens must be perfectly transparent and must not reflect light, then this relation will be true.[15]

We can see that the source of light will attract any optical system which decreases the divergence of rays in a beam of light which passed through this system. A lens is not the only device to possess this property (can you suggest others?). Unfortunately, their efficiency falls the farther they are from the source of light. We don't mention here the difficulties facing the manufacturer of the very long-focus lenses needed to pull a vehicle towards the Sun over distances of the order of millions of kilometres. We have been interested in the principles.

4.20. Electrons are, we all know, found in atoms in different states, each corresponding to specific energy. When passing from a higher energy state to a lower energy one the electron releases the excess energy as electromagnetic radiation. Depending on the frequency, an observer perceives a colour.

In metals, due to thermal excitation energy the electrons farthest from nucleus (in chemistry these are called valence electrons) can easily pass

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[15] We can easily show that if the source of light is on the principal optical axis, the total change in the momentum of light incident on the lens in the plane perpendicular to an optical axis vanishes. Therefore the lens is exposed to a force directed parallel to the optical axis.

to an excited state and back to the normal or ground state, thus liberating the energy in the form of light.

This is not the case with quartz and glass. Here all the electrons are tightly bound with atomic nuclei and only change their energy state with difficulty. To produce noticeable luminescence needs a much higher temperature.

4.21. The theory of relativity forbids the relative displacement of two material objects at a velocity exceeding that of light. However, the intersection point of two rulers is only a geometrical image whose velocity (relative to the Earth or any other object) is not limited by the theory of relativity.

Nevertheless it first seems that placing a metal ring at the intersection point (Fig. S.4.11), we

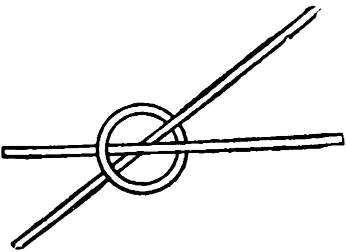

Fig. S.4.11

would find a contradiction with the theory of relativity since a material object associated with a geometrical point of intersection must move with it and at the same velocity at that!

However, don't forget that, according to relativity, an increase in the velocity of a body brings about change in its mass, i.e.

$$m=\frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\ \text{(see Problem 4.1).}$$

Thus, as the moving ruler gradually accelerates the mass of the ring builds up, so further increase in its velocity (and hence that of the ruler) becomes more difficult, while the attainment of the velocity of light is impossible.

Here is what M. Gardner, a famous American popularizer of science wrote in *Relativity for the million*, which we suggest everyone interested in physics should read.

"Although signals cannot be sent faster than the speed of light, it is possible to observe certain types of motion that, relative to the observer, will have a speed faster than light. Imagine a gigantic pair of scissors, the blades as long as from here to the planet Neptune. The scissors begin to close with uniform speed. As this happens, the point where the cutting edges intersect will move toward the points of the scissors with greater and greater velocity. Imagine yourself sitting on the motionless pin that joins the blades. Relative to your inertial frame, the point of intersection of the blades will soon be moving away from you with a speed greater than that of light. Of course, it is not a material object that is moving, but a geometrical point.

Perhaps this thought occurs to you: Suppose that the handles of the scissors are on the earth and the point of intersection of the blades is at Neptune. As you wiggle the handles slightly, the intersection point jiggles back and forth.

Could you not, then, transmit signals almost instantaneously to Neptune? No, because the impulse that moves the blades has to pass from molecule to molecule, and this transmission must be slower than light. There are no absolutely rigid bodies in general relativity. Otherwise you could simply extend a rigid rod from the earth to Neptune and send messages instantaneously by wiggling one end. There is no way that the giant pair of scissors, or any other type of so-called rigid object, could be used for transmitting a signal faster than the speed of light.

4.22. As the velocity of the right-hand lever approaches the velocity of light, the mass of the lever increases infinitely (cf. the solutions to Problems 4.1 and 4.21), and this prevents it from reaching the velocity of light, to say nothing of exceeding it.

4.23. The calculation convincingly demonstrates the absurd conclusions that can be drawn by mechanically applying mathematical formulas without seeing into the underlying physics.

Radium is a member of a radioactive family. In the chain of transformations it comes between thorium which decays to radium, and radon which is the decay product of radium. The radium now on Earth is not the remainder of the colossal initial quantity calculated in the problem.

At present we know of three natural radioactive families. These are the uranium, thorium and actinium series which are named after the first member in the chain of transformations. A fourth, the neptunium, family is composed of artificially produced isotopes not found on the Earth. Due to their fairly long half-lives, the progenitors of the first three families can be found in nature. Their half-lives are

uranium	4.5×10^{9} years,
thorium	1.4×10^{10} years,
actinium	7.1×10^{8} years.

However the members of their families can only be detected in nature due to the continuous production in the process of radioactive decay of other elements.

At the moment of the Earth's birth only a minor amount of neptunium would have been present (though even this is very doubtful from the viewpoint of nuclear physics), but its half-life is "only" 2.2×10^6 years and this is too short for neptunium to have lasted until now.

4.24. Let us consider a gas contained in a piston-plugged cylinder. As long as the piston is at rest, the average velocity of the gas molecules v remains constant (if, of course, there is no heat supplied to the gas), since the collisions between molecules and cylinder walls as well as the piston are elastic and the velocity after a collision remains unchanged.

However, if the piston is pushed into the cylinder at a certain constant velocity u, then the molecules striking the piston get a velocity $v + u$ relative to it. These molecules are reflected with the same velocity relative to the piston. However, since the piston moves with a velocity u relative to the cylinder, then after the reflection the velocity of the molecules relative to the cylinder is $u + (u + v)$ $v + 2u$, i.e. it is increased by $2u$.

Charged particles are accelerated in space in the same manner. If a proton flying away from the Earth at a velocity v arrives at an agglomerate of interstellar gas carrying a magnetic field

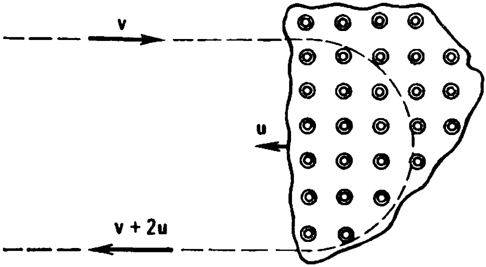

Fig. S.4.12

and moving with a velocity u towards the Earth, then after being "reflected" by the magnetic field (Fig. S.4.12) the proton will return to the Earth with a velocity $v + 2u$. True, the proton can be trapped by a magnetic field whose velocity vector is oriented outward of the Earth and can be slowed down, but an exact calculation demonstrates that the moving particles meet, per unit time, more accelerating than decelerating fields, so the net effect is acceleration.

4.25. Nuclear reactions, like chemical ones, are classed into those that release energy, exothermic reactions, and those that absorb energy, endothermic reactions. The first reaction in the problem is exothermic and proceeds spontaneously. The second reaction is endothermic and for a proton to transform to a neutron, positron, and neutrino requires a huge, on a microworld scale, amount of energy.

According to the modern concepts resulting from the theory of relativity, an increase in the energy of a body is accompanied by an increase in its mass. A proton capable of generating the neutron-positron-neutrino triad must have as large mass as the mass of a proton in the ground state plus twice the mass of an electron (or a positron since the masses of both particles are identical). Therefore the law of mass conservation remains valid. The same is true of the law of charge conservation, as can be seen in these reactions.

4.26. During conventional (electron) β-decay one of the neutrons composing the parent nucleus is transformed into three particles (proton, electron, and antineutrino):

$$^1_0n \rightarrow {}^1_1p + {}^0_{-1}e + {}^0_0\tilde{\nu}.$$

The proton remains in the daughter nucleus, while the electron and the antineutrino are ejected. Thus the nuclear charge is a unity larger than the parent nucleus, and the daughter nucleus is one cell to the right of the parent nucleus in the periodic table.

During a positron β-decay the following reaction takes place in a parent nucleus

$$^1_1p \rightarrow {}^1_0n + {}^0_{+1}e + {}^0_0\nu,$$

after which the neutron is left in the daughter nucleus, while both the positron and neutrino are ejected. Thus, a positron β-decay decreases the nuclear charge by unity and the daughter nucleus lies a cell to the left of the parent nucleus. Electron decay occurs in neutron-rich (proton-deficient) nuclei, while the positron reaction occurs in proton-rich (neutron-deficient) nuclei. The reactions were treated in Problem 4.25.